Lectures 8: Maximum likelihood I.
(nonlinear least square fits)

χ^2 fitting procedure!

example: testing coin making machine

Model for motivating nonlinear least squares fitting (χ2 fitting)

Manufacturer prints coins noticing that the printing machine produces biased heads. This can be measured by tossing n coins from the batch and measuring the binomial probability p of the batch. For convenience of some analysis $2p - 0.4$ is determined by measuring $2n_{head}/n - 0.4$ which turns out to be the function of the temperature where the machine operates (temperature x is recorded for the measurement). The results also depend on five parameters $b_1 \dots b_5$ of the mechanical construction of the printing machine. A smart theorist comes up with a model how the value of p depends on the temperature x and the five parameters $b_1 \dots b_5$:

$$f(x) = b_1 \exp(-b_2 x) + b_3 \exp\left(-\frac{1}{2}\frac{(x-b_4)^2}{b_5^2}\right)$$

f(x)=2p-0.4 is the measured value of 2p-0.4 as a function of temperature x

Manufacturer wants to determine the parameters $b_1 \dots b_5$ so that they can operate the machine at the temperature where $2p - 0.4 = 0.6$ so that $p=0.5$ and the coins are unbiased. This will require to fit the five parameters $b_1 \dots b_5$ of the machine based on the available data at many temperatures. **How do we do that?**

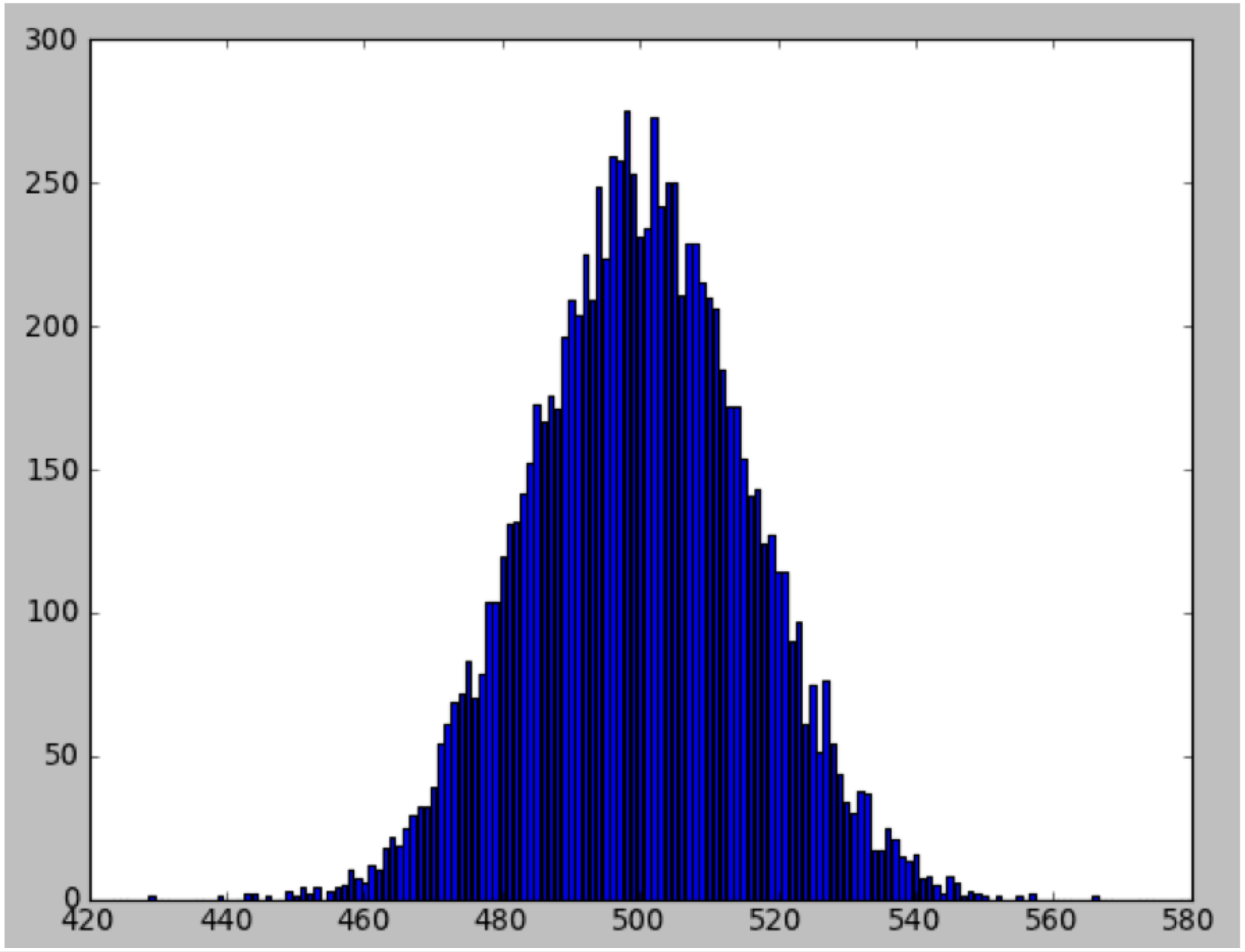
Data are collected at various temperatures x_i.

At each temperature x_i the value $y_i = 2n^{(i)}_{heads}/n - 0.4$ is measured to approximate $2p - 0.4$ from n coin tosses

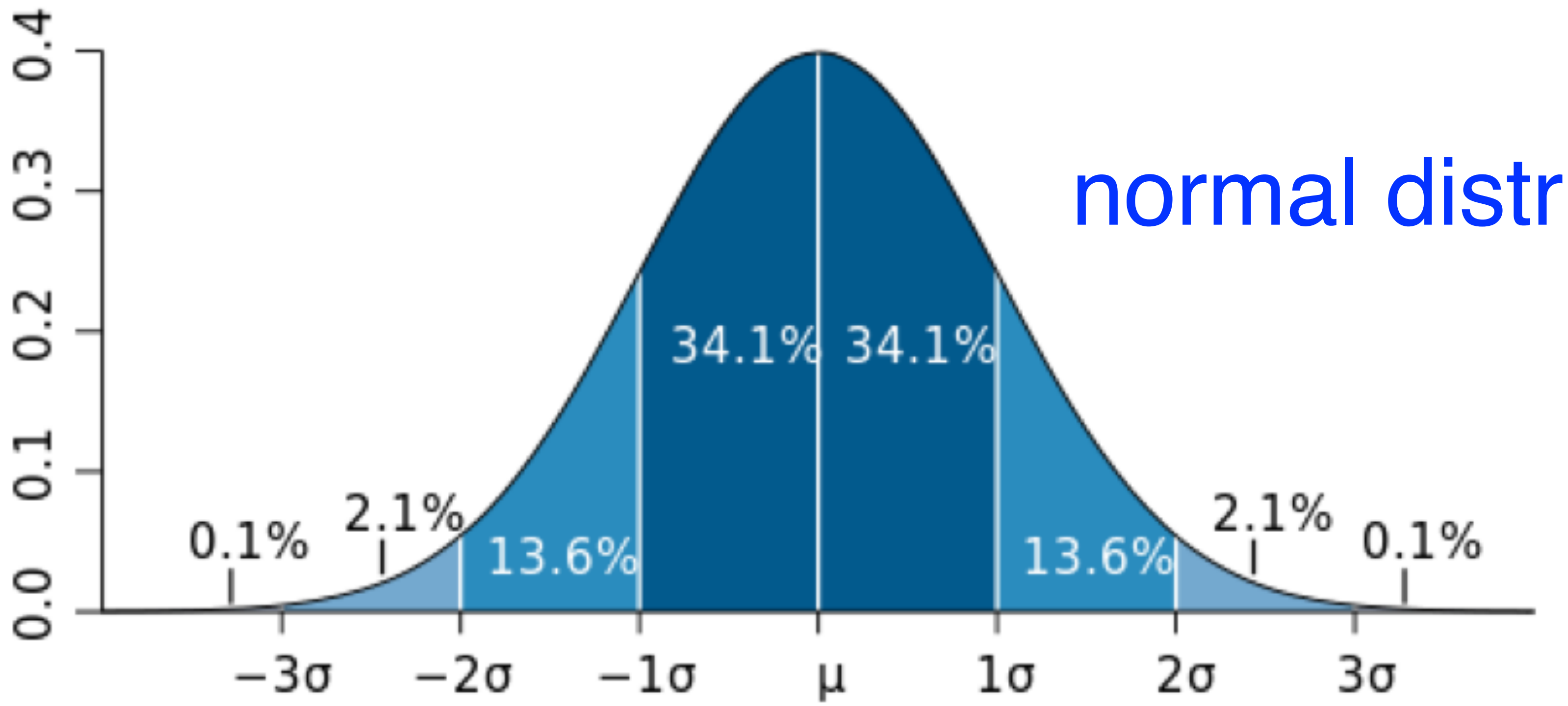
But y_i has some error e_i

What is the error?

central limit theorem:



10,000 trials of 1,000 tosses



normal distribution (bell curve)

Weighted Nonlinear Least Squares Fitting
a.k.a. χ^2 Fitting
a.k.a. Maximum Likelihood Estimation of Parameters (MLE)
a.k.a. Bayesian parameter estimation
(with uniform prior and maybe some other normality assumptions)

these are not all exactly identical, but they're real close!

$$y_i = y(\mathbf{x}_i | \mathbf{b}) + e_i$$

measured values supposed to be a model, plus an error term

$$e_i \sim N(0, \sigma_i)$$

the errors are Normal, either independently

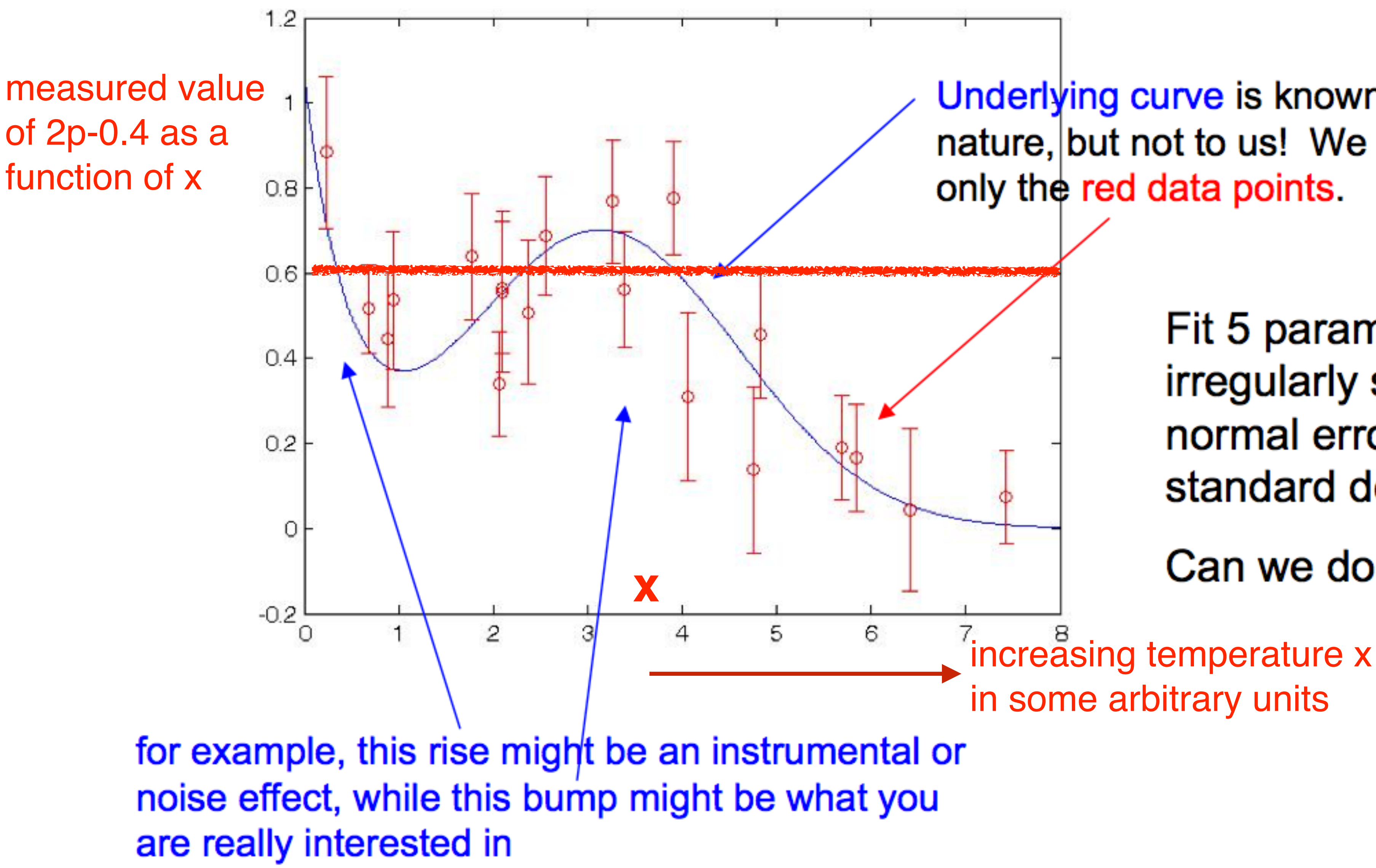
$$\mathbf{e} \sim N(0, \mathbf{\Sigma})$$

or else with errors correlated in some known way (e.g., multivariate Normal)

We want to find the parameters of the model **b** from the data.

An example might be something like fitting a known functional form to data

$$f(x) = b_1 \exp(-b_2 x) + b_3 \exp\left(-\frac{1}{2}\frac{(x-b_4)^2}{b_5^2}\right)$$



Fit 5 parameters from 20 irregularly space points, with normal errors of known standard deviations.

Can we do it? How well?