8-11. $$\frac{n_2}{n_1} = \frac{g_2 e^{-E_2/kT}}{g_1 e^{-E_1/kT}} = \frac{g_2}{g_1} e^{-(E_2-E_1)/kT}$$

$$e^{(E_2-E_1)/kT} = \frac{g_2}{g_1} \times \frac{n_1}{n_2} = (E_2 - E_1)/kT = \ln\left(\frac{g_2}{g_1} \times \frac{n_1}{n_2}\right)$$

$$T = \frac{E_2 - E_1}{k \ln\left[(g_2/g_1)(n_1/n_2)\right]} = \frac{10.2 eV}{(8.617 \times 10^{-5} eV/K) \ln(4 \times 10^6)} = 7790 K$$

8-12. $$\frac{n_2}{n_1} = \frac{g_2}{g_1} e^{-(E_2-E_1)/kT} = \frac{3}{1} e^{-\left[\frac{4 \times 10^{-3} eV}{(8.617 \times 10^{-5} eV/K)(300K)}\right]} = 2.57$$

8-17. For hydrogen: $E_n = -\frac{mk^2e^4}{2\hbar^2}\frac{1}{n^2} = -\frac{13.605687}{n^2}eV$ using values of the constants accurate to six decimal places.

$E_1 = -13.605687 eV$

$E_2 = -3.401422 eV \qquad E_2 - E_1 = 10.204265 eV$

$E_3 = -1.511743 eV \qquad E_3 - E_1 = 12.093944 eV$

(a) $\frac{n_2}{n_1} = \frac{g_2}{g_1} e^{-(E_2 - E_1)/kT} = \frac{8}{2} e^{-10.20427/0.02586} = 4e^{-395} = 4 \times 10^{-172} \approx 0$

$\frac{n_3}{n_1} = \frac{g_3}{g_1} e^{-(E_3 - E_1)/kT} = \frac{18}{2} e^{-12.09394/0.02586} = 9e^{-468} = 9 \times 10^{-203} \approx 0$

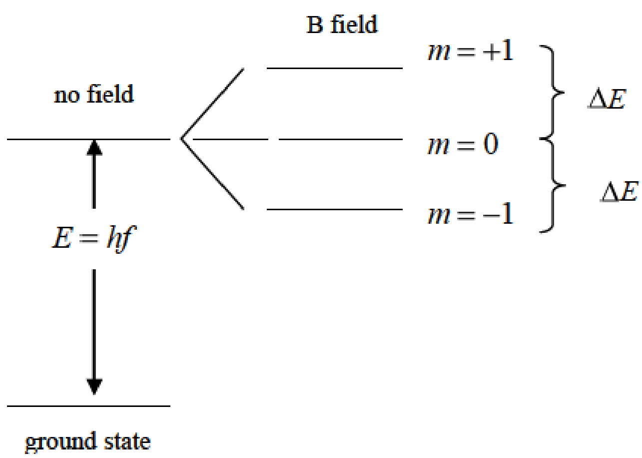
(b) $\frac{n_2}{n_1} = 0.01 = 4e^{-10.20427/kT} \quad \rightarrow \quad e^{-10.20427/kT} = 0.0025$

$$-10.20427/kT = \ln\ 0.0025\ = -5.99146$$

$$T = \frac{10.20427eV}{5.99146\ \ 8.61734\times10^{-5}eV\bullet K} = 19{,}760K$$

(c) $\frac{n_3}{n_1} = 9e^{-12.09394/\ 8.61734\times10^{-5}\ \ 19{,}760} = 0.00742 = 0.7\%$

8-18.



Neglecting the spin, the $3p$ state is doubly degenerate: $\ell = 0{,}1$ hence, there are two m = 0 levels equally populated.

$E = hf\ = hc/\lambda = 1.8509eV \qquad \lambda = 670.79nm$

$$\Delta E = \frac{e\hbar B}{2m_e} = 2.315\times10^{-4}eV$$

(a) The fraction of atoms in each m-state relative to the ground state is: (Example 8-2)

$$\frac{n_{+1}}{n} = e^{-1.8511/0.02586} = e^{-71.58} = 10^{-31.09} = 8.18\times10^{-32}$$

$$\frac{n_0}{n} = 2\times e^{-1.8509/0.02586} = 2e^{-71.57} = 2\times10^{-31.08} = 1.64\times10^{-31}$$

$$\frac{n_0}{n_{-1}} = e^{-1.8507/0.02586} = e^{-71.56} = 10^{-31.08} = 8.30\times10^{-32}$$

(b) The brightest line with the B-field "on" will be the transition from the m = 0 level, the center line of the Zeeman spectrum. With that as the "standard", the relative intensities will be: $8.30/16.4/8.18 \quad \rightarrow \quad 0.51/1.00/0.50$

8-21. Assuming the gasses are ideal gases, the pressure is given by: $P = \frac{2}{3}\frac{N\langle E\rangle}{V}$ for classical, FD, and BE particles. P_{FD} will be highest due to the exclusion principle, which, in effect, limits the volume available to each particle so that each strikes the walls more frequently than the classical particles. On the other hand, P_{BE} will be lowest, because the particles tend to be in the same state, which in effect, is like classical particles with a mutual attraction, so they strike the walls less frequently.

8-23. $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m\langle E\rangle}} = \frac{h}{\sqrt{2m \;\; 3kT/2}} = \frac{h}{3mkT^{\;\;1/2}}$

The distance between molecules in an ideal gas $V/N^{\;\;1/3}$ is found from

$$PV = nRT = nRT \;\; N_A/N_A \;\; = NkT \rightarrow \;\; V/N^{\;\;1/3} = \;\; kT/P^{\;\;1/3}$$

and equating this to λ above, $kT/P^{\;\;1/3} = \frac{h}{3mkT^{\;\;1/2}}$

$\frac{kT}{P} = \frac{h^3}{3mkT^{\;\;3/2}}$ and solving for T, yields: $T^{5/2} = \frac{P}{k}\frac{h^3}{3mk^{\;\;3/2}}$

$$T = \left[\frac{Ph^3}{k \;\; 3mk^{\;\;3/2}}\right]^{2/5} = \left[\frac{101kPa \;\;\; 6.63\times10^{-34}J\bullet s^{\;\;3}}{3 \;\; 2\times1.67\times10^{-27}kg \;\;\; 1.38\times10^{-23}J/K^{\;\;5/2}}\right]^{2/5} = 4.4K$$

8-26. $T_C = \frac{h^2}{2mk}\left[\frac{N}{2\pi \;\; 2.315 \;\; V}\right]^{2/3}$ (Equation 8-48)

The density of liquid *Ne* is 1.207 g/cm^3, so

$$\frac{N}{V} = \frac{1.207g/cm^3 \;\;\; 6.022\times10^{23}molecules/mol \;\;\; 10^6cm^2/m^3}{20.18g/mol} = 3.601\times10^{28}/m^3$$

$$T = \frac{6.626\times10^{-34}J\bullet s^{\;\;2}}{2 \;\; 20u\times1.66\times10^{-27}kg/u \;\;\; .381\times10^{-23}J/K}\left[\frac{3.601\times10^{28}m^3}{2\pi \;\; 2.315}\right]^{2/3} = 0.895K$$

Thus, T_C at which ^{20}Ne would become a superfluid is much lower than its freezing temperature of 24.5*K*.

8-28. $\langle E\rangle = \dfrac{hf}{e^{hf/kT}-1}$ (Equation 8-60)

(a) For $T = 10hf/k$; $hf = kT/10 \rightarrow \langle E\rangle = \dfrac{hf}{e^{1/10}-1} = \dfrac{kT/10}{0.1051} = 0.951kT$

(b) For $T = hf/k$; $hf = kT \rightarrow \langle E\rangle = \dfrac{hf}{e^{1}-1} = \dfrac{kT}{1.718} = 0.582kT$

(c) For $T = 0.1hf/k$; $hf = 10kT \rightarrow \langle E\rangle = \dfrac{hf}{e^{10}-1} = \dfrac{10kT}{2.20\times10^{4}} = 4.54\times10^{-4}kT$

According to equipartition $\langle E\rangle = kT$ in each case.

8-29. $C_V = 3N_A k\left(\dfrac{hf}{kT}\right)^2 \dfrac{e^{hf/kT}}{\left(e^{hf/kT}-1\right)^2}$ As $T\rightarrow\infty$, hf/kT gets small and

$e^{hf/kT} \approx 1 + hf/kT + \cdots$

$C_V = 3N_A k\left(\dfrac{hf}{kT}\right)^2 \dfrac{1+hf/kT+\cdots}{\left(hf/kT\right)^2} \approx 3N_A k = 3N_A\left(R/N_A\right) = 3R$

The rule of Dulong and Petit.

8-31. $C_V = 3R\left(\dfrac{hf}{kT}\right)^2 \dfrac{e^{hf/kT}}{\left(e^{hf/kT}-1\right)^2}$ (Equation 8-62)

At the Einstein temperature $T_E = hf/k$,

$C_V = 3R\left(1\right)^2 \dfrac{e^{1}}{\left(e^{1}-1\right)^2} = 3R\left(0.9207\right) = 3\left(8.31J/K\bullet mol\right)\left(0.9207\right)$

$= 22.95K/K\bullet mol = 5.48cal/K\bullet mol$

8-35. Approximating the nuclear potential with an infinite square well and ignoring the Coulomb repulsion of the protons, the energy levels for both protons and neutrons are given by $E_n = \left(n^2h^2\right)/8mL^2$ and six levels will be occupied in ^{22}Ne, five levels with 10 protons and six levels with 12 neutrons.

$E_F\left(\text{protons}\right) = \dfrac{\left(5\right)^2\left(1240MeV\bullet fm\right)^2}{8\left(1.0078u\times931.5MeV/u\right)\left(3.15fm\right)^2} = 516MeV$

$$E_F\ \text{neutrons} = \frac{6^{2}\ 1240 MeV \bullet fm^{2}}{8\ 1.0087u \times 931.5 MeV/u\ \ 3.15 fm^{2}} = 742 MeV$$

$$\langle E \rangle\ \text{protons} = 3/5\ E_F = 310 MeV$$

$$\langle E \rangle\ \text{neutrons} = 3/5\ E_F = 445 MeV$$

As we will discover in Chapter 11, these estimates are nearly an order of magnitude too large. The number of particles is not a large sample.

8-36. $E_1 = h^2/8mL^2$. All 10 bosons can be in this level, so E_1 total $= 10h^2/8mL^2$.

8-39. For a one-dimensional well approximation, $E_n = n^2h^2 \big/ 8mL^2$. At the Fermi level E_F, $n = N/2$, where N = number of electrons.

$$E_F = \frac{(N/2)^2 h^2}{8mL^2} = \frac{h^2}{32m}\left(\frac{N}{L}\right)^2$$

where N/L = number of electrons/unit length, i.e., the density of electrons. Assuming 1 free electron/*Au* atom,

$$\frac{N}{L} = \left[\frac{N_A \rho}{M}\right]^{1/3} = \left[\frac{(6.02\times10^{23} electrons/mol)(19.32 g/cm^3)(10^2 cm/m)^3}{197 g/mol}\right]^{1/3} = 3.81\times10^9 m^{-1}$$

$$E_F = \frac{(6.63\times10^{-34} J\bullet s)^2 (3.81\times10^9 m^{-1})^2}{32(9.11\times10^{-31} kg)(1.602\times10^{-19} J/eV)} = 1.37 eV$$

This is the energy of an electron in the Fermi level above the bottom of the well. Adding the work function to such an electron just removes it from the metal, so the well is $1.37 eV + 4.8 eV = 6.2 eV$ deep.