Assignment III.

Bootstrap and Jackknife

due: November 28, 2016

problem 1 PHYS 139/239

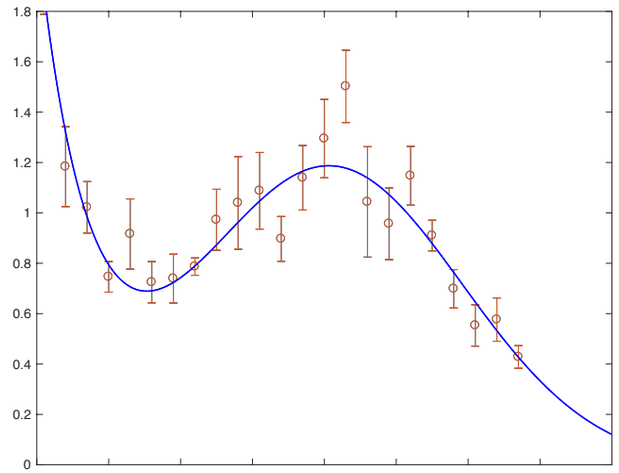
We measure in an experiment at 23 values of x_i the outcome y_i from normal distributions where the results are listed in the data.txt file:

x	y	y error
0.100000000000000	1.955692474636036	0.166896282383792
0.400000000000000	1.183586547503424	0.158551780782385
0.700000000000000	1.022128862295741	0.102145122199102
1.000000000000000	0.746134082944572	0.060820536337125
1.300000000000000	0.916188421395087	0.139506053368529
1.600000000000000	0.724682156536752	0.081793212333357
1.900000000000000	0.739127499035786	0.096894347069944
2.200000000000000	0.786742524422711	0.034353661707974
2.500000000000000	0.972558512530457	0.121213729440151
2.800000000000000	1.039776955766267	0.183845107945299
3.100000000000000	1.087705062846587	0.152064123528747
3.400000000000000	0.896727858969629	0.088835443972525
3.699999999999999	1.139381591276074	0.128022842446142
4.000000000000000	1.294829163615035	0.155588445889791
4.299999999999999	1.502261299770580	0.143493932937373
4.600000000000000	1.043529911555928	0.219186627495748
4.899999999999999	0.956827376670183	0.142469078945670
5.199999999999999	1.147387265711086	0.116683264235504
5.499999999999999	0.909994065501967	0.060876724854546
5.799999999999999	0.698671186235582	0.076323301379691
6.100000000000000	0.553227945238010	0.082132016628130
6.399999999999999	0.576371045690540	0.085922021448737
6.699999999999999	0.427880507687987	0.044877728959367

problem 1 PHYS 139/239

(A) assuming that the experiment is described by the theoretical function f(x) of five parameters, calculate the mean value of $b_3 b_5$ and calculate the error from linear error propagation.

$$f(x) = b_1 \exp(-b_2 x) + b_3 \exp\left(-\frac{1}{2}\frac{(x-b_4)^2}{b_5^2}\right)$$

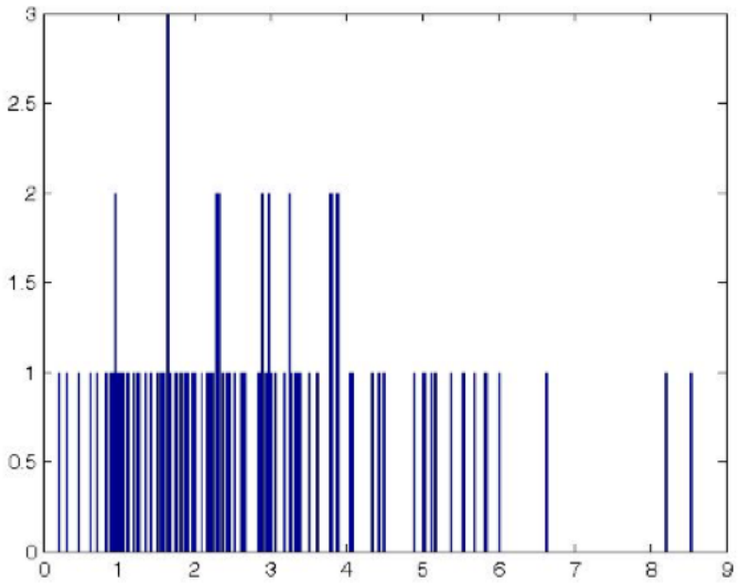


(B) Calculate the mean value of $b_3 b_5$ and calculate the error from the posterior distribution of $b_3 b_5$

problem 2 PHYS 139/239

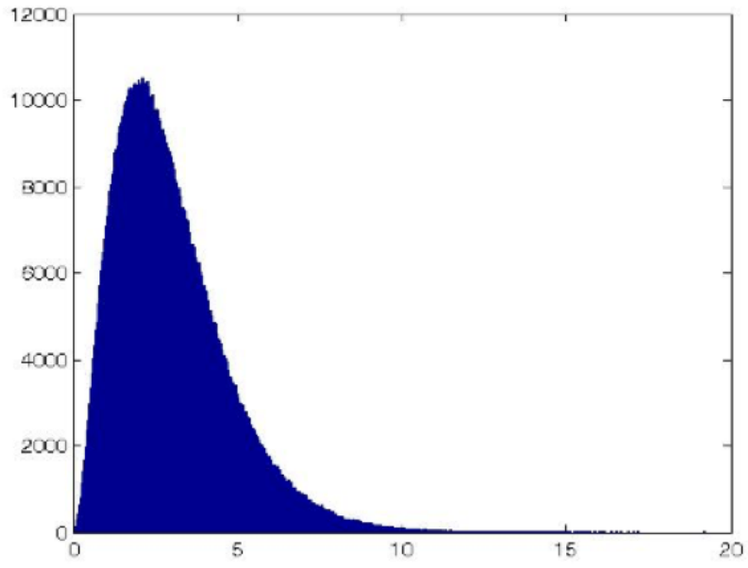
(A) Create the visible data set with 100 data, like on the left plot, from the Gamma distribution Gamma(4,1). Your sampling will look different. Also create the histogram of the invisible Gamma distribution on the right side plot. Your distribution will look different. Calculate the mean and the variance of the data and compare with the analytic values.

Visible side (sample):



These happen to be drawn from a Gamma distribution.

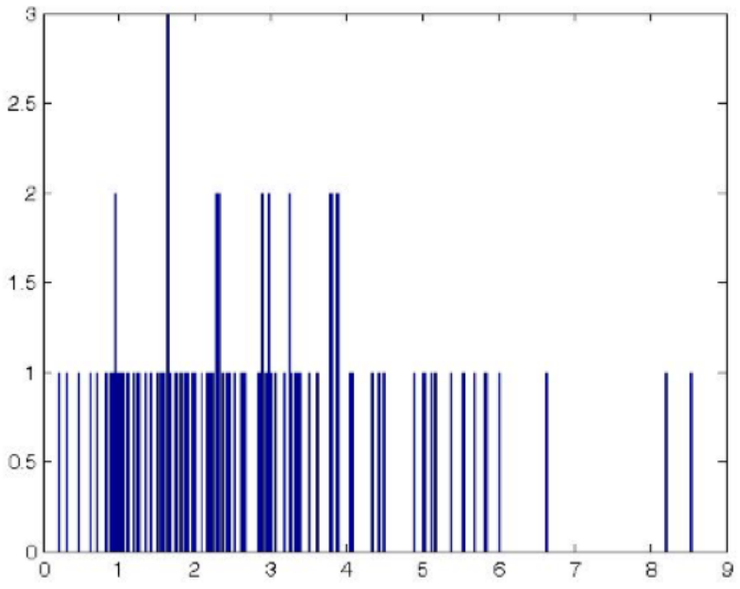
Hidden side (population):



problem 2 PHYS 139/239

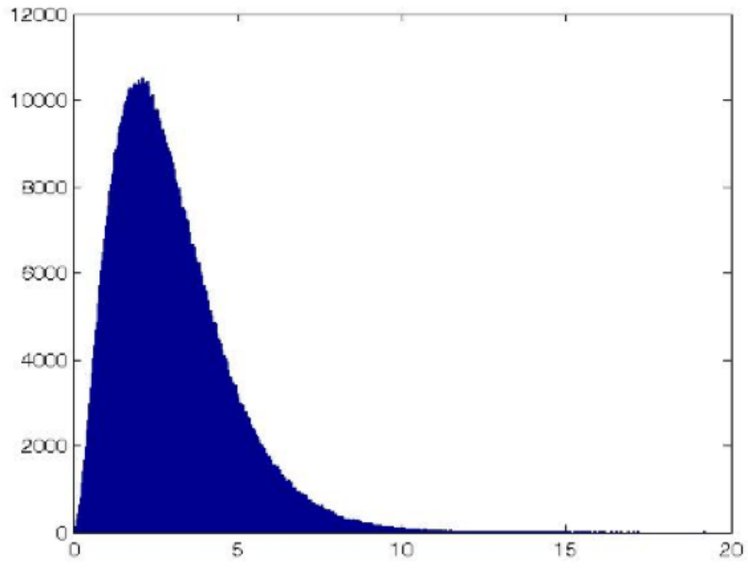
(B) Calculate the mean and the error of the ratio x^2/x^4 accurately with large sampling on the “invisible” right side of the sampling and compare with bootstrap sampling of the visible 100 data.

Visible side (sample):



These happen to be drawn from a Gamma distribution.

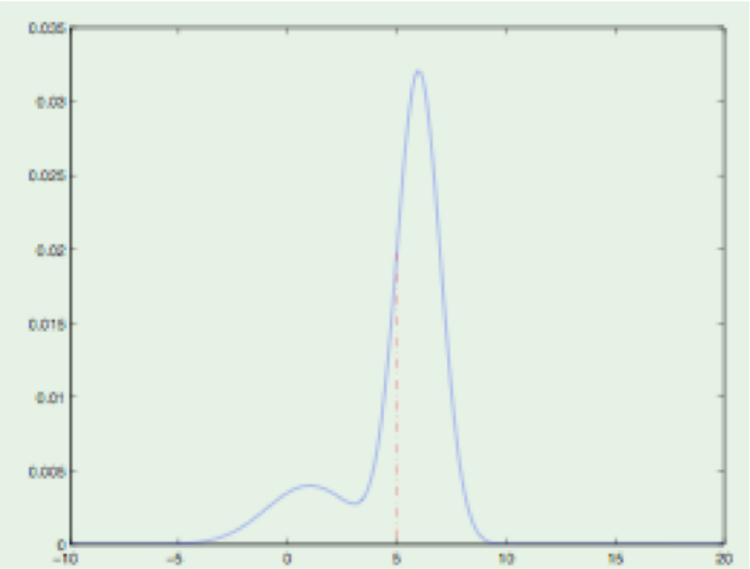
Hidden side (population):



problem 3 PHYS 239

Repeat Problem 2 with the superposition of two normal distributions for the ratio x^2/x^4:

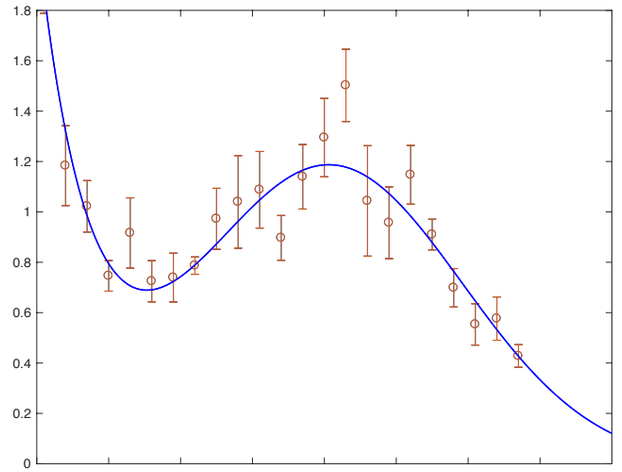
$F(x) = 0.3 \cdot N(\mu=1,\sigma=2) + 0.7 \cdot N(\mu=4,\sigma=1)$. Your distribution is expected to be similar to the plot.



problem 4 PHYS 139/239

Calculate from the bootstrap procedure of your data in Problem 1 the mean value of $b_3 b_5$ and calculate the bootstrap error.

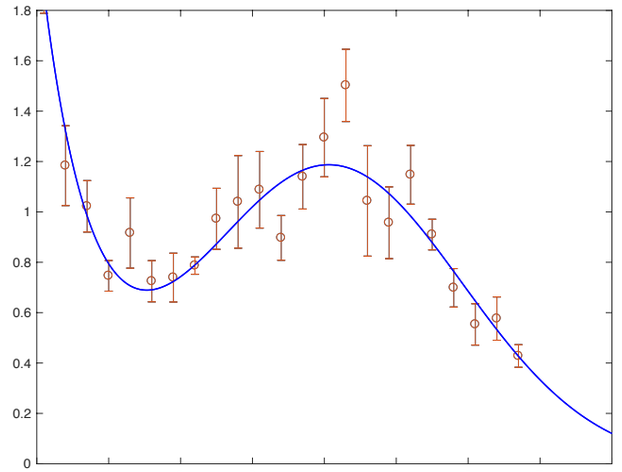
$$f(x) = b_1 \exp(-b_2 x) + b_3 \exp\left(-\frac{1}{2}\frac{(x-b_4)^2}{b_5^2}\right)$$



problem 5 PHYS 139/239

Calculate from the jackknife procedure of your data in Problem 1 the mean value of b_3b_5 and calculate the jackknife error. Can you estimate the jackknife bias on b_3b_5 ?

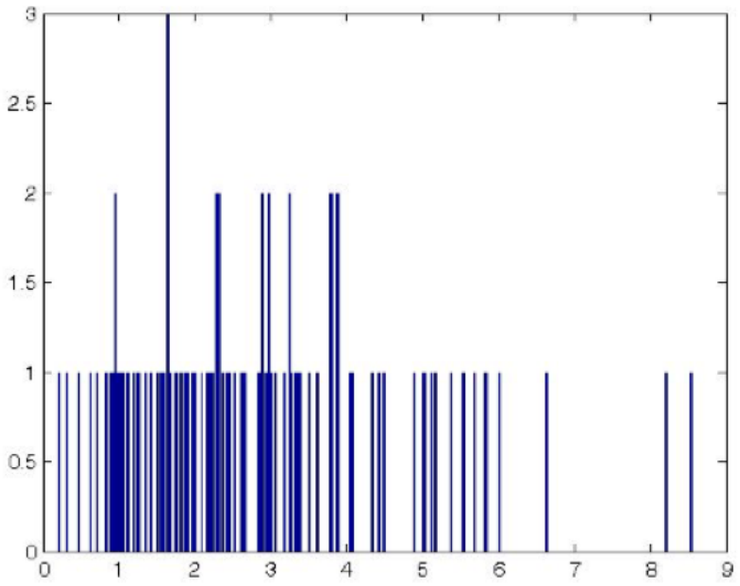
$$f(x) = b_1 \exp(-b_2 x) + b_3 \exp\left(-\frac{1}{2}\frac{(x-b_4)^2}{b_5^2}\right)$$



problem 6 PHYS 139/239

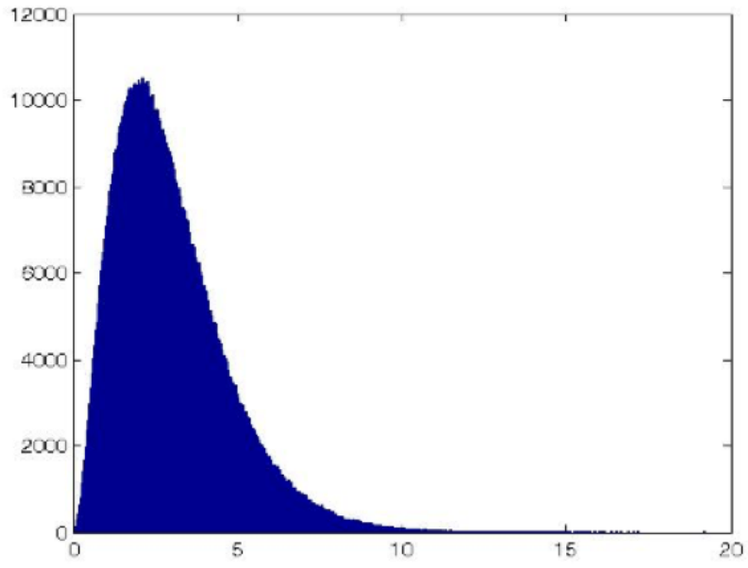
Compare the mean and the error of the ratio x^2/x^4 you obtained accurately with large sampling on the "invisible" right side of the sampling in Problem 2 and compare with jackknife sampling of the visible 100 data.

Visible side (sample):



These happen to be drawn from a Gamma distribution.

Hidden side (population):



problem 7 PHYS 139/239

Jackknife analysis of part (B) of Problem 2 for mean/median

problem 8 PHYS 239

Jackknife analysis for x^2/x^4 of Problem 3

with the F(x) = 0.3 · N(μ=1,σ=2) + 0.7 · N(μ=4,σ=1).