

PHYSICS 220 : GROUP THEORY
PROBLEM SET #1

- [1] Show that the following is true for 2-cycles: $(jk)(kl)(jk) = (jl)$.
- [2] Show that S_n is isomorphic to a subgroup of A_{n+2} , and describe how to elicit this isomorphism.
- [3] Show that A_4 is not simple.
- [4] Find $\langle D_n, D_n \rangle$.
- [5] Find the center of the quaternion group.
- [6] Describe the Lie algebras $\mathfrak{e}(n)$ and $\mathfrak{p}(n, 1)$ for the Euclidean and Poincaré groups.
- [7] Show that the set of all $n \times n$ real upper triangular matrices is a matrix Lie group. Describe how to go about constructing the inverse of any element.
- [8] Find a basis for the Lie algebra $\mathfrak{so}(3, \mathbb{R})$. Show that you can choose a normalization $\text{Tr}(X^a X^b) = -2\delta^{ab}$. Find the structure constants.
- [9] Show that for any traceless 2×2 matrix X , its exponential is given by

$$\exp(X) = \cos \sqrt{\Delta} I + \frac{\sin \sqrt{\Delta}}{\sqrt{\Delta}} X \quad , \quad (1)$$

where $\Delta = \det X$.