

Physics 4A
Lecture 5: Jan. 22, 2015

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Interpreting Newton's 2nd Law


- Vector equation: $\sum \vec{F} = m\vec{a}$
 $\Rightarrow \sum \vec{F}_x = m\vec{a}_x, \sum \vec{F}_y = m\vec{a}_y, \sum \vec{F}_z = m\vec{a}_z$
- refers to *external* forces exerted on the object by the environment
 - impossible for object to affect its own motion by applying force on itself!
 - cant lift yourself to ceiling by pulling up you belt
- Valid only in inertial frames of reference
- Only true when mass $m = \text{constant}$
 - think of acceleration of a fuel tank due to a force when the tank is leaking fluid ??

Newton's 3rd Law

- Force acting on a body is always a result of its interaction with another body. Forces always come in pairs
- Newton's third law:
 - When a force due to object B acts on object A, then an equal & opposite force acts on B

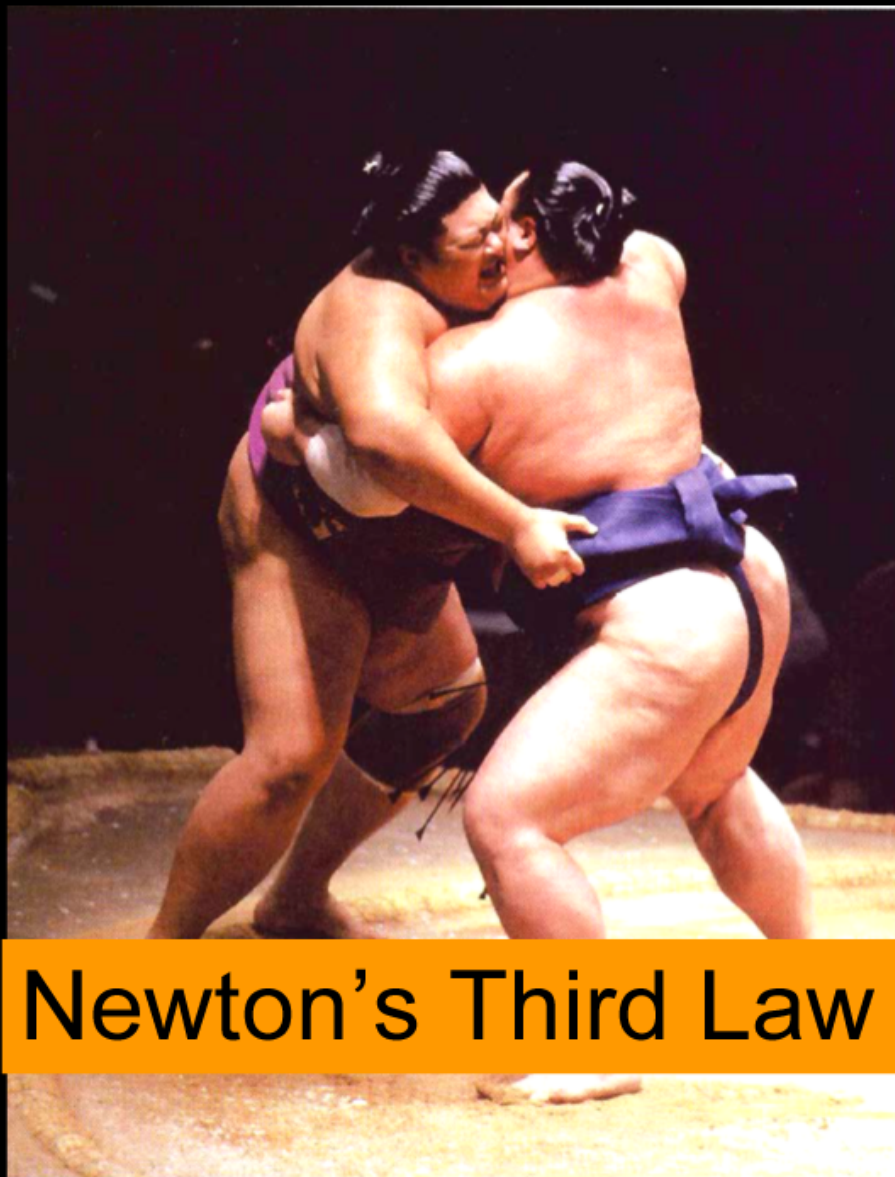
$$\vec{F}_{BA} = -\vec{F}_{AB}$$


this force
acts on A


this force
acts on B

action-reaction pair

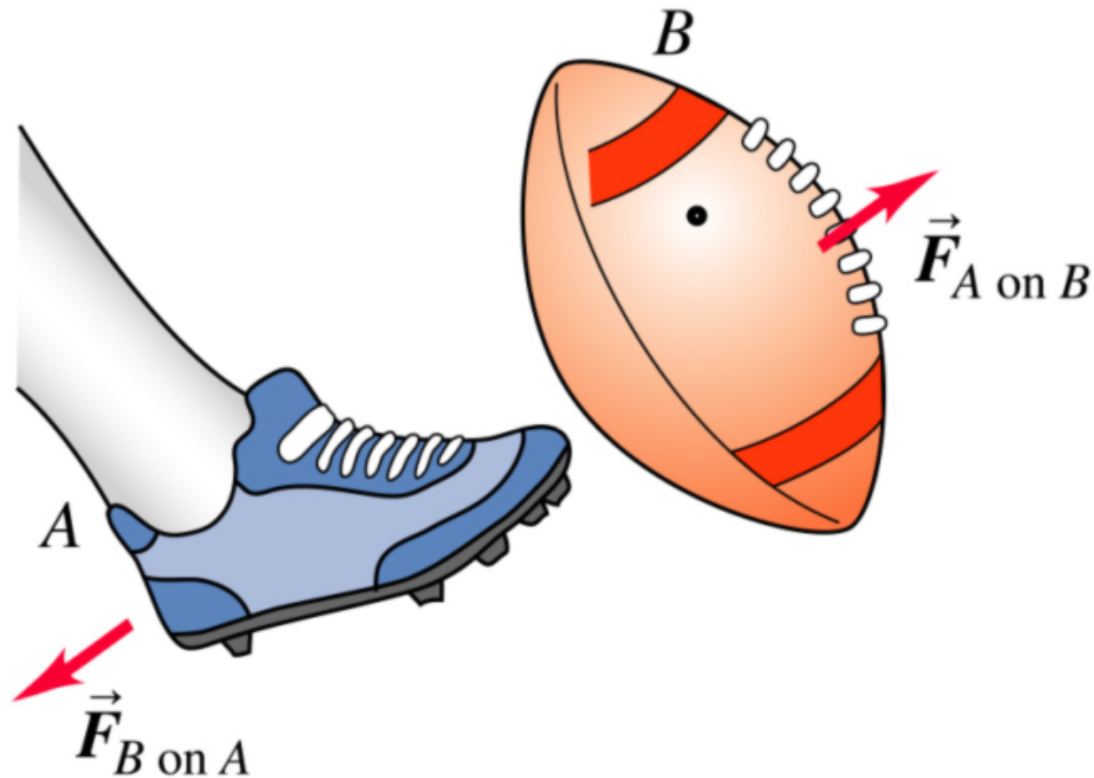
choice of *which* one
is action or reaction
is arbitrary ! Forces
are equal & opposite



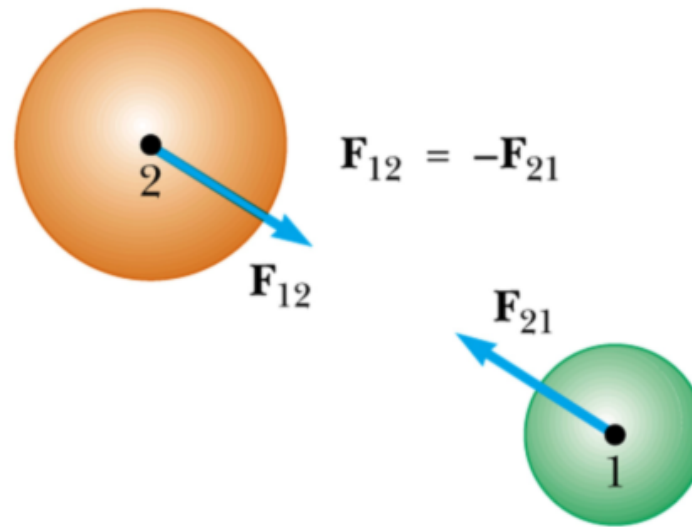
Newton's Third Law

Action-Reaction Pair

The pair of forces are simultaneous, equal & opposite and act on different bodies



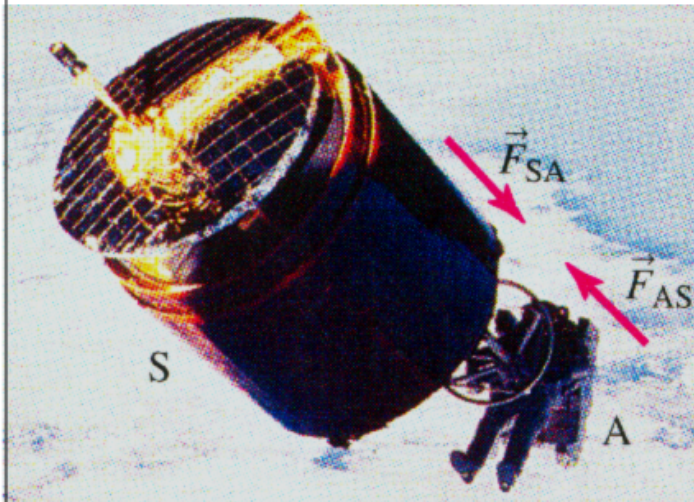
Newton's Third Law



Action Equals Reaction,
Acts On Different Bodies

Astronaut (A) and Satellite (S) are in an environment where they can be said to form an isolated system with no external force acting on the system. Astronaut tugs on satellite with force of 10N towards her (to the right).

What is the force on the astronaut?



Apply Newtons 3rd law:

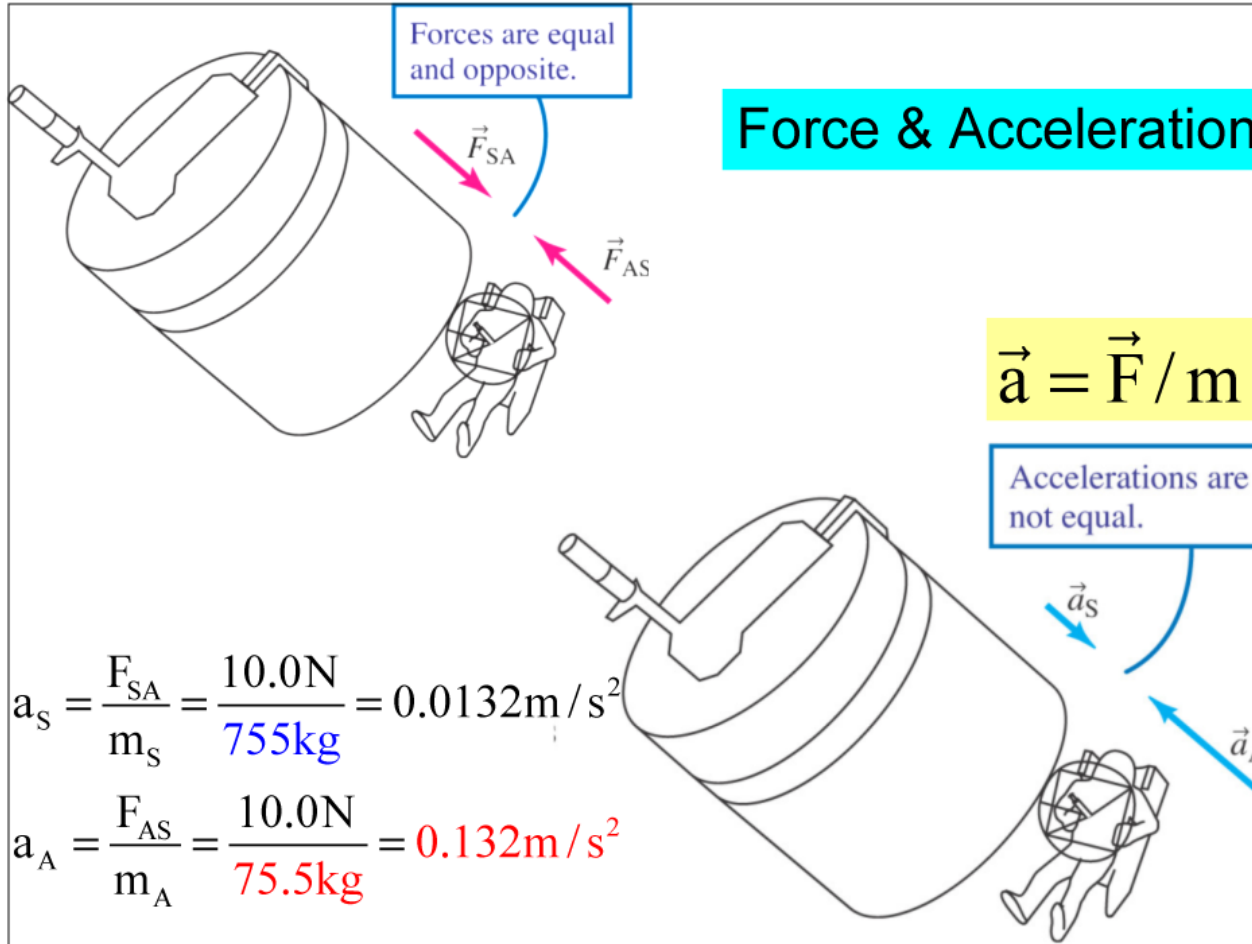
Force on Satellite due to Astronaut = \vec{F}_{SA}

Force on Astronaut due to Satellite = \vec{F}_{AS}

$$= -\vec{F}_{SA}$$

= -10N towards left

Force & Acceleration



Quick Question:

- What happens when you jump in the air? Which is the most accurate statement?
- A) It is the upward force exerted by the ground that pushes you up, but this force can never exceed your weight
- B) You are able to spring up because the earth exerts a force on you that is stronger than the downward force you exert on earth
- C) Since the ground is stationary, it cannot exert the upward force necessary to propel you into the air. Instead, it is the internal forces of your muscles acting on your body itself that propels your body into the air.
- D) When you push down on the earth with a greater force than your weight, the earth will push back on you with the same magnitude and thus propel you into the air.

Rocket Propulsion System

When valve of a CO₂ extinguisher is opened (due to pressure inside) force is exerted on mass of CO₂ gas causing it to be accelerated from the fire extinguisher. **By 3rd law, expelled gas exerts equal and opposite force on fire-extinguisher**

If the extinguisher is held rigidly by the man on the cart, force due to gas acts on the entire system

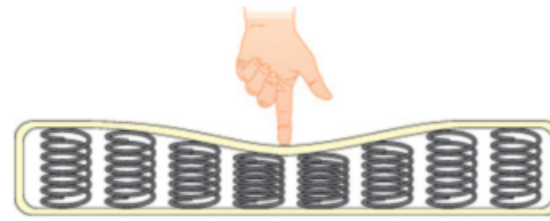
Gas goes to the right,
man+cart+extinguisher
accelerate towards left

rocket accelerates matter in
one direction and it itself
accelerates in oppo. dir.



The Normal Force

When a body presses against a surface, the surface deforms and pushes on body with a **Normal force** that is \perp to surface



\vec{F}_g = Gravitational force due to the block on table

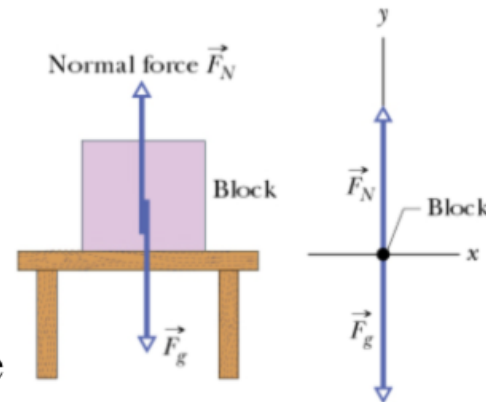
\vec{F}_N = Normal force due to the table **on the Block**

Using Newton's Second law, In general:

$$\vec{F}_N - \vec{F}_g = m\vec{a}_y \Rightarrow \vec{F}_N = m(\vec{g} + \vec{a}_y)$$

If the table-block system is not accelerating relative

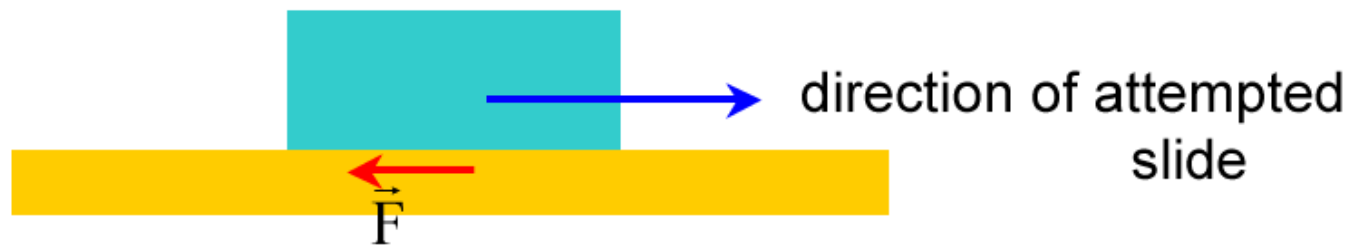
to ground \Rightarrow $\boxed{\vec{F}_N = m\vec{g}}$



Frictional Force

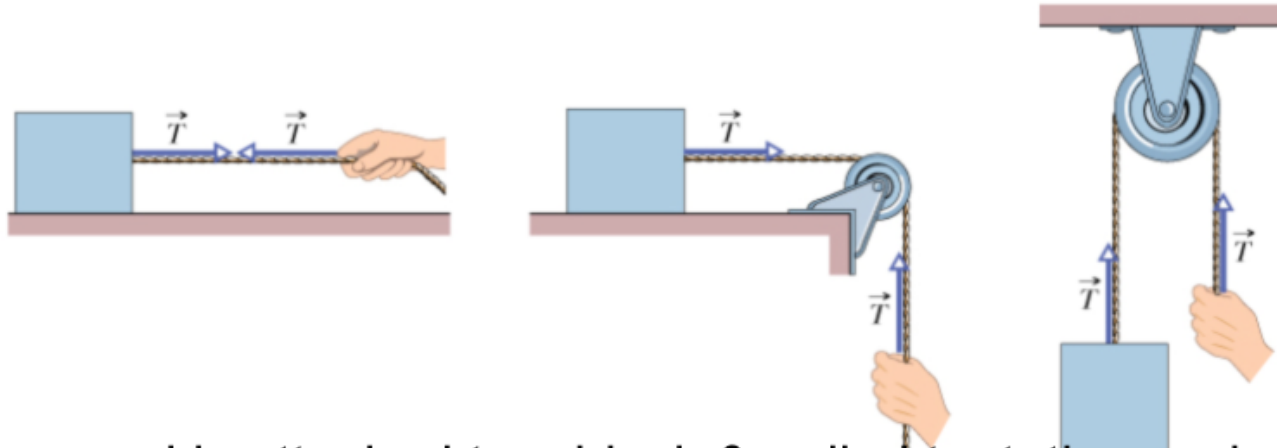
An attempt to slide a body over a surface leads to a resistive force due to bonding between body & surface

This resistance can be modeled as a single force F (**frictional force**) directed along the surface, opposite to direction of intended motion



Sometimes we assume surface to be frictionless $\Rightarrow F=0$

Tension Force On a Cord



When a cord is attached to a block & pulled taut, the cord pulls on the block with a force T directed away from the block & along the cord. T is called **Tension force or just Tension**.

Tension in the cord is the magnitude T of force on block

A cord is sometimes idealized to be massless & unstretchable and exists only as a connection between two bodies

Frictionless, massless pulley is another idealization. If a cord wraps halfway around a pulley, net force on pulley from cord $=2T$

Pop quiz:

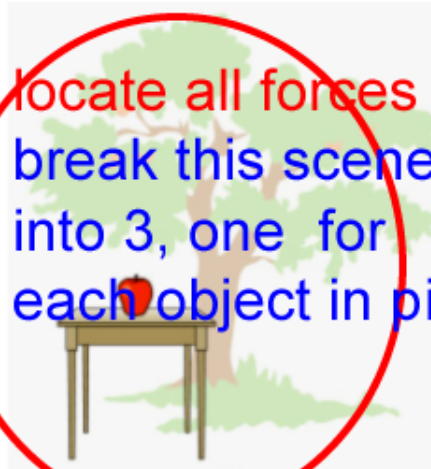
- A steel rod A of mass 2 kg is rigidly attached to a metal block B of mass 5 kg and is pulling it up so it accelerates vertically at 2 m/s^2 . The force in the link between A and B is:

A) $7g \text{ N}$

B) $7g + 14 \text{ N}$

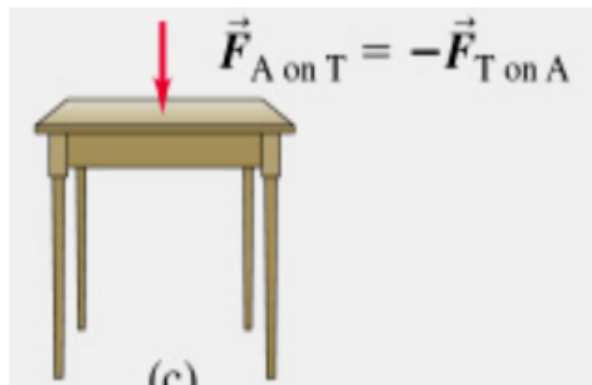
C) $5g + 10 \text{ N}$

D) $5g \text{ N}$



locate all forces
break this scene
into 3, one for
each object in pic

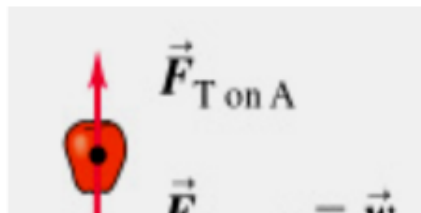
Forces on Table



which are the
3rd law
action-reaction
pairs?

What are the forces
involved ?

Forces on Apple



Forces on Earth

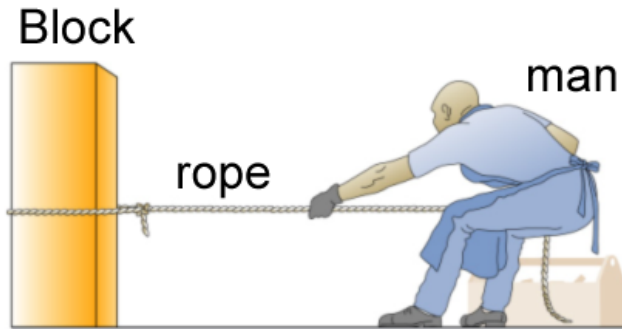


$$\vec{F}_{A \text{ on } T} = -\vec{F}_{T \text{ on } A}$$

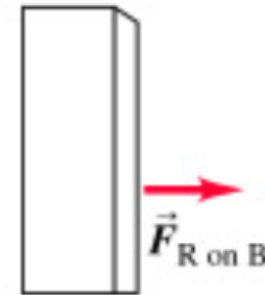
$$\vec{F}_{A \text{ on } E} = -\vec{F}_{E \text{ on } A}$$

They act on
different bodies

Man Pulls A Block With A Rope



Now examine
horizontal forces
on **each** object



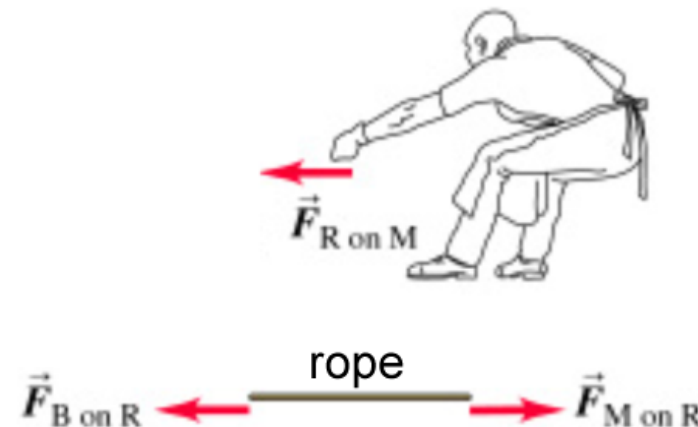
Examine all the forces on
all objects in this picture

Action-Reaction pairs:

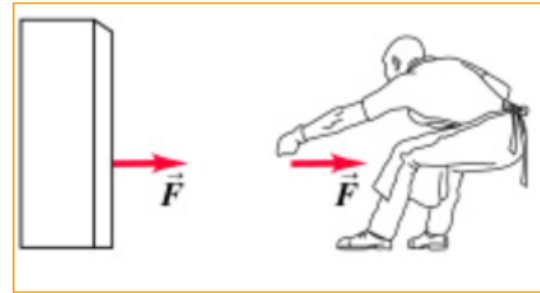
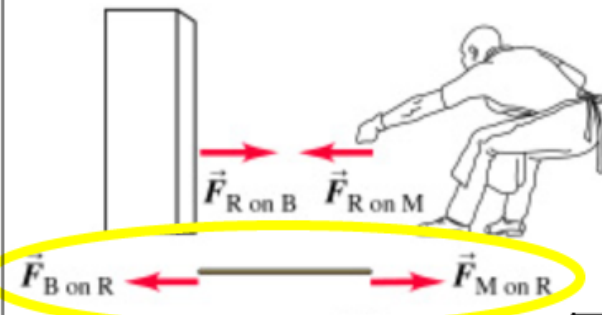
$$\vec{F}_{R \text{ on } M} = -\vec{F}_{M \text{ on } R}$$

$$\vec{F}_{B \text{ on } R} = -\vec{F}_{R \text{ on } B}$$

Action & Reaction act
on different bodies



Man Pulls A Block With A Rope



when
 $a_R=0$
 or
 $m_R=0$

Apply 2nd Law to Rope: $\sum \vec{F} = \vec{F}_{\text{MonR}} + \vec{F}_{\text{BonR}} = m_{\text{rope}} \vec{a}_{\text{rope}}$

If R & B are accelerating, then rope not in equilibrium $\vec{F}_{\text{MonR}} \neq \vec{F}_{\text{BonR}}$

If rope in equilibrium, $\vec{a}_{\text{rope}} = 0$ & $\vec{F}_{\text{MonR}} = \vec{F}_{\text{BonR}}$ (2nd law)

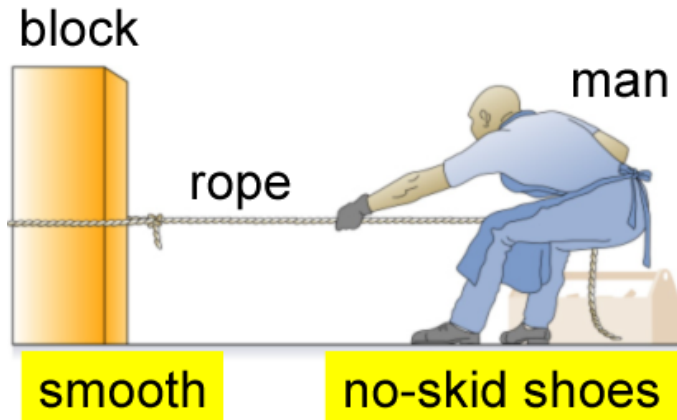
$$\text{If } \vec{a}_{\text{rope}} \neq 0 \text{ but } m_{\text{rope}} \approx 0 \Rightarrow \sum \vec{F} = \vec{F}_{\text{MonR}} + \vec{F}_{\text{BonR}} = m_{\text{rope}} \vec{a}_{\text{rope}} = 0$$

$$\text{so } \vec{F}_{\text{BonR}} = -\vec{F}_{\text{MonR}} \text{ (2nd law) \& } \vec{F}_{\text{BonR}} = -\vec{F}_{\text{RonB}} \text{ (3rd law)}$$

$\Rightarrow \vec{F}_{\text{RonB}} = \vec{F}_{\text{MonR}} \Rightarrow$ rope "transmits" to the block,

without change, the force man exerts on rope (special case $m_R=0$)

Is There A Paradox Here ?



If the man pulls as hard on the B&R system as the B&R system pulls back on him, then why does the B&R move **but not the man?**

?

Dont confuse 3rd law with 2nd law ! **3rd law says nothing about motion, just relates forces on different bodies**

2nd Law: Net Force acting on the body cause acceleration

If the man does not move, its because of FRICTION between his boot and the surface. Friction force acting on man's no-skid shoes is much larger than that acting on the "marble" block

Net force acting on man is **zero**, not so on the block due to friction!



Engine can not push the car forward since it's a part of it !

Engine drives either the front or back axle of car which causes tires to rotate. Tires in turn push against the road surface thru force of friction f between tire and road surface.

3rd law \Rightarrow road must then push against the tires with equal and oppositely directed force = $-f$. This external force causes car to accelerate \Rightarrow Vroom Vroom !...what happens if tire is "bald" ?

Be Careful When Analyzing Motion

Newton's laws very simple but applying them to different scenarios can be difficult ! Some tips:

1. 1st & 2nd law applies to a **specific body**. So amongst various choices first identify the body to focus on.
2. Only forces acting on **THAT body** matter ! need to identify all forces on the chosen body, then find $\sum \vec{F}$
3. Only **members** of action-reaction pair that act on **body** of focus matter. Disregard forces that this body exerts on **OTHER** bodies in the environment

Free-Body Diagrams

Diagram showing the body by itself, **Free of surrounding**

Use Vectors to denote magnitude and direction of forces applied on it by all OTHER bodies that interact with it

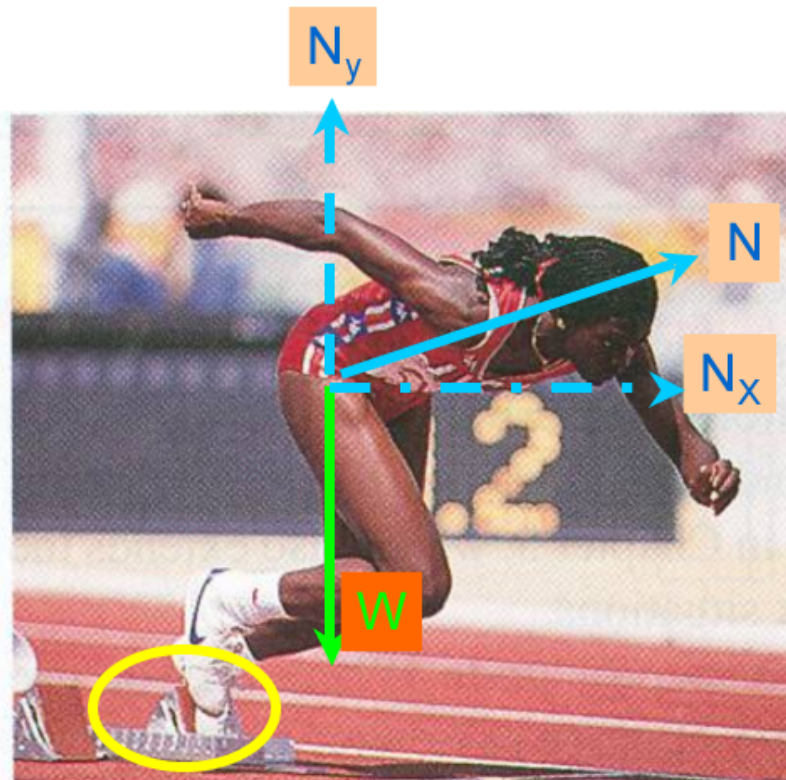
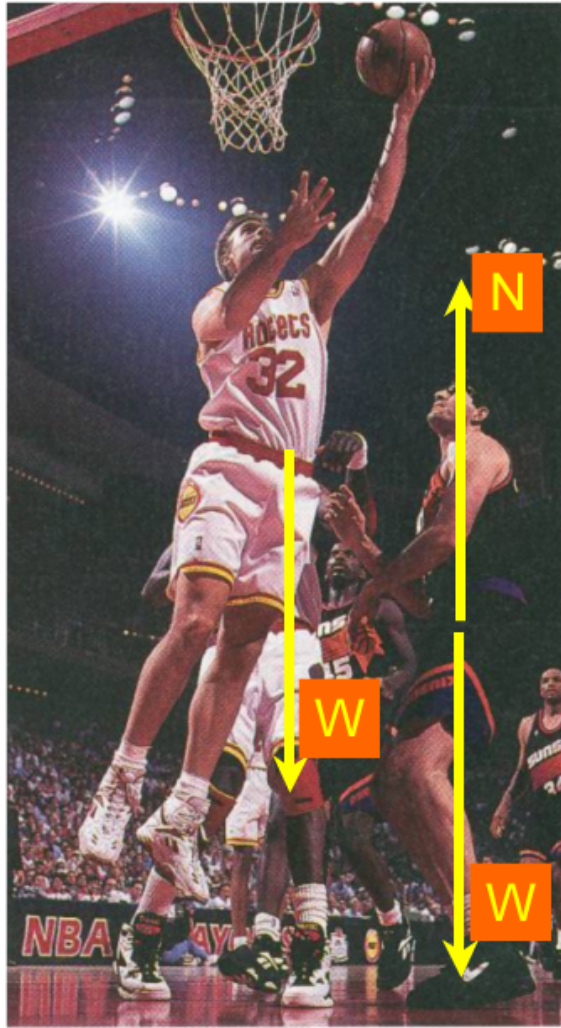
Dont include any force that this body exerts on another

Never include 2 forces from an action-reaction pair in the *same* free-body diagram (they act on diff bodies)

Forces that a body exerts on itself is never included since it cant affect motion !

What does a Free-body diagram look like: **already using it !**

Some Free Body Diagrams

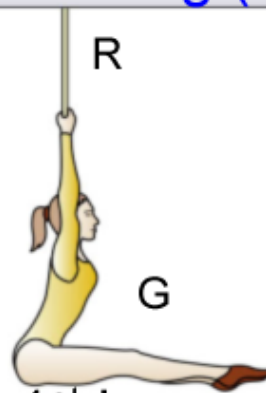


She sprints because $N_x = ma_x$

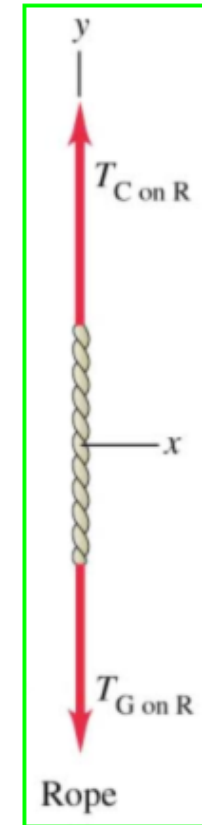
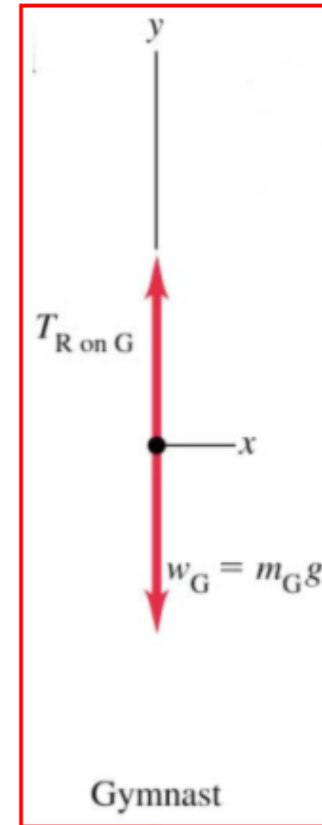
She is in vertical equilibrium

Gymnast ($m_G = 50 \text{ kg}$) hangs from ceiling with **massless** rope. What is her weight?
 Force exerted by rope on gymnast?
 Tension of rope ? **Ceiling (C)**

$$w_G = m_G g = 490 \text{ N}$$



Equilibrium \Rightarrow apply 1st law
 Relating forces \Rightarrow apply 3rd law

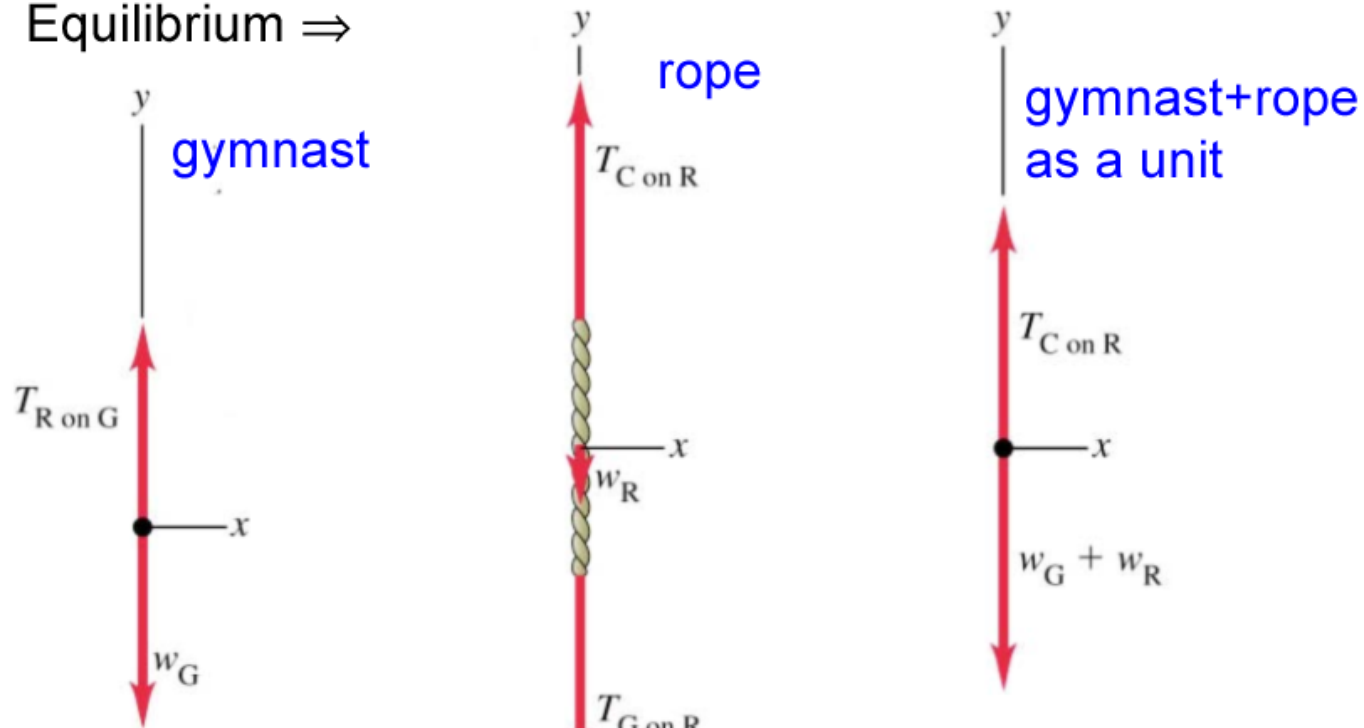


$$\text{Gymnast: } \sum F_y = T_{R \text{ on } G} + (-w_G) = 0 \Rightarrow T_{R \text{ on } G} = 490 \text{ N}$$

$$\text{Rope: } \sum F_y = T_{C \text{ on } R} + (-T_{G \text{ on } R}) = 0 \Rightarrow T_{C \text{ on } R} = T_{G \text{ on } R} = 490 \text{ N}$$

Tensions at each end if rope has weight = 120N

Equilibrium \Rightarrow



$$\text{Gymnast: } \sum F_y = T_{R \text{ on } G} - w_G = 0 \Rightarrow T_{R \text{ on } G} = T_{G \text{ on } R} = w_G = 490\text{N}$$

$$\text{Rope: } \sum F_y = T_{C \text{ on } R} - T_{G \text{ on } R} - w_R = 0 \Rightarrow T_{C \text{ on } R} = T_{G \text{ on } R} + w_R = 610\text{N}$$

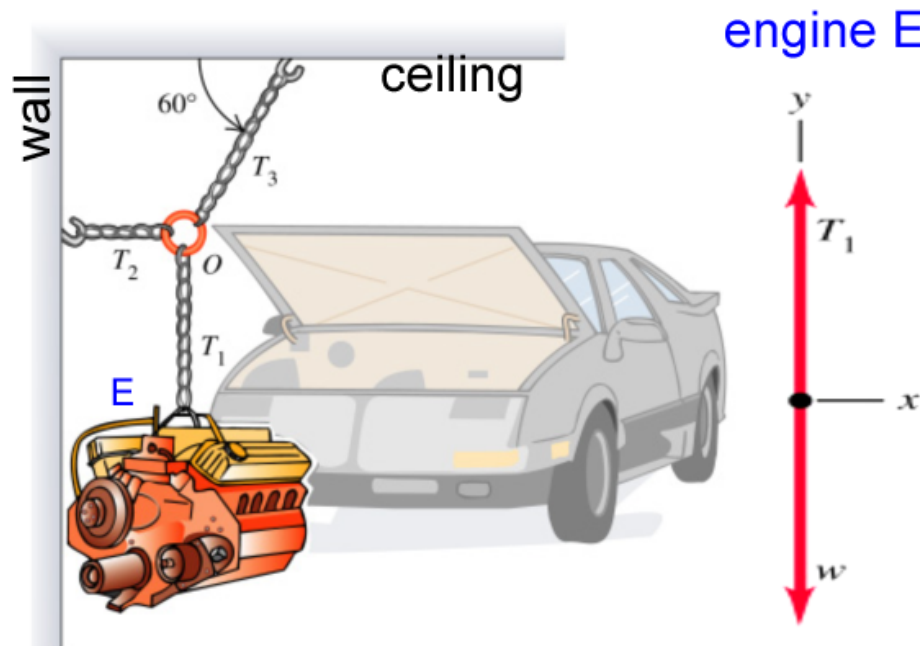
Tension is different at the two ends of rope (about G) since $m_R \neq 0$

Equilibrium Situation in 2D

Engine of weight w hangs from chain linked at ring O to 2 other chains: **one fastened to ceiling** & other to wall.

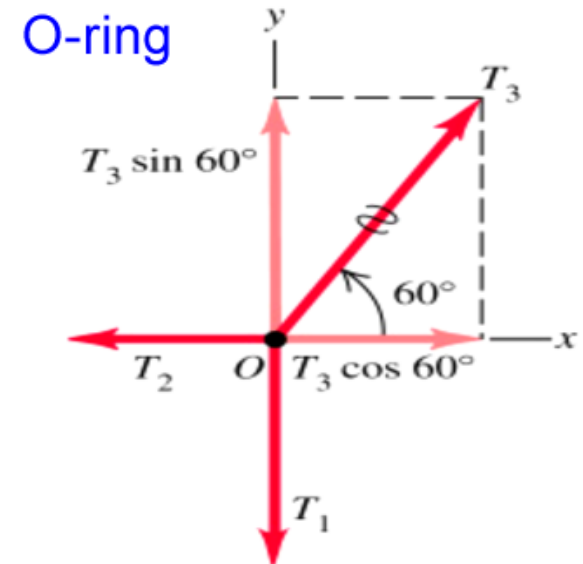
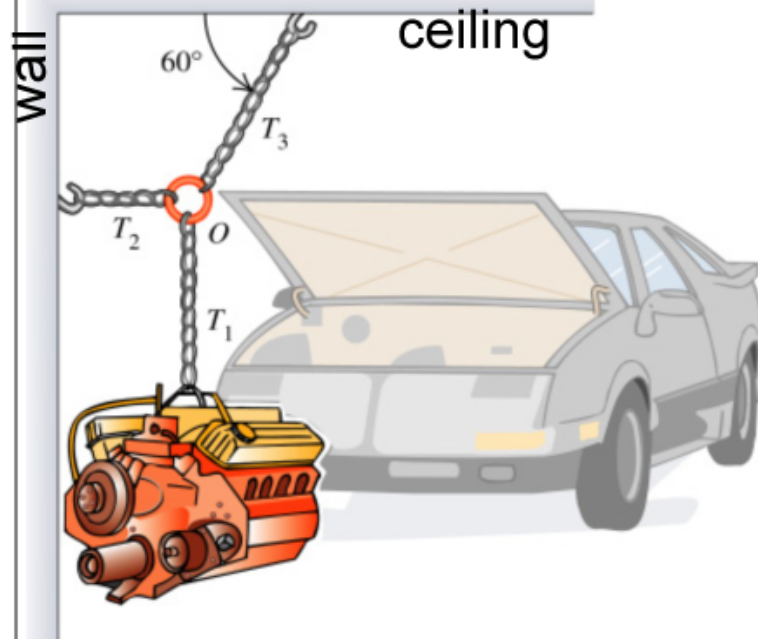
Find tension in each chain, assuming **massless** ring and chains

All bodies in equilibrium \Rightarrow can use 1st law to find T_1, T_2, T_3
Draw FB diagrams for bodies in equilibrium: Engine, O-ring



Horiz. & slanted chains
not attached to E
 \Rightarrow don't exert force on E
Engine in Equilibrium \Rightarrow
 $\sum F_y = T_1 - w = 0$
 $\Rightarrow \boxed{T_1 = w}$

Equilibrium in 2 Dimensions



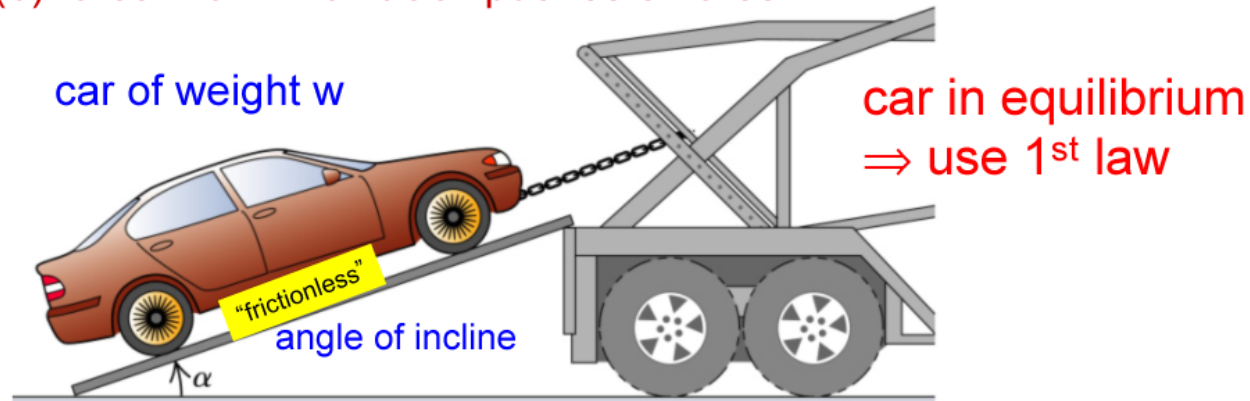
$$\sum F_x = T_3 \cos 60^\circ - T_2 = 0 \quad \& \quad \sum F_y = T_3 \sin 60^\circ - T_1 = 0$$

$$\text{Since } T_1 = w \Rightarrow T_3 = \frac{T_1}{\sin 60^\circ} = 1.155w; \quad T_2 = \frac{w \cos 60^\circ}{\sin 60^\circ}$$

How hard was that if setup correctly ?

Inclined Plane Dynamics

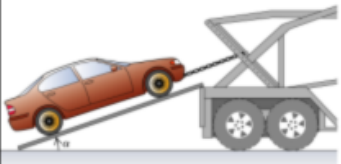
Car rests on tracks of ramp, is attached to truck by a chain. What is (a) tension on chain ?
(b) force with which track pushes on tires ?



ramp exerts force on each of 4 tires. Lump these into single force \perp to track & ignore forces \parallel to track (frictionless tracks)

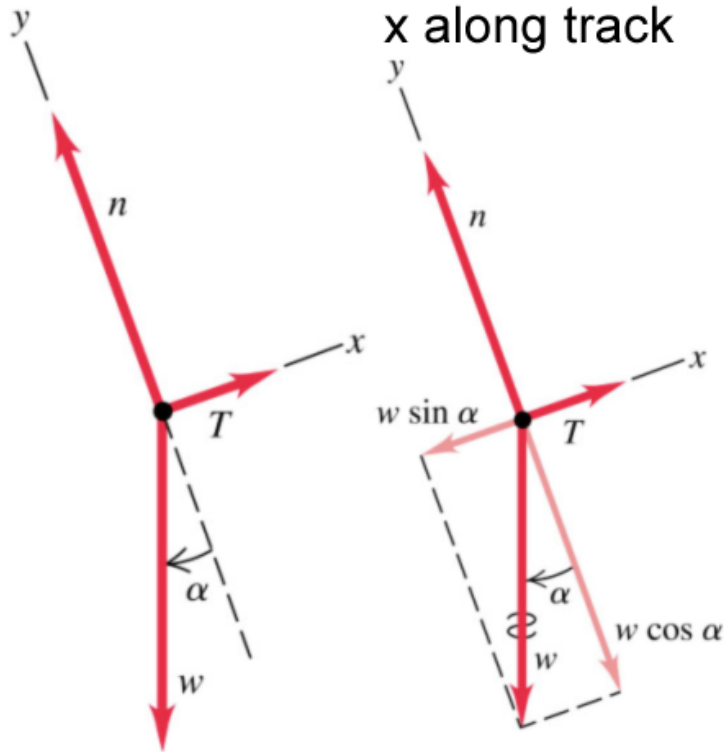
Different forces act in different direction. Set up X-Y coord. syst and write 1st law in each direction \Rightarrow 2 equations

We have 2 unknowns: Tension T and Normal force N on car



Free Body Diagram For Car

Choose coordinate system, with
x along track



Balance forces along x & y
car in equilibrium \Rightarrow 1st law

$$\sum F_x = T - w \sin \alpha = 0$$

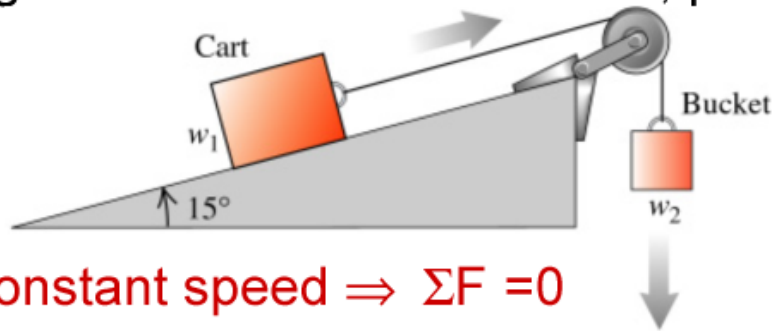
$$\sum F_y = n - w \cos \alpha = 0$$



Magnitude of $T = w \sin \alpha$
& Normal Force $n = w \cos \alpha$

Motion Along An Inclined Plane

Cart (w_1) being pulled up a inclined plane by a bucket (w_2). If both objects move with same constant speed, what is the relation between weights w_1 & w_2 ? Ignore friction on the surface, pulley & assume massless cable.



Draw
FB
diags

constant speed $\Rightarrow \Sigma F = 0$

The Cart in equilibrium

$$\Sigma F_x = T - w_1 \sin 15^\circ = 0$$

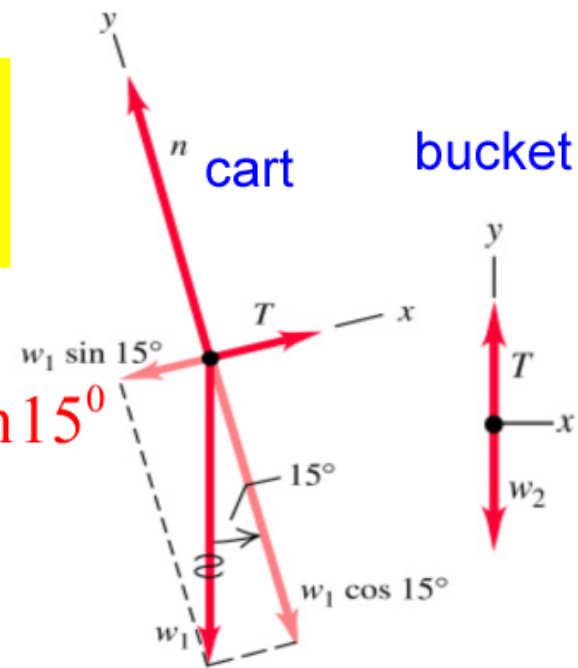
$$\Sigma F_y = n - w_1 \cos 15^\circ = 0$$

$$\Rightarrow w_2 = w_1 \sin 15^\circ$$

$$\text{or } \frac{w_2}{w_1} = 0.26$$

The Bucket in equilibrium

$$\Sigma F_y = 0 \Rightarrow T = w_2$$



Ice Sailing With 2nd Law !

Iceboat at rest. Steady wind pushes the sail. 4s after wind starts, boat's speed = 22km/h. What constant force F_W does wind exert on the sail? Mass of boat+sailor = 200kg. Ignore friction.

draw free-body diagram



$$\sum F_x \Rightarrow F_W = ma_x$$

$$\text{Speed } v_x = v_{x0} + a_x t$$

$$\Rightarrow a_x = \frac{v_x - v_{x0}}{t} = 1.5 \text{ m/s}^2$$

$$\Rightarrow F_W = ma_x = 300 \text{ N}$$

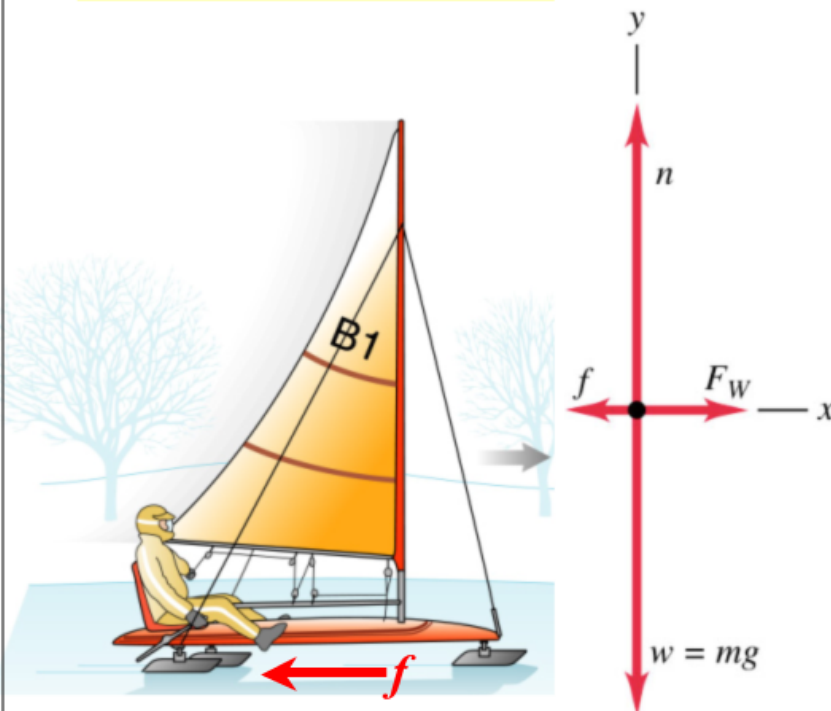
$$\sum F_y = 0 \Rightarrow n = mg$$

$$\Rightarrow \text{Normal } n = 1960 \text{ N}$$

Ice Sailing On Surface With Friction

Iceboat at rest. Steady wind pushes the sail. Boat accelerates at $a_x = 1.5 \text{ m/s}^2$ in presence of constant horizontal frictional force $f = 100 \text{ N}$. What is F_w here ?

draw the free-body diagram



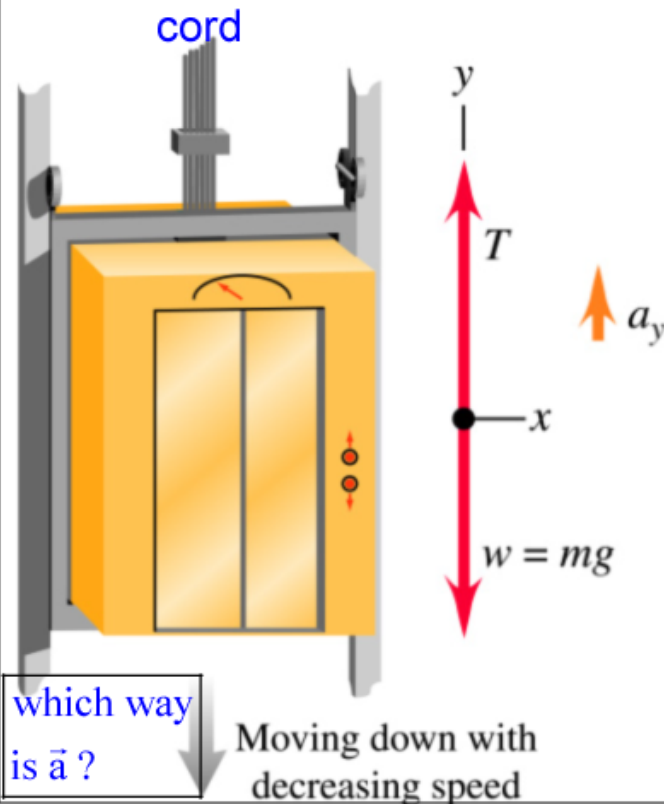
$$\sum F_x \Rightarrow F_w - f = ma_x$$

$$\Rightarrow F_w = ma_x + f$$

$$\begin{aligned} \Rightarrow F_w &= (200 \text{ kg})(1.5 \text{ m/s}^2) \\ &\quad + 100 \text{ N} \\ &= 400 \text{ N} \end{aligned}$$

Tension & Weight In An Elevator

Woman in elevator moving down with speed 10m/s comes to a stop after decelerating for 25m. If mass of (woman+elevator)=800kg, find the tension T in elevator cord as the elevator decelerates.



$$\text{Elevator: } \sum F_y \Rightarrow T - w = ma_y$$
$$\Rightarrow T = w + ma_y = (g + a_y)m$$

What is a_y ?

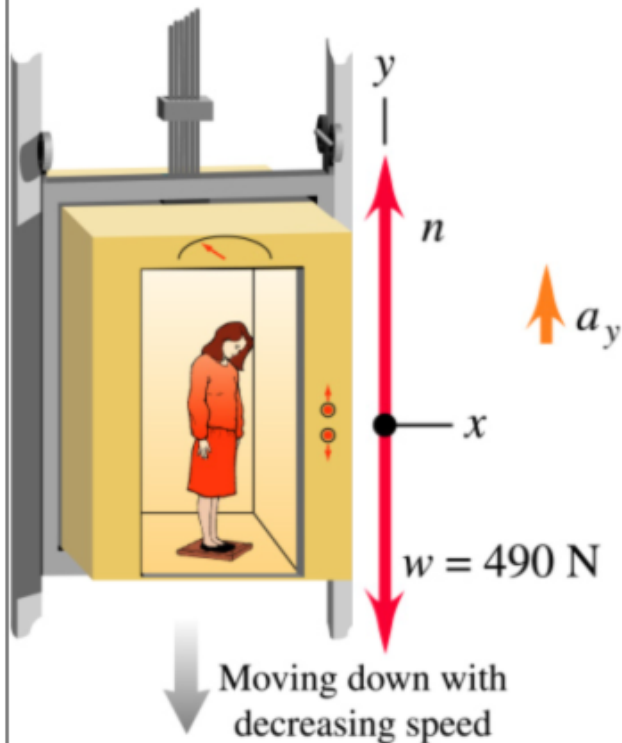
$$\text{Use } v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

$$\Rightarrow a_y = \frac{v_y^2 - v_{0y}^2}{2(y - y_0)} = +2\text{m/s}^2$$

$$\Rightarrow T = (g + a_y)m = 9440\text{N}$$

Apparent Weight In An Elevator

A 50.0 kg woman stands on a measuring scale while riding in the elevator. As it decelerates to a stop, what is her weight force ?



Woman: $m=50\text{kg}$

$$\sum F_y \Rightarrow n - w = ma_y$$

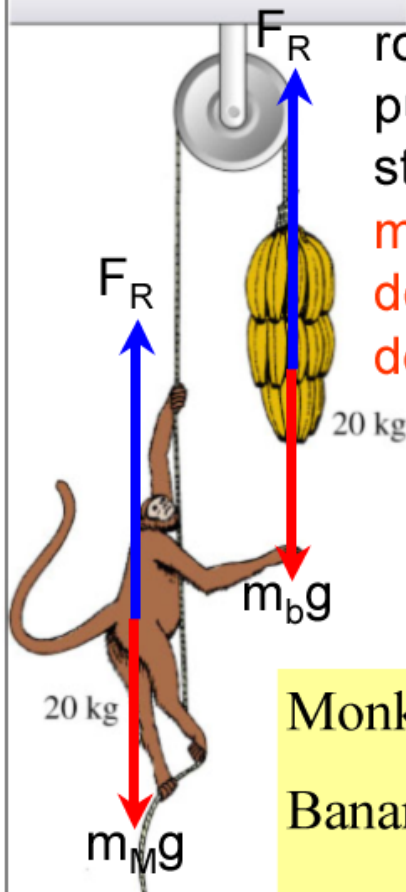
$$\Rightarrow n = m(g + a_y) = 590\text{N}$$

As elevator decelerates, scale pushes up on woman with $n=590\text{N}$. \Rightarrow Woman pushes down on scale with equal & oppo force

\Rightarrow her apparent weight on scale =590N

$\Rightarrow 100\text{N}$ more than her normal weight !!
This happens only during deceleration

Monkey (M) & Banana Bunch (B)



20kg monkey holds a massless rope that passes over frictionless pulley attached to 20kg banana bunch. Monkey starts to climb up rope to get it. (a) Do the bananas move up, down or stay at rest (b) as monkey climbs does the distance between M & B increase, decrease or stay same?

When monkey pulls rope with force F_M , rope pulls on M. with equal & opposite force $F_R = -F_M$. Massless rope \Rightarrow Force of rope on Monkey = $F_R = -F_M$

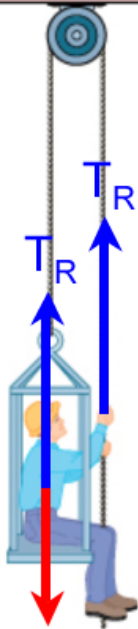
$$\text{Monkey: } \sum F_y = F_R - m_M g = m_M a_M$$

$$\text{Bananas: } \sum F_y = F_R - m_B g = m_B a_B$$

$$\text{Since } m_M = m_B \Rightarrow \boxed{a_M = a_B}$$

Both accelerate upwards when monkey pulls!

Can You Raise Yourself ?



Man(M) dangles a massless rope (R) over a massless, frictionless pulley and pulls rope. (Man+chair)'s mass=95kg. **With what force must the man pull rope if he wants to rise up with constant velocity?**

Man pulls rope up with force $F_M \Rightarrow$ Rope pulls back

Rope tension $T_R = -F_M$ since R & P massless & frictionless

Total upward force on Man+chair due to its 2 contact points with rope is $T_R + T_R$. Apply 2nd law on Man \Rightarrow

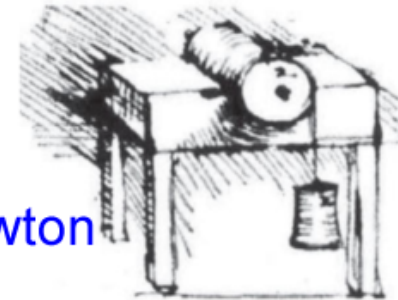
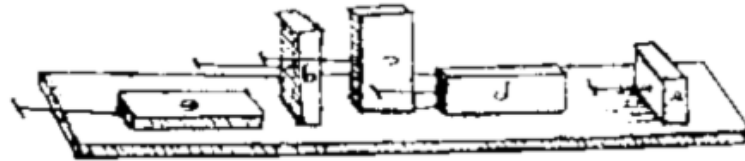
$(m_M + m_C)g$

$\Rightarrow 2T_R - m_{M+C}g = m_{M+C}a$. When $a = 0$ (const velocity)

$$\Rightarrow \boxed{T_R = -F_M = \frac{1}{2} m_{M+C}g} = \left(\frac{95\text{kg}}{2}\right)(9.80\text{m/s}^2) = 466\text{N}$$

Frictional Forces

First Quantitative study by Leonardo da Vinci (1452-1519).
~200 years before Newton's studies of dynamics



200 years before Force was even *defined* by Newton
da Vinci observed:

area in contact has no effect on friction

if load on an object is doubled, its friction is also doubled

Scientists are still studying the initial fleeting instants of sliding
to understand e.g. **nanomachines** & **seismic signals in earthquake**

Quantitative Properties of Friction

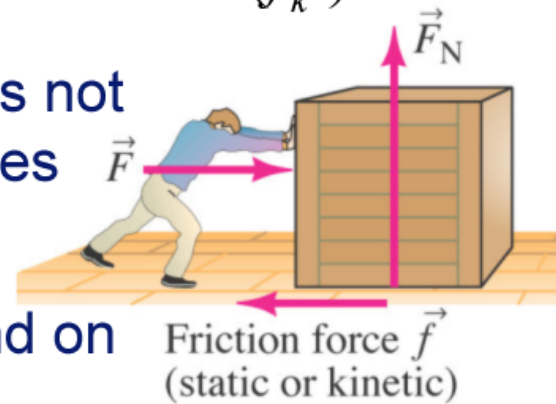
Two kind of friction: Static (\vec{f}_s) and kinetic (\vec{f}_k)

Static friction applies when object is not moving along surface; kinetic applies when object is moving

Expt \Rightarrow Both types of friction depend on the normal force on the object

$$\vec{f} \perp \vec{n} \ \& \ \vec{f} \propto \vec{n}$$

\vec{f} always acts to oppose relative motion of surfaces



Properties Of Friction

Expt \Rightarrow When a force \vec{F} is applied to a body, the resulting frictional force has three properties:

1. If the body does not move, \vec{f}_s & component of \vec{F} \parallel to surface balance each other: $\vec{f}_s = -\vec{F}_{\parallel}$

2. $|\vec{f}_s|$ has a maximum value $f_{s,max}$ $f_{s,max} = \mu_s F_n$

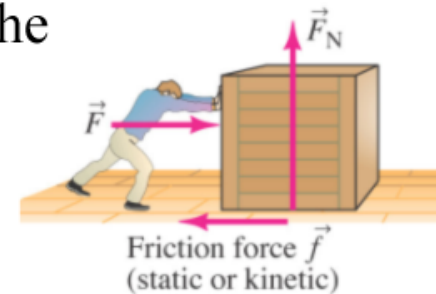
μ_s = coefficient of Static friction, F_n = normal force on surface

If $|\vec{F}_{\parallel}|$ exceeds $f_{s,max}$ then body starts sliding along surface

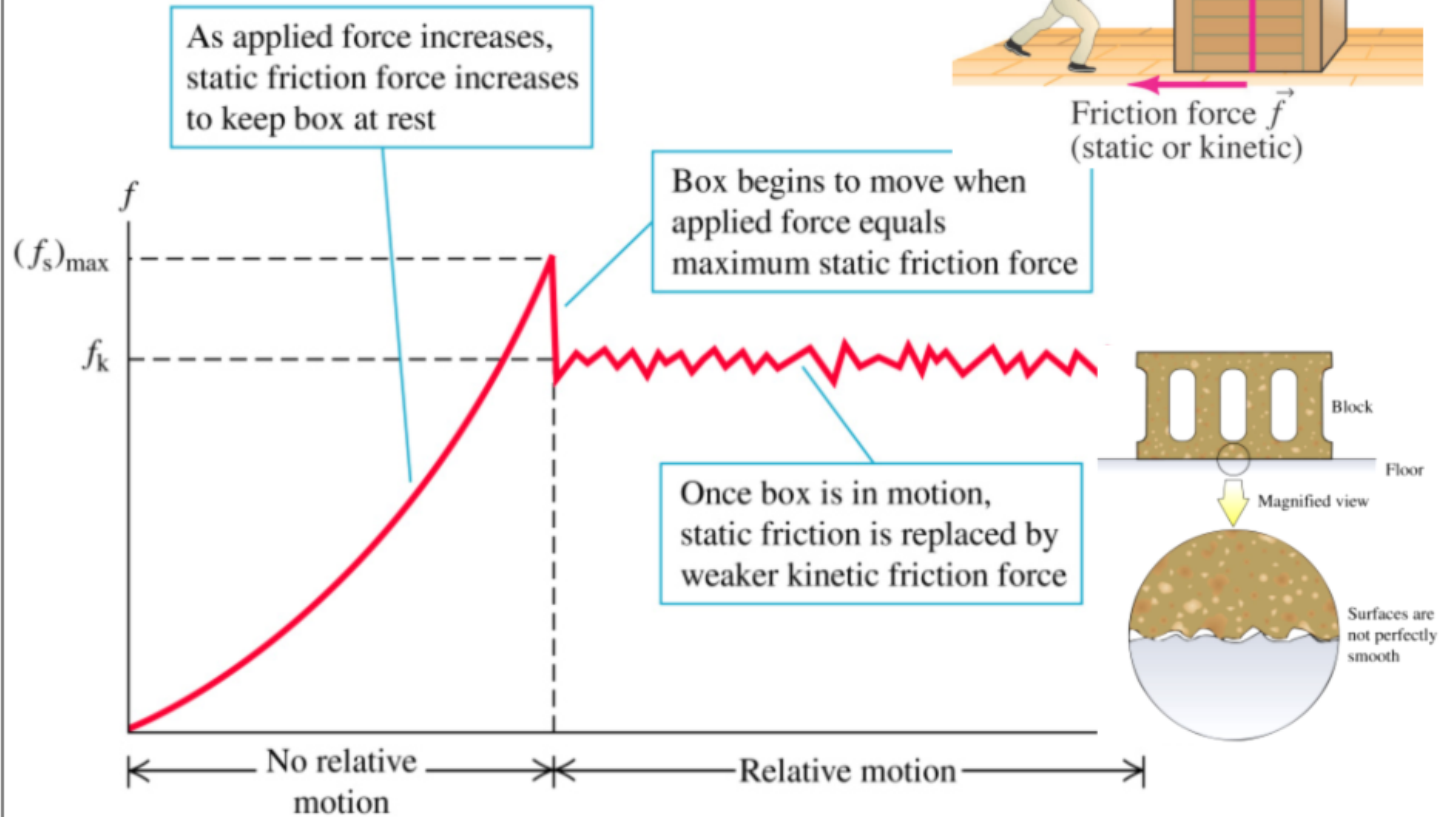
3. When body starts sliding, friction drops to value f_k

$f_k = \mu_k F_n$ μ_k = coefficient of Kinetic friction

F_n is a measure of how firmly body presses against surface

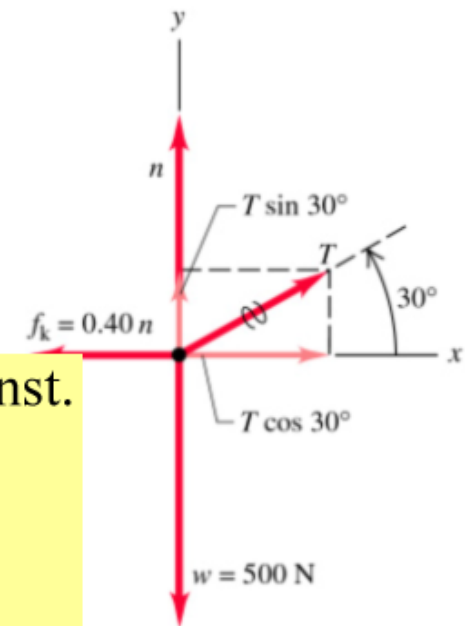
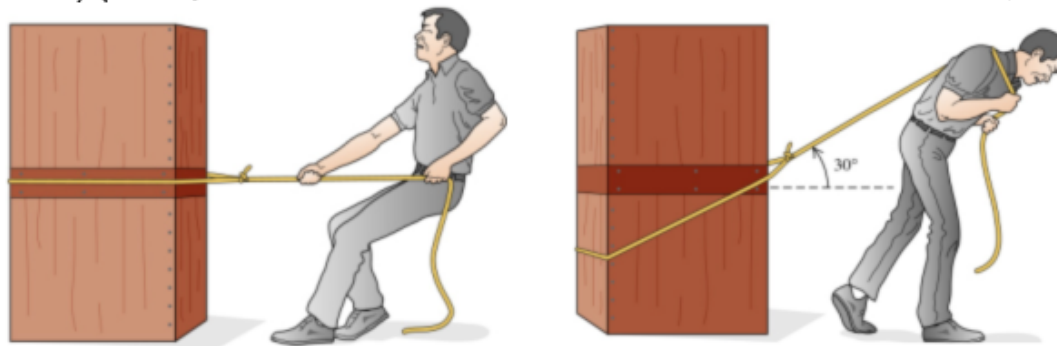


Friction Pictorially



Getting An Angle On Kinetic Friction

Is it harder or easier to keep box moving by pulling it at an angle $\theta = 30^\circ$ above horizontal? \Rightarrow What is T now?



System in equilibrium since box moving with $v = \text{const.}$

$$\sum F_x = T \cos \theta - f_k = 0 \Rightarrow T \cos \theta = \mu_k n$$

$$\sum F_y = T \sin \theta + n - w = 0 \Rightarrow n = w - T \sin \theta$$

Solve for T and n : Start by substituting for n in top eqn.

$$\Rightarrow T = \frac{\mu_k w}{\cos \theta + \mu_k \sin \theta}; \text{ for } \theta = 30^\circ \Rightarrow T = 188 \text{ N} < 200 \text{ N}$$

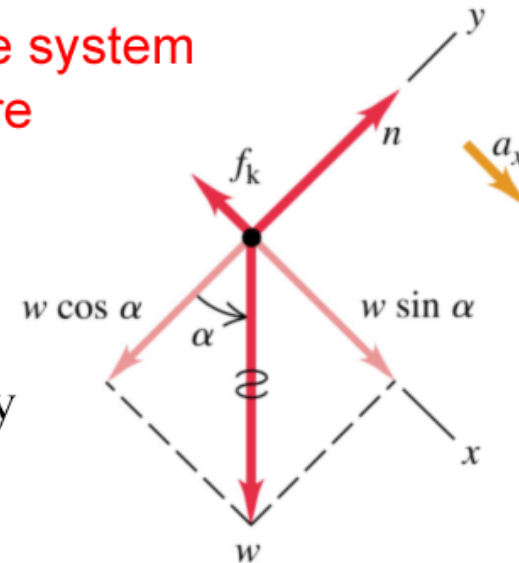
angled is better since n is smaller!

Sledding in the winter Wonderland

Sleigh weighing w accelerates down a snow slope (angle α w.r.t horizontal).
 μ_k is coeff. of kinetic friction. Express acceleration a in terms of g, w, α & μ_k



pick coordinate system
with care



Sleigh accelerates \Rightarrow apply 2nd law along x, y

$$\sum F_x = mg \sin \alpha - f_k = ma_x; \text{ with } f_k = \mu_k n$$

$$\sum F_y = n - mg \cos \alpha = 0 \Rightarrow n = mg \cos \alpha,$$

$$\Rightarrow f_k = \mu_k n = \mu_k mg \cos \alpha$$

substituting in expression for $\sum F_x \Rightarrow$

$$mg \sin \alpha - \mu_k mg \cos \alpha = ma_x \Rightarrow a_x = g(\sin \alpha - \mu_k \cos \alpha)$$

Quick question

- Which one of these statements is NOT true:
- (A) It is easier to keep a block on a horizontal surface moving than to get it to move.
- (B) A heavier object will start to slide down a slippery slope before a lighter one
- (C) Friction will exactly cancel a horizontal force trying to slide a body on a horizontal surface until it exceeds μW (W = weight of body, μ = coeff of static friction)

Drag Force !

When body falls fast enough through the fluid to cause turbulence then resistive DRAG force $f \propto v^2$: $f = Dv^2$

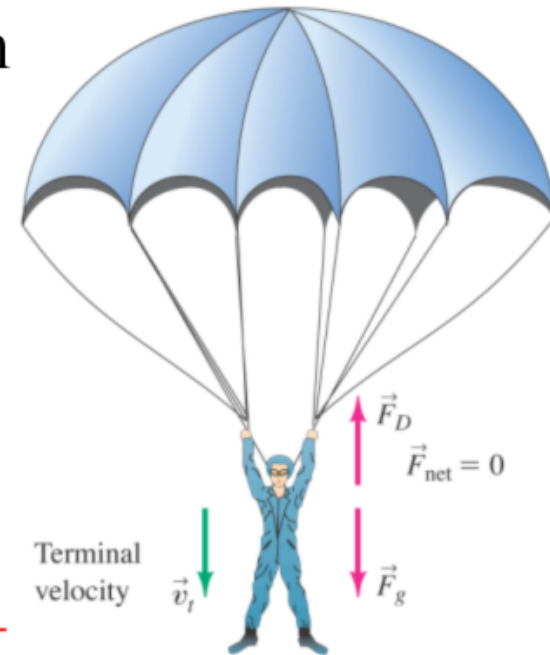
$D = \text{constant}$, depends on shape & size of the body and properties of fluid

Drag force rises to negate weight force

Ultimately: $\sum F_y = mg - Dv_t^2 = 0 \Rightarrow v_t = \sqrt{\frac{mg}{D}}$

This is why sheet of paper falls slower than if it were crumpled!

Steel sphere, radius R , falls faster in air than a plastic ball of same radius. Same D , different $m \Rightarrow$ different v_t



Terminal Velocity & Sky Diving !

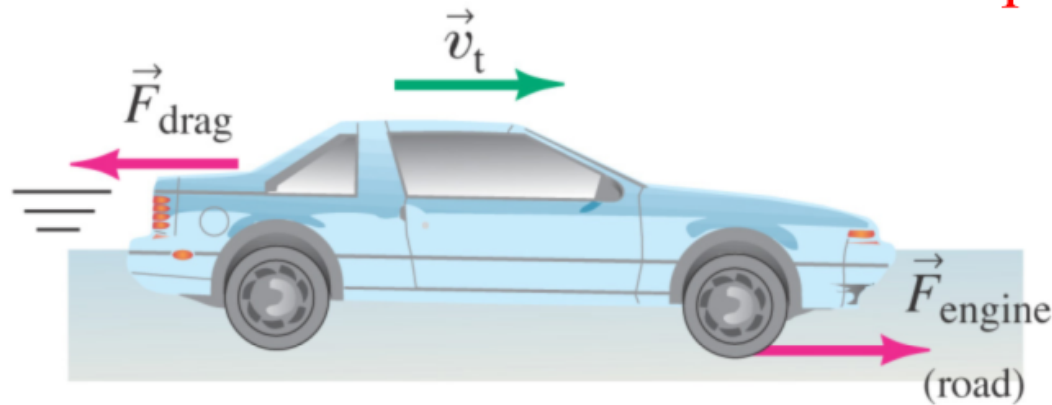


By changing position of arm/legs skydivers can change D and thus adjust terminal speed of their fall

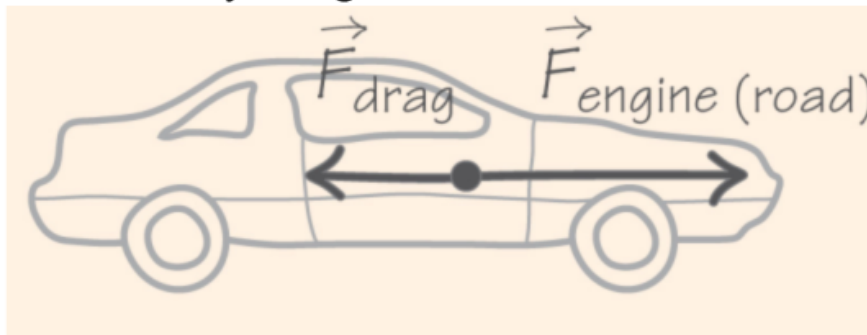
Dropped separately, \rightarrow these divers have to change their v_t in order in order to arrive at same location at same time and link up !!



$f = Dv^2 \Rightarrow$ Drag force rises quickly ! Engine has to do more work with increased $v \Rightarrow$ larger gas bill \Rightarrow One of reasons behind recommended speed limit



Free body diagram

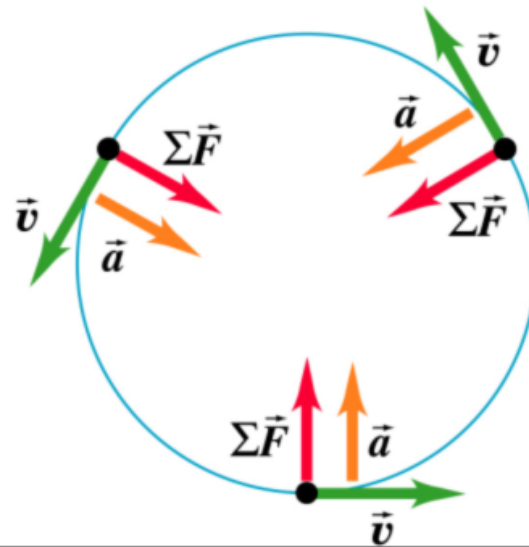


Force In Uniform Circular Motion

Object of mass m in uniform circular motion of radius r

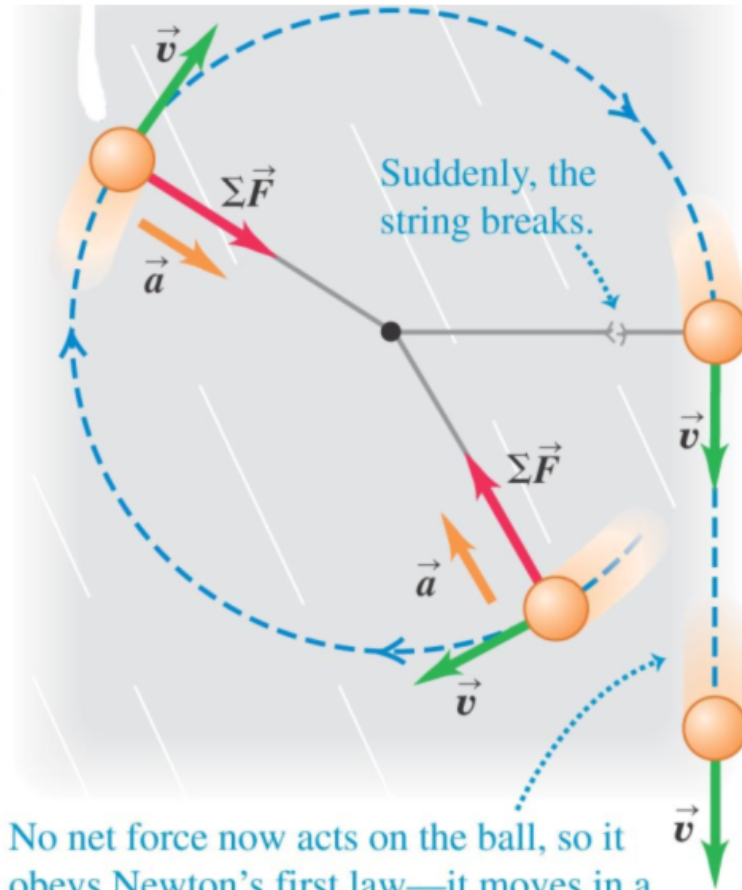
experiences centripetal acceleration: $a_r = \frac{v^2}{r}$

$$\text{2nd law : } \vec{a}_r = \frac{\vec{F}}{m} \Rightarrow F = m \frac{v^2}{r}, \text{ directed along } \vec{a}_r$$



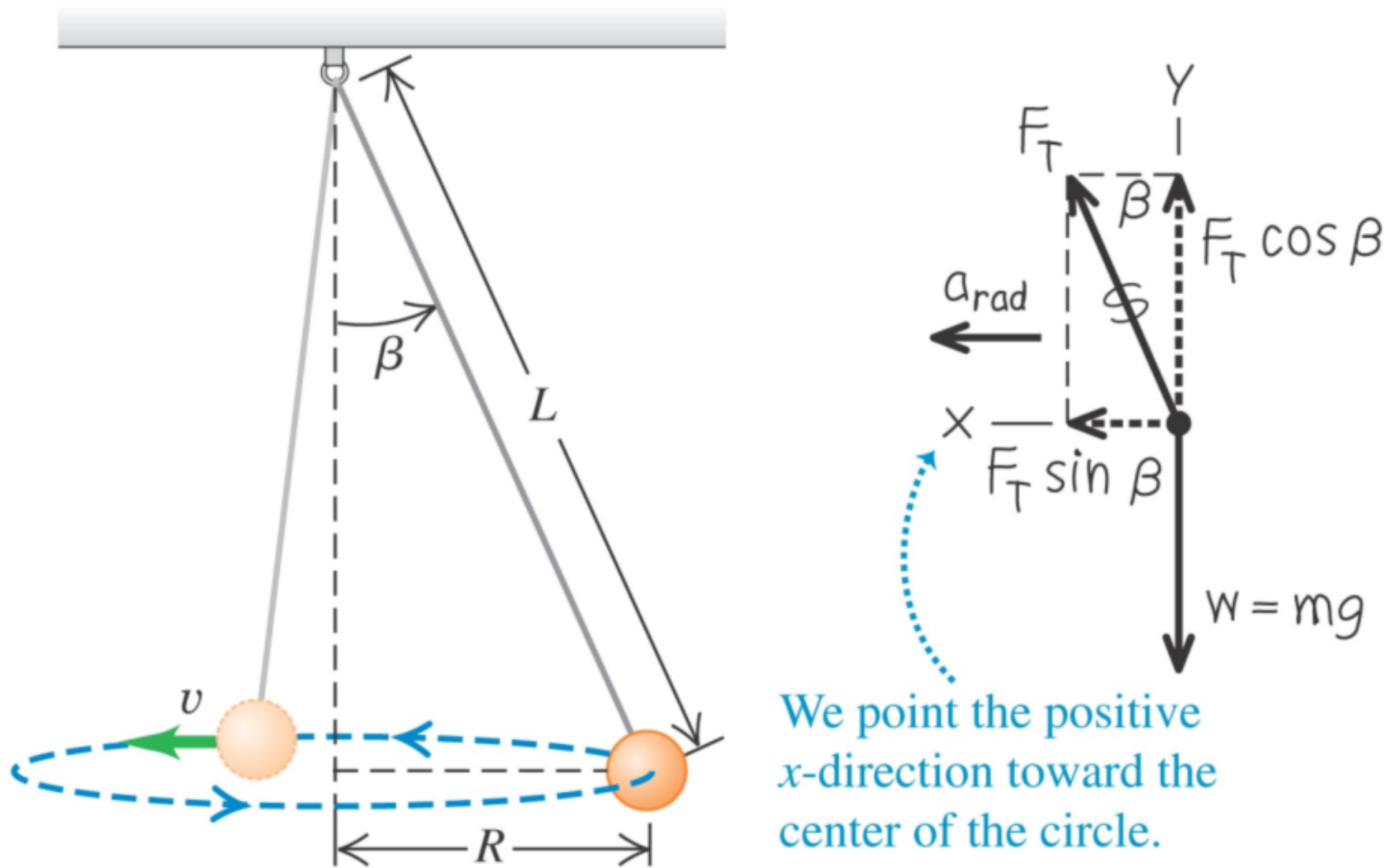
Understanding Circular Motion

A ball attached to a string whirls in a circle on a frictionless surface.



No net force now acts on the ball, so it obeys Newton's first law—it moves in a straight line at constant velocity.

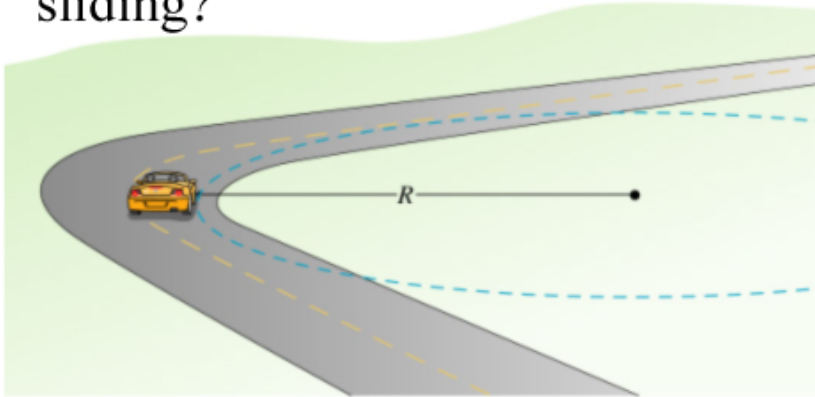
Conical Pendulum



We point the positive x -direction toward the center of the circle.

Circular Motion: Car Rounding A Flat Curve

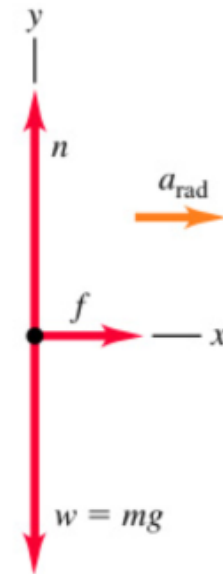
Car is rounding a flat, **unbanked** curve with radius R . If coeff. of static friction between road/tire is μ_s , what is the max. speed v_{\max} , that car can take the curve without sliding?



$$a_{\text{rad}} = \frac{v^2}{R}$$

$$v_{\max} \Rightarrow a_{\text{rad}/\max}$$

Only horizontal force is due to friction f by road



Frictional force must point towards center of \odot to cause a_{rad}

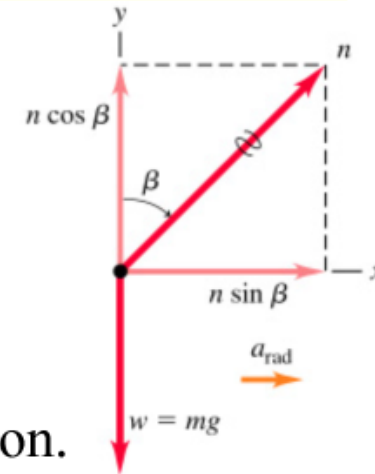
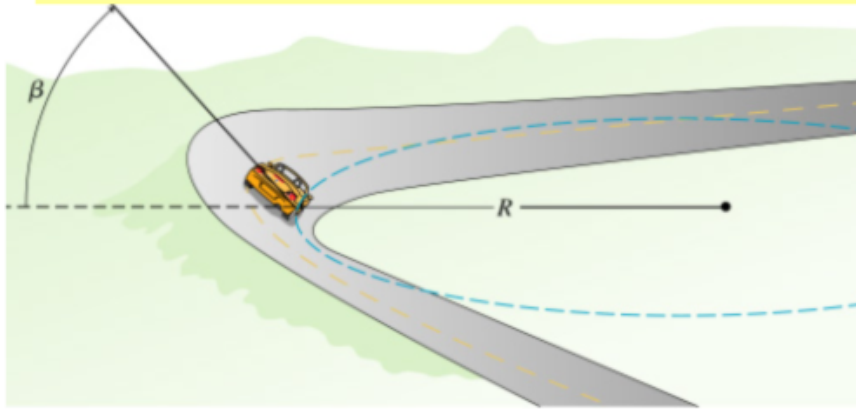
If the car **barely does not slip** radially, f must be **static friction**

Max. static friction available: $f_{\max} = \mu_s n = \mu_s mg \Rightarrow \mu_s mg = \frac{mv_{\max}^2}{R}$

$$\Rightarrow v_{\max} = \sqrt{\mu_s g R}$$

Rounding A Banked Curve Without Friction

Car on icy road \Rightarrow "No" friction. Road is banked at angle β w.r.t horizontal. What is β such that car with speed v can safely bank without sliding off?



Horizontal component of \vec{n} causes the circular motion.

$$\sum F_x = n \sin \beta = m a_{\text{rad}} ; \sum F_y = n \cos \beta - mg = 0 \Rightarrow n = mg / \cos \beta$$

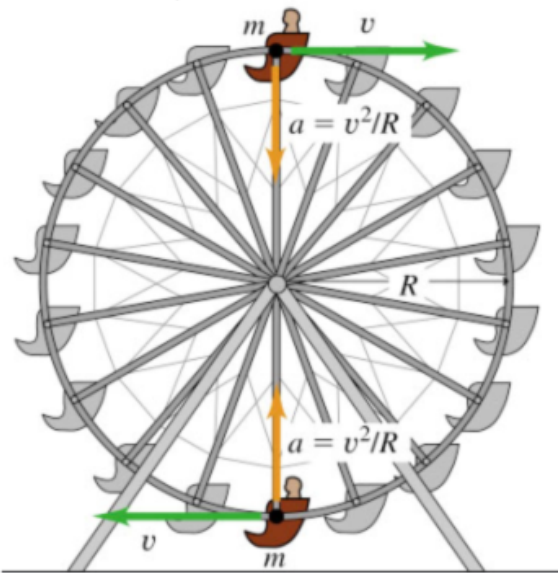
Substitute in 1st eq. $\Rightarrow \tan \beta = \frac{a_{\text{rad}}}{g}$

But $a_{\text{rad}} = \frac{v^2}{R} \Rightarrow \boxed{\tan \beta = \frac{v^2}{gR}} \leftarrow \text{note dependence on } v \text{ \& } R$

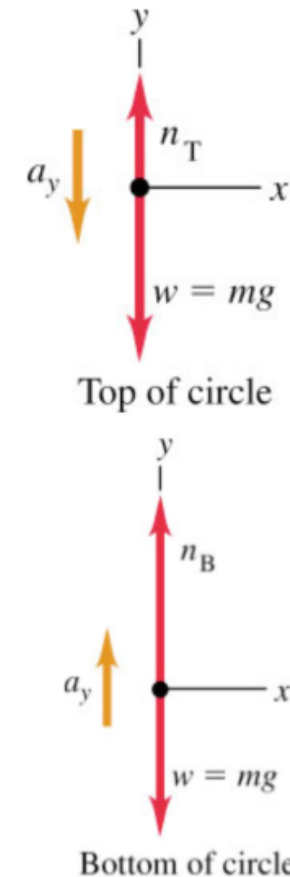
Hence the speed limit posted on banked curves

Uniform Circular Motion in Vertical Circle

Man on a ferris wheel. Wheel moves in vertical circle of rad. R with constant speed v .
 If seat stays vertical, what's the force seat exerts on man at the top/bottom of circle?



Force by seat on man is the **normal force n_T**
 Look at free body diag and apply 2nd law for circular motion
In chosen coord. system, upwards is +y



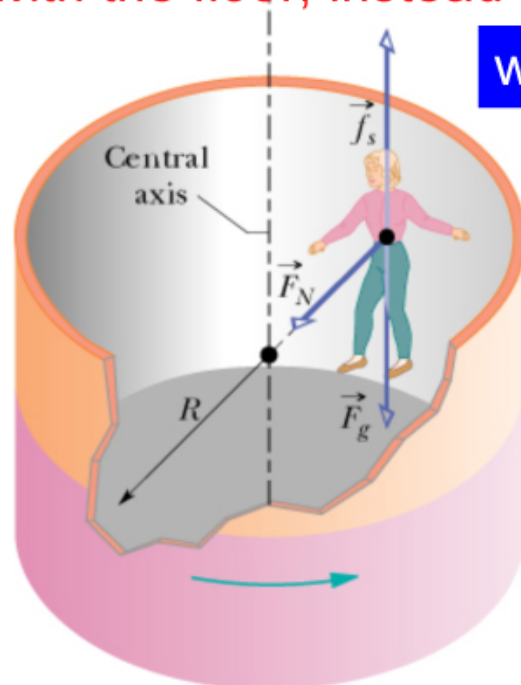
$$\text{Top: } \sum F_y = n_T - mg = -\frac{mv^2}{R} \Rightarrow n_T = m\left(g - \frac{v^2}{R}\right)$$

$$\text{Bottom: } \sum F_b = n_b - mg = +\frac{mv^2}{R} \Rightarrow n_b = m\left(g + \frac{v^2}{R}\right)$$

Rotor Of Death ?

Rotor: hollow cylinder that rotates rapidly around its central axis
Before ride begins, man enters cylinder and stands on a floor up against cylinder wall. Door is then closed and rotor starts spinning. Man, wall and floor spin in unison.

At some rotor speed, the floor falls away ! But Man does not fall with the floor, instead is pinned to wall !!



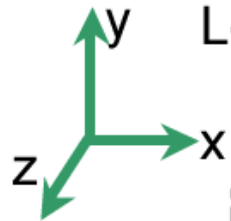
why does Man not fall ? What's the Miracle ?

friction !

Suppose $\mu_s = 0.4$ between man & wall and cylinder radius $R = 2.1\text{m}$.

What minimum speed v must cylinder/man have if man is to **not start falling down when the floor drops**

Rotor Of Death ! (not)



Look at Man's free-body diagram along y and radially (z)

Apply 2nd law on Man:

$$\sum \vec{F}_y : 0 = f_s - w \Rightarrow \mu_s n - w = 0 \Rightarrow \boxed{n = mg / \mu_s}$$

$$\sum \vec{F}_z : \boxed{n = mv^2 / r}; \text{ substitute for n in above eqn.}$$

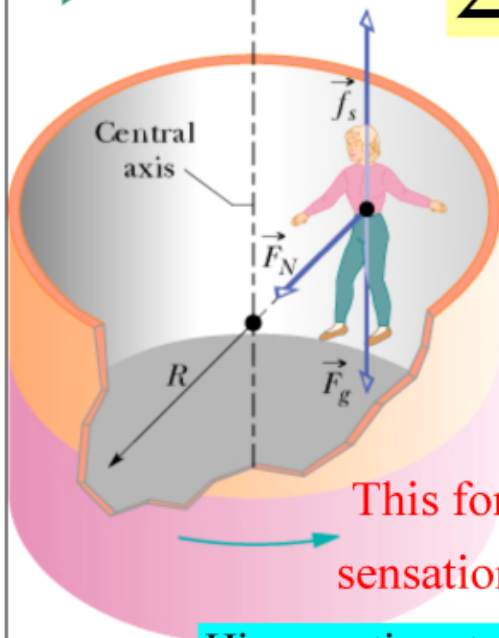
$$\Rightarrow v = \sqrt{\frac{gR}{\mu_s}} = \sqrt{\frac{(9.8m/s^2)(2.1m)}{0.40}} = 7.2m/s$$

Note: v is independent of mass of man

$$\text{Centripetal Force on Man} = n = \frac{mv^2}{R} = 1200N$$

This force is directed towards central axis, yet man has a sensation that the force directed against him is radially outwards!

His sensation stems from fact that he is in an **accelerated/non-inertial** frame of reference. In such Frame of reference, forces can be illusory. This illusion is main part of the Rotor's attraction !!



Question 1

- Of everything covered in this course so far, I have the most trouble with:
- (A) Vectors
- (B) projectiles
- (C) Ropes, pulleys and sliding blocks
- (D) Circular motion

Question 2

- I find that the material in this course is being covered;
- (A) Too fast
- (B) Too slowly
- (C) At about the right speed.