Assignment I.

problems for probability concepts

due: October 12, 2016

Code in Matlab (strongly encouraged) or Python

problem 1   PHYS 139/239

In the programming language of your choice, how would you simulate flips of a fair, two-sided coin?

Do it. Simulate 10^6 flips. How many heads do you get?

problem 2 PHYS 139/239

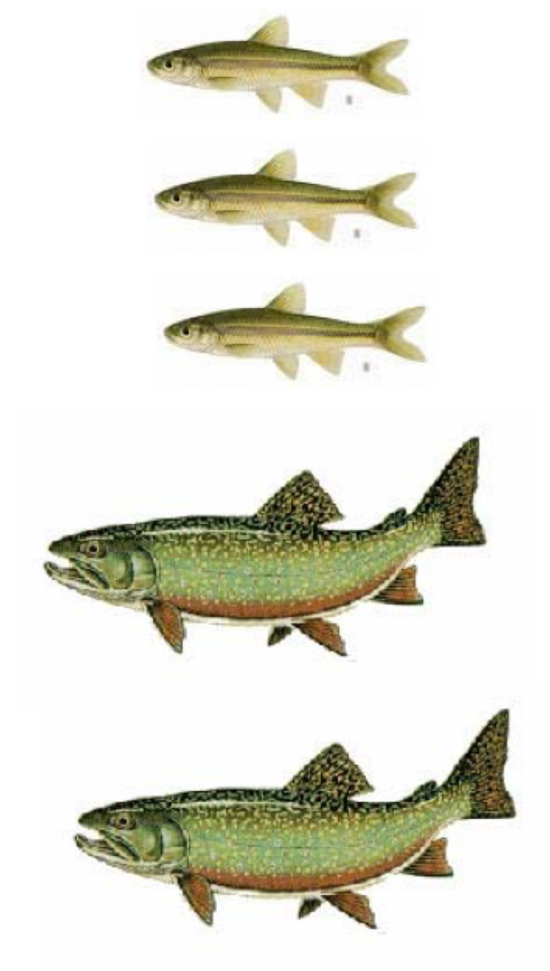
- You are an oracle that, when asked, says "yes“ with probability $P = 1/4$ and "no" with probability $1\text{-}P = 3/4$.
- How do you do this using only a fair, two-sided coin?
- Same problem with $P = 1/3$.
- Same problem with $P = 1/\pi$ PHYS 239 only

problem 3 PHYS 139/239

In the programming language of your choice, how would you simulate draws from two weird 7-sided dice whose faces (showing 1 through 7 spots) have probabilities proportional to: $1{:}e{:}\pi{:}4{:}5{:}6{:}e^{\pi}$ respectively?

– Do it. Simulate 10^6 throws of the dice. How many times is the sum of the two dice equal to 8?
– What should it be (in expectation) analytically?

problem 4 PHYS 139/239



Example: A barrel has 3 minnows and 2 trout, with equal probability of being caught. Minnows must be thrown back. Trout we keep.

What is the probability that the 2nd fish caught is a trout?

$$H_1 \equiv \text{1st caught is minnow, leaving } 3+2$$
$$H_2 \equiv \text{1st caught is trout, leaving } 3+1$$
$$B \equiv \text{2nd caught is a trout}$$

$$\begin{aligned} P(B) &= P(B|H_1)P(H_1) + P(B|H_2)P(H_2) \\ &= \tfrac{2}{5}\cdot\tfrac{3}{5} + \tfrac{1}{4}\cdot\tfrac{2}{5} = 0.34 \end{aligned}$$

How many draws are needed to distinguish between P=0.34 (probability that second fish is a trout) and P=1/3?

Do a convincing simulation.

PHYS 239: repeat with 5 minnows and 3 trout

problem 5 PHYS 139/239

Example: The Monty Hall or
Let's Make a Deal Problem

- Three doors
- Car (prize) behind one door
- You pick a door, but don't open it yet
- Monty then opens one of the other doors, <u>always revealing no car</u> (he knows where it is)
- You now get to switch doors if you want
- Should you?
- Most people reason: Two remaining doors were equiprobable before, and nothing has changed. So doesn't matter whether you switch or not.

Work out the case for 5 doors when Monty opens two doors

PHYS 239 only: generalize the problem to N doors where Monty opens n doors.

problem 6   PHYS 139/239

Simulate the Knight/Troll/Gnome problem 100,000 times.

Plot (fraction of safe crossings so far) vs. (number of simulated trials so far) to confirm that this fraction converges to the probability calculated in the segment

problem 7 PHYS 239

- Each one of the boxes shown contains 10 items.
- Most of the boxes contain 2 valuable items and 8 worthless items, but a few have 9 valuable items and just 1 worthless item.
- You pick a box, reach in, and select one item at random. It is valuable and you keep it!

- Q1. What have you learned about the number of boxes that originally had 9 valuable items?
- Q2. What is the probability that the next item that you select at random *from the same box* will be valuable?