PHYSICS 4E QUIZ 5 (take-home) SPRING QUARTER 2016
PROF. HIRSCH MAY 28-31

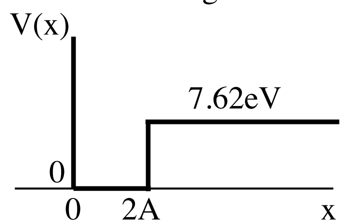
Show all steps in your calculations. Justify all answers. Write clearly.

$hc = 12{,}400 eVA,\ k_B = 1/11{,}600 eV/K,\ m_e c^2 = 511{,}000 eV,\ \mu_B = 5.79\times10^{-5} eV/T$

$ke^2 = 14.4 eVA,\ \hbar c = 1973\ eVA,\ m_p c^2 = 938.28 MeV,\ \hbar^2/(2m_e) = 3.81 eVA^2$

Problem 1 (15 pts)
An electron is in the ground state of the 1-dimensional potential shown in the figure

$V(x)=\infty$ for $x<0$
$V(x)=0$ for $0<x<2A$
$V(x)=7.62eV$ for $x>2A$.



(A means Angstrom = 10^{-10}m)
(a) Find its energy in eV, accurate to 2 decimal places. Use continuity of wavefunction and derivative.
(b) At which position x is this electron most likely to be found?
(c) Find its wavefunction for all x, including normalization.
(d) If you check whether the electron is or is not in this 'box' (i.e. in the region $0<x<2A$) 1000 times, how many times are you going to find that the box is empty?
(e) How much more likely is it to find this electron in the region $1A<x<2A$ than in the region $0<x<1A$?

Problem 2 (15 pts+3 extra credit)
An electron moves in a three-dimensional potential V(x,y,z) given by
$V(x,y,z)=\alpha(x^2+y^2)$ for $0<z<a$, any x,y
$V(x,y,z)=\infty$ for $z<0$ or $z>a$, any x, y
With $\alpha=2eVA^{-2}$, a=5A.
(a) Show that the wavefunction is separable.
(b) Find the ground state energy in eV.
(c) Find the ground state wavefunction $\Psi(x,y,z)$ expressed in terms of α, a and m_e (electron mass). You don't have to give normalization, leave it as a constant.
(d) For the electron in the ground state, how much more likely is it to find it at (x,y,z)=(0,0,2.5) than at (x,y,z)=(1,1,1.25)?
(e) Suppose there are 10 electrons in this potential, assume they don't interact with each other, they have spin but ignore spin-orbit coupling. Find the total energy of the system in eV.

(f) Same as (e) for 10 α-particles, The mass of an α-particle is m_α=3,727 MeV, the spin is 0.
(g) For extra credit: find the energy (in eV) and the wavefunction of the lowest energy state for one electron in this potential that has orbital angular momentum in the z direction $L_z = \hbar$. Justify your answer.