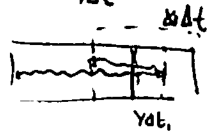
Two events simultaneous in one inertial system are not, in general, simultaneous in another
S

$\Delta t = \sqrt{h^2 + (v\Delta t)^2}/c \quad \rightarrow \quad \Delta t = \frac{h}{c}\underbrace{\left(\frac{1}{\sqrt{1-v^2/c^2}}\right)}_{=\gamma} \qquad \Delta\bar{t} = \sqrt{1-\frac{v^2}{c^2}}\,\Delta t$

Moving clocks run slow (time dilation)



$\Delta t_1 = \frac{\Delta x + v\Delta t_1}{c} \qquad \Delta t_2 = \frac{\Delta t - v\Delta t_2}{c} \qquad \Delta t = \Delta t_1 + \Delta t_2 = \frac{2\Delta x}{c}\frac{1}{1-v^2/c^2}$

$\Delta\bar{t} = \frac{\Delta t}{\gamma}$

$\Rightarrow \quad \Delta\bar{x} = \gamma\Delta x$

Moving objects are shortened

Lorentz transformations

Galil.

$\bar{x} = x - vt \qquad \bar{x} = \gamma(x - vt) \qquad x = \quad +$

$\bar{y} = y \qquad \bar{y} = y \qquad \Delta\bar{t} = \frac{1}{\gamma}\Delta t$

$\bar{z} = z \qquad \bar{z} = z$

$\bar{t} = t \qquad \bar{t} = \gamma\left(t - \frac{v}{c^2}x\right) \qquad t = \quad +$

Einstein's velocity addition rule

$u = \frac{dx}{dt} \qquad d\bar{x} = \gamma(dx - v\,dt) \qquad d\bar{t} = \gamma\left(dt - \frac{v}{c^2}dx\right) \qquad \bar{u} = \frac{d\bar{x}}{d\bar{t}} = \frac{dx/dt - v}{1 - \frac{v}{c^2}\frac{dx}{dt}} = \frac{u - v}{1 - \frac{uv}{c^2}}$

Structure of spacetime

$x^0 \equiv ct \qquad \beta \equiv \frac{v}{c} \qquad x^{1,2,3} = \{x, y, z\}$

$\bar{x}^0 = \gamma(x^0 - \beta x^1)$
$\bar{x}^1 = \gamma(x^1 - \beta x^0)$
$\bar{x}^2 = x^2 \quad \bar{x}^3 = x^3$

$\bar{x} \quad \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = x$

$\bar{x}^\mu = \sum_0^3 \Lambda^\mu{}_\nu x^\nu \qquad \bar{a}^\mu = \sum \Lambda^\mu{}_\nu a^\nu$

$a^\mu b_\mu = -a^0b^0 + a^1b^1 + \dots \qquad a_\mu = (a_0, a_1, a_2, a_3) \equiv (-a^0, a^1, a^2, a^3)$

summation convention

$a_\mu = \sum g_{\mu\nu} a^\nu \qquad g_{\mu\nu} = {}^{-1}\ {}_1\ {}_1\ {}_1$ Minkowski metric.

$a^\mu a_\mu > 0 \quad a_\mu$ is spacelike — inertial system in which 2 events simultaneous

< 0 timelike — same point $\Delta x^\mu \equiv x^\mu_A - x^\mu_B$

$= 0$ lightlike — connected by light signal. $I \equiv (\Delta x)^\mu (\Delta x)_\mu = -(\Delta x^0)^2 + \dots = -c^2t^2 + d^2$

spacetime diagrams

Relativistic Mechanics

$d\tau = \sqrt{1 - u^2/c^2}\,dt$

proper time (own)

ordinary velocity $\vec{u} = \frac{d\vec{l}}{dt}$ proper velocity $\vec{\eta} \equiv \frac{d\vec{l}}{d\tau} \qquad \vec{\eta} = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}\vec{u}$

$\eta^\mu \equiv \frac{dx^\mu}{d\tau} \qquad \eta^0 = \frac{dx^0}{d\tau} = c\frac{dt}{d\tau} = \frac{c}{\sqrt{1-u^2/c^2}}$

$\bar{\eta}^0 = \gamma(\eta^0 - \beta\eta^1)$
$\bar{\eta}^1 = \gamma(\eta^1 - \beta\eta^0)$
$\bar{\eta}^2 = \eta^2 \quad \bar{\eta}^3 = \eta^3$

$\bar{\eta}^\mu = \Lambda^\mu{}_\nu \eta^\nu$ proper velocity 4-vector. 4-velocity.

more convenient $d\tau$ invariant

ordinary veloc $\bar{u}_x = \frac{d\bar{x}}{d\bar{t}} = \frac{u_x - v}{(1 - vu_x/c^2)}$

$\bar{u}_y = \frac{d\bar{y}}{d\bar{t}} \quad \frac{u_y}{\gamma(1 - vu_x/c^2)}$

$\bar{u}_z = \frac{d\bar{z}}{d\bar{t}} \quad u_z/\gamma(1 - v u_x/c^2)$

Relativistic energy & momentum.

$\vec{p} \equiv m\vec{\eta} = \dfrac{m\vec{u}}{\sqrt{1-u^2/c^2}}$ $\qquad p^\mu \equiv m\eta^\mu \qquad p^0 = m\eta^0 = \gamma_u mc \qquad E \equiv \gamma_u mc^2$

$E_{rest} \equiv mc^2 \qquad E_{kin} \equiv E - mc^2 = mc^2(\gamma_u - 1)$

In every closed system, ← no external forces,
the total relativistic energy & momentum are conserved.

$p^\mu p_\mu = -(p^0)^2 + (\vec{p}\cdot\vec{p}) = -m^2c^2 \qquad E^2 - p^2c^2 = m^2c^4$

Relativistic Kinematics.

Relativistic Dynamics

work-energy thm: net work done on a particle = increase in its kin. ene.

$\vec{F} = \dfrac{d\vec{p}}{dt} \qquad W \equiv \int \vec{F}\cdot d\vec{l} = \int \dfrac{d\vec{p}}{dt}\cdot d\vec{l} = \int \dfrac{d\vec{p}}{dt}\cdot\dfrac{d\vec{l}}{dt}\,dt = \int \dfrac{d\vec{p}}{dt}\cdot\vec{u}\,dt.$

$$\frac{d\vec{p}}{dt}\cdot\vec{u} = \frac{d}{dt}\left(\frac{m\vec{u}}{\sqrt{1-u^2/c^2}}\right)\cdot\vec{u} = \frac{m\vec{u}}{(1-u^2/c^2)^{3/2}}\cdot\frac{d\vec{u}}{dt} = \frac{d}{dt}\left(\frac{mc^2}{\sqrt{1-u^2/c^2}}\right) = \frac{dE}{dt}.$$

$$W = \int \frac{dE}{dt}\,dt = E_{final} - E_{initial}$$