Physics 4E **Quiz 1 Solutions** Spring 2016

Problem 1 $\langle |v_x| \rangle = 200\, m/s$

(a) $f(v_x) = \left(\frac{m}{2\pi kT}\right)^{1/2} e^{-\frac{m v_x^2}{2kT}}$

$$\langle |v_x| \rangle = 2\int_0^\infty dv_x\, f(v_x) = 2\left(\frac{m}{2\pi kT}\right)^{1/2} \int_0^\infty dv_x\, v_x\, e^{-\frac{m v_x^2}{2kT}}$$

Use that $\int_0^\infty dx\, x\, e^{-\lambda x^2} = \frac{1}{2\lambda}$, $\lambda = \frac{m}{2kT}$ $\Rightarrow$

$$\langle |v_x| \rangle = 2\left(\frac{m}{2\pi kT}\right)^{1/2} \frac{1}{\cancel{2}} \cdot \frac{\cancel{2} kT}{m} = \left(\frac{\cancel{4}^{\,2} \cancel{m}}{\cancel{2\pi kT}} \frac{(kT)^{\cancel{2}}}{m^{\cancel{2}}}\right)^{1/2} = \left(\frac{2}{\pi}\frac{kT}{m}\right)^{1/2}$$

$$\boxed{\langle |v_x| \rangle = \left(\frac{2}{\pi}\frac{kT}{m}\right)^{1/2}}$$ we know $\frac{1}{2} m \langle v_x^2 \rangle = \frac{1}{2} kT \Rightarrow \sqrt{\langle v_x^2 \rangle} = \left(\frac{kT}{m}\right)^{1/2}$

$$\Rightarrow \sqrt{\langle v_x^2 \rangle} = \left(\frac{\pi}{2}\right)^{1/2} \langle |v_x| \rangle = 1.253 \times 200\, \frac{m}{s} = \boxed{250.7\, m/s}\ (a)$$

(b) $$\frac{f(400 m/s)}{f(200 m/s)} = \frac{e^{-\frac{m}{2kT} \cdot 4v^2}}{e^{-\frac{m}{2kT} \cdot v^2}} = e^{-\frac{m}{2kT} \cdot 3v^2}$$ with $v = 200\,\frac{m}{s}$

$$\frac{3m}{2kT} \cdot v^2 = \frac{3\cancel{m}}{\cancel{2kT}} \cdot \frac{\cancel{2}}{\pi}\frac{\cancel{kT}}{\cancel{m}} = \frac{3}{\pi}, \quad e^{-\frac{3}{\pi}} = 0.385$$

$$\Rightarrow f(400 m/s) = 0.385\, f(200 m/s) = \boxed{385 \text{ molecules}}$$

(c) $$\frac{300}{200} = \left(\frac{T+100}{T}\right)^{1/2} \Rightarrow T + 100 = 2.25\, T \Rightarrow \boxed{T = 80 K}$$

Problem 2

$\lambda_m = 6000\,\text{Å}$.

$$\lambda_m T = \frac{hc}{4.965\,k_B} \Rightarrow T = \frac{12{,}400 \times 11{,}600}{4.965\,k_B \times 6000}\,K = 4828.5K$$

$$\boxed{T = 4828.5\,K}$$

Power emitted $P = \sigma T^4 A$, A = area. $\sigma = 5.6703 \times 10^{-8} \frac{W}{m^2 K^4}$

$$A = \frac{40\,W \quad m^2 K^4}{5.6703 \times 10^{-8}\,W \cdot 4828.5^4 K^4} = 1.30 \times 10^{-6}\,m^2$$

$$\Rightarrow \boxed{A = 1.30\,mm^2} \text{ (a)}$$

b) $P \propto \frac{1}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$

$\lambda_1 = 6000\,\text{Å}$, $\lambda_2 = 3000\,\text{Å}$

$\frac{hc}{\lambda_1 kT} = 4.965$, $\frac{hc}{\lambda_2 kT} = 9.93$

$$\frac{P(3000\,A)}{P(6000\,A)} = \left(\frac{6000}{3000}\right)^5 \frac{e^{4.965} - 1}{e^{9.93} - 1}$$

we can neglect the (-1)

$$\Rightarrow \frac{P(3000\,A)}{P(6000\,A)} \cong 2^5 \times e^{-4.965} = 32 \times 0.0698 = \boxed{0.22 = 22\%}$$

(c) The photons with $\lambda = 3000\,\text{Å}$ have twice the energy of photons with $6000\,\text{Å}$

$$\Rightarrow \frac{\#\text{ of photons at } 3000\,\text{Å}}{\#\text{ of photons at } 6000\,\text{Å}} = \frac{1}{2} \frac{P(3000\,A)}{P(6000\,A)} = 0.11 = 11\%$$

$\Rightarrow$ for every 10,000 photons at $6000\,\text{Å}$, $\boxed{1{,}110 \text{ photons at } 3000\,\text{Å}}$

Problem 3

$$\lambda' - \lambda = \lambda_c (1 - \cos\theta) \qquad \lambda_c = \frac{h}{m_e c} = 0.0243\,\mathring{A}$$

range of λ' is between λ (for $\theta = 0$) and $\lambda + 2\lambda_c$ (for $\theta = 180°$)

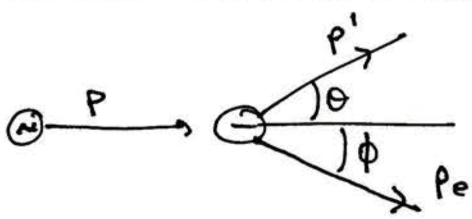
if range of λ' is between λ and $3\lambda \Rightarrow$ $\boxed{\lambda = \lambda_c = 0.0243\,\mathring{A}}$

(b) For $\lambda' = 2\lambda = 2\lambda_c \Rightarrow$ $\boxed{\theta = \pi/2 = 90°}$

(b) $$\frac{hc}{\lambda} - \frac{hc}{2\lambda} = \frac{hc}{2\lambda} = K_e = \frac{hc \cdot m_e c}{2h} = \frac{m_e c^2}{2}$$

$$\Rightarrow \boxed{K_e = \frac{m_e c^2}{2} = 255,500\,eV}$$

(d)



$$p_e \cos\phi = p - p'\cos\theta$$
$$p_e \sin\phi = p'\sin\theta$$
$$\Rightarrow \tan\phi = \frac{p'\sin\theta}{p - p'\cos\theta} = \frac{\frac{1}{\lambda'}\sin\theta}{\frac{1}{\lambda} - \frac{1}{\lambda'}\cos\theta} = \frac{\frac{1}{2}\sin\theta}{1 - \frac{1}{2}\cos\theta}$$

$$\Rightarrow \tan\phi = \frac{\sin\theta}{2 - \cos\theta} = \frac{1}{2} \Rightarrow \boxed{\phi = 0.464\,rad = 26.6°}$$

(e) extra credit:

For $\lambda' = 3\lambda_c \Rightarrow K_e = \frac{hc}{\lambda} - \frac{hc}{3\lambda} = \frac{2}{3}\frac{hc}{\lambda_c} = \frac{2}{3} m_e c^2$

$$\Rightarrow E_e = K_e + m_e c^2 = \frac{5}{3} m_e c^2 = \gamma m_e c^2$$

$$\Rightarrow \gamma = \frac{5}{3} = \frac{1}{\sqrt{1 - v^2/c^2}} \Rightarrow 1 - \frac{v^2}{c^2} = \frac{1}{\gamma^2} \Rightarrow \frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}}$$

$$\Rightarrow \frac{v}{c} = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5} = \boxed{0.8 = \frac{v}{c}}$$