PRINT Midterm soln.

(field)
Radiation reaction

Radiation

total power radiated! power, energy carried away

Larmor formula

Energy lost under rad. react. force $\bar{F}_{rad} \cdot \bar{v} = -\frac{\mu_0 q^2 a^2}{6\pi c} = -P$ wrong only considered radiation field.
incorrect instantaneously $\int$ over infinite sphere. no veloc. field

velocity fields do carry energy!!! energy exchange between particle & velocity fields as in accelerates & decel

– on avg correct $\int_{t_1}^{t_2}$ (periodic motion) $\rightarrow \int a^2 dt = \int_{t_1}^{t_2} \frac{d\bar{v}}{dt} \cdot \frac{d\bar{v}}{dt} dt = \bar{v} \cdot \frac{d\bar{v}}{dt}\Big|_{t_1}^{t_2} - \int_{t_1}^{t_2} \frac{d^2\bar{v}}{dt^2} \cdot \bar{v} \, dt$. $\int$

$$\int_{t_1}^{t_2} \underbrace{\left(\bar{F}_{rad} - \frac{\mu_0 q^2}{6\pi c}\dot{\bar{a}}\right)}_{=0} \cdot \bar{v} \, dt = 0$$

only time avg. of parallel comp.

Abraham-Lorentz formula for the radiation reaction force.

how about component $\perp$ to $\bar{v}$

(?) Suppose part. subj to no ext. forces $F_{rad} = ma \Rightarrow a(t) = a_0 e^{t/\tau}$ electron 6×10^{-24} s
spontaneous increase

but $a_0 = 0$ ✓ however excluding runaway solutions $\Rightarrow$ response before force acts.
acausal preacceleration

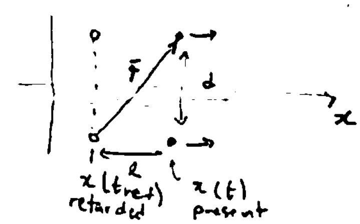
The mechanism responsible for rad. reaction

singularity: fields blow up at the particle. consider extended charge distribution

Lorentz use spherical charge distr.

net-force of the charge on itself.
self-force

"Dumbbell"



$$\bar{E}_1 \overset{\text{due to (2)}}{=} \frac{q/2}{4\pi\epsilon_0} \frac{r}{(\bar{r}\cdot\bar{u})^3}\left((c^2 + \bar{r}\cdot\bar{a})\bar{u} - \bar{r}\cdot\bar{u}\,\bar{a}\right)$$

$\bar{u} = c\hat{r} - \bar{v}$
$(\bar{v}(t_r) = 0)$

$\bar{u} = c\hat{r}$ & $\bar{r} = l\hat{x} + d\hat{y} \Rightarrow \bar{r}\cdot\bar{u} = cr$ $\quad \bar{r}\cdot\bar{a} = la$ $\quad r = \sqrt{l^2+d^2}$ $\quad \bar{a} = a\hat{x}$

y comp. cancel. $u_x = \frac{cl}{r} \Rightarrow E_{1x} = \frac{q}{8\pi\epsilon_0 c^2} \frac{lc^2 - ad^2}{(l^2+d^2)^{3/2}} (= E_{2x})$ $\quad \bar{F}_{self} = \frac{q}{2}(\bar{E}_1 + \bar{E}_2) = q \cdot E_{1x}$

Expand in powers of d, then size $\rightarrow 0$ all positive powers $\rightarrow 0$

$$x(t) = x(t_r) + \dot{x}(t_r)(t-t_r) + \tfrac{1}{2}\ddot{x}(t_r)(t-t_r)^2 + \tfrac{1}{3!}\dddot{x}(t_r)(t-t_r)^3 + \cdots, \qquad T = t - t_r$$

$$l = x(t) - x(t_r) = \tfrac{1}{2}aT^2 + \tfrac{1}{6}\dot{a}T^3 + \cdots \qquad (cT)^2 = l^2 + d^2$$

$$\Rightarrow d = \sqrt{(cT)^2 - l^2} = cT\sqrt{1 - \left(\frac{aT}{2c} + \frac{\dot{a}T^2}{6c} + \cdots\right)^2} = cT - \frac{a^2}{8c}T^3 + (\ )T^4 + \cdots$$

solve for $T(d)$ systematic procedure reversion of series but here first terms

$$d \cong cT \Rightarrow T \cong \frac{d}{c} \Rightarrow d \cong cT - \frac{a^2 d^3}{8c\,c^3} \Rightarrow T \cong \frac{d}{c} + \frac{a^2 d^3}{8c^5} \cdots$$

$$T = \frac{1}{c}d + \frac{a^2}{8c^5}d^3 + (\ )d^4 + \cdots$$

$$l = \frac{a}{2c^2}d^2 + \frac{\dot{a}}{6c^3}d^3 + (\ )d^4 + \cdots \qquad \Rightarrow \bar{F}_{self} = \frac{q^2}{4\pi\epsilon_0}\left[-\frac{a}{4c^2 d} + \frac{\dot{a}}{12c^3} + (\ )d + \cdots\right]\hat{x}$$

at t_r

$$a(t_r) = a(t) + \dot{a}(t)(t_r - t) + \cdots = a(t) - \dot{a}(t)T + \cdots = a(t) - \dot{a}(t)\frac{d}{c} + \cdots, \quad \cdots \tfrac{1}{4}\frac{\dot{a}}{c^3}$$

First term: $m = 2m_0 + \frac{1}{4\pi\epsilon_0}\frac{q^2}{4dc^2}$ as in special rel. repulsion enhances

$$\bar{F}_{self} = \frac{q^2}{4\pi\epsilon_0}\left[-\frac{a(t)}{4c^2 d} + \frac{\dot{a}(t)}{3c^3} + (\ )d + \cdots\right]\hat{x}$$

to Newton's law

potential energy $\frac{1}{4\pi\epsilon_0}\frac{q/2}{d}$, $E = mc^2$

2nd term. $\frac{\mu_0 q^2 \dot{a}}{12\pi c}$ $\neq$ Abraham-L by a factor of 2 self-force — Rad react is due to the force of the charge on itself

$d \ll \lambda \ll r$

$V \sim \omega \frac{\cos\theta}{r} \sin[t]$ $\quad E \approx \omega^2 \frac{\sin\theta}{r} \cos(t)\ \hat{\theta}$ $\quad \hat{\phi}$

$I_0 = q_0\omega$

$A \sim \omega \frac{1}{r} \sin[t]\hat{z}$ $\quad B \sim \omega^2 \frac{\sin\theta}{r} \cos[t]\ \hat{\phi}$ $\quad \hat{\theta}$

$m_0 = \pi b^2 I_0$

$p_0 = q_0 d \quad d = \pi b$

$\langle S \rangle \sim \omega^4 \frac{\sin^2\theta}{r^2} \hat{r}$

$A \sim \omega \frac{\sin\theta}{r} \sin[t]\hat{\phi}$

$\frac{P_{mag}}{P_{elec}} = \left(\frac{m_0}{p_0 c}\right)^2 = \left(\frac{\omega b}{c}\right)^2 \xrightarrow{\text{small}}$ small.

Arbitrary source. $\vec{p} = \int r' \rho(\vec{r}, t_0)\, d\tau'$ $\quad t_0 = t - \frac{r}{c}$

$$\bar{A}(\bar{r}, t) \cong \frac{\mu_0}{4\pi} \frac{\dot{\bar{p}}(t_0)}{r} \qquad V(\bar{r}, t) \cong \frac{1}{4\pi\epsilon_0}\left[\frac{Q}{r} + \frac{\hat{r}\cdot\bar{p}(t_0)}{r^2} + \frac{\hat{r}\cdot\dot{\bar{p}}(t_0)}{rc}\right]$$

$$\bar{E}(\bar{r}, t) \cong \frac{\mu_0}{4\pi r}\left[\hat{r} \times (\hat{r} \times \ddot{\bar{p}})\right] \qquad \bar{B}(\bar{r}, t) \cong -\frac{\mu_0}{4\pi r c}(\hat{r} \times \ddot{\bar{p}})$$

$$\bar{S} \sim \left[\ddot{p}(t_0)\right]^2 \frac{\sin^2\theta}{r^2}\hat{r} \qquad P_{rad} \cong (\ddot{p}(t_0))^2$$

Next term in r' $\to$ magnetic dipole, electric quadrupole

Point charge $\quad \boxed{\bar{u} = c\hat{r} - \bar{v}}$

$$\bar{E} = \frac{q}{4\pi\epsilon_0} \frac{r}{(\bar{r}\cdot\bar{u})^3}\left((c^2 - v^2)\bar{u} + \bar{r} \times (\bar{u} \times \bar{a})\right) \qquad B = \frac{1}{c}\hat{r} \times \bar{E} \qquad \bar{S} = \frac{1}{\mu_0 c}\left(E^2\hat{r} - \hat{r}\cdot\bar{E}\,\bar{E}\right)$$

$$P = \frac{\mu_0 q^2 a^2}{6\pi c} \qquad P = \frac{\mu_0 q^2 \gamma^6}{6\pi c}\left(a^2 - \left|\frac{\bar{v} \times \bar{a}}{c}\right|^2\right) \qquad \gamma = 1/\sqrt{1 - v^2/c^2}$$