Physics 4A
Lecture 3: Jan. 13, 2015

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UCSD Physics

Equations needed to solve problems involving constant acceleration

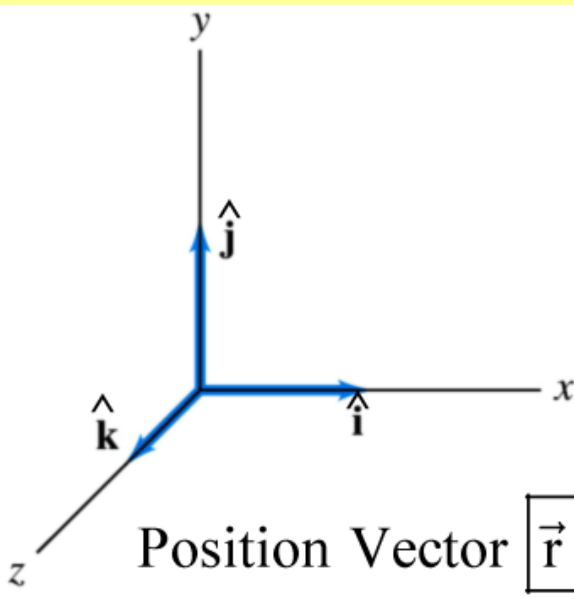
(1) $$v = v_0 + at$$

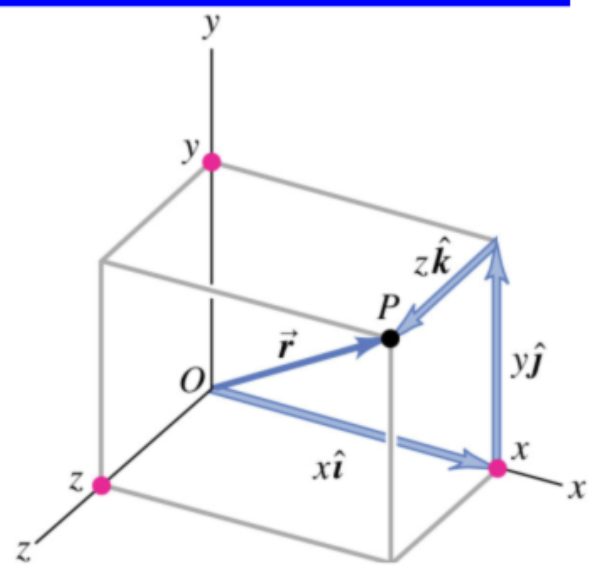
(2) $$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

(3) $$v^2 = v_0^2 + 2a(x - x_0)$$

Generalizing Motion From 1D → 3D

Cartesian coordinate system in 3D





Position Vector $\boxed{\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}}$

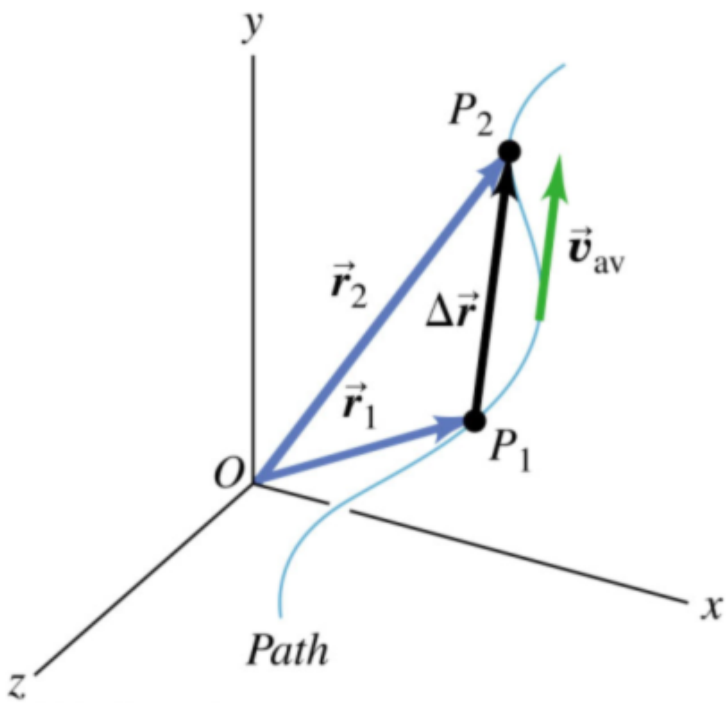
Path of particle moving in 3D space is a curve

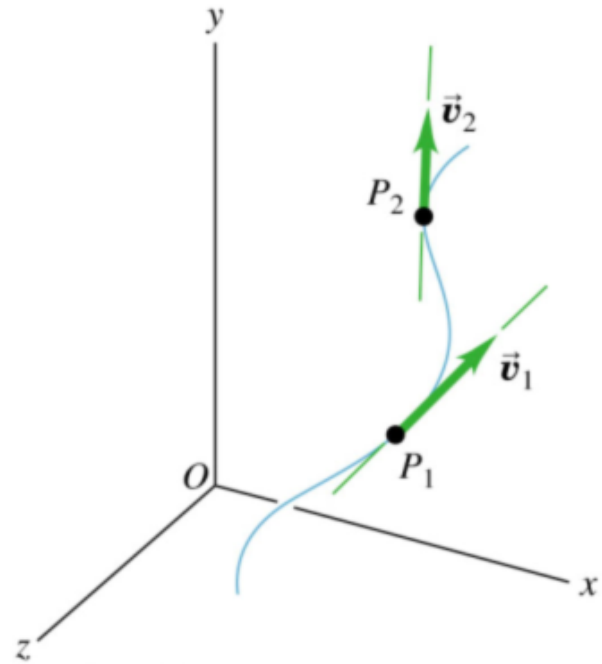
When particle moves from $P_1(x_1,y_1,z_1) \rightarrow P_2(x_2,y_2,z_2)$

displacement $\Delta\vec{r} = (x_2\text{-}x_1)\,\hat{i} + (y_2\text{-}y_1)\,\hat{j} + (z_2\text{-}z_1)\,\hat{k}$

$$= \Delta x\,\hat{i} + \Delta y\,\hat{j} + \Delta z\,\hat{k}$$

Velocity in 3 Dimensions





Average Velocity Vector

$$\vec{V}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t}$$

Instant. Velocity Vector

$$\vec{V} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

at every point along the path,vector

$\vec{V}$ is tangent to the path at that point

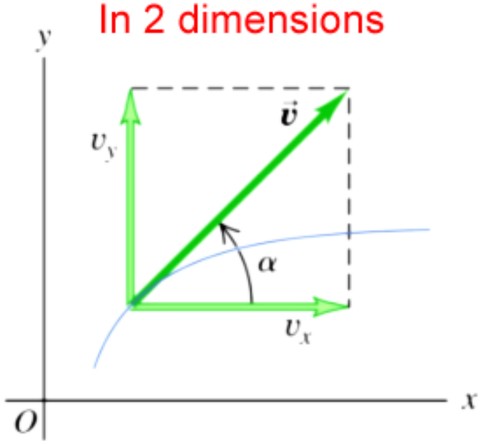
Components of Velocity Vector

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\,\hat{i} + \frac{dy}{dt}\,\hat{j} + \frac{dz}{dt}\,\hat{k}$$

$$= v_x\,\hat{i} + v_y\,\hat{j} + v_z\,\hat{k}$$

$$\text{Speed} = v = |\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

In 2 dimensions



$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\,\hat{i} + \frac{dy}{dt}\,\hat{j} = v_x\,\hat{i} + v_y\,\hat{j}$$

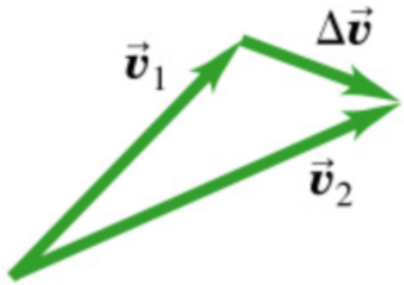
$$\text{Speed} = v = |\vec{v}| = \sqrt{v_x^2 + v_y^2}$$

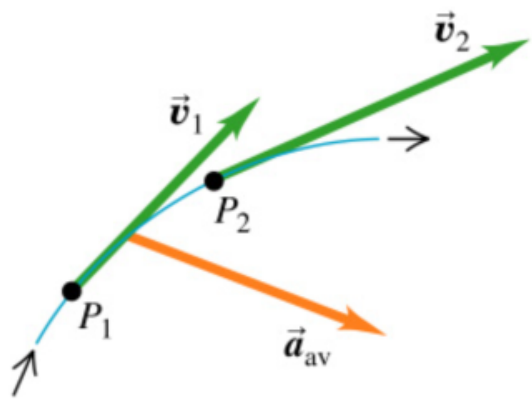
$$\tan\alpha = \frac{v_y}{v_x}$$

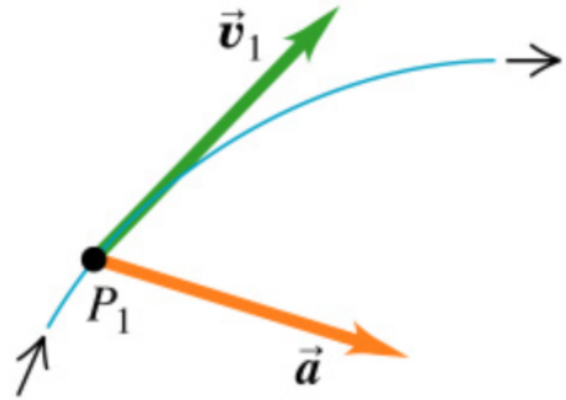
Acceleration In 3 Dimension

Vector subtraction

$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$







Average Acceleration

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t}$$

Instantaneous Acceleration

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

$\vec{a}$ always points towards the *concave* side of the curved path

When moving in a curved path

$\vec{a} \neq 0$ even if speed is constant

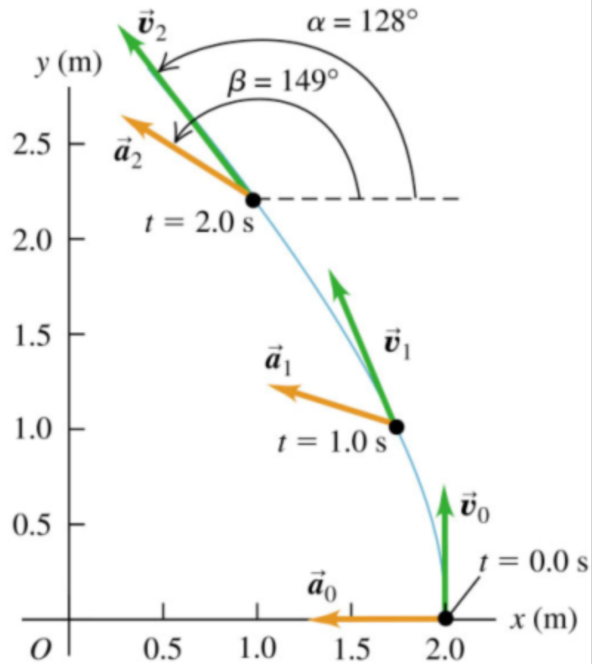
Components of Acceleration Vector

$$\vec{a} = \frac{dv_x}{dt}\,\hat{i} + \frac{dv_y}{dt}\,\hat{j} + \frac{dv_z}{dt}\,\hat{k}$$

$$= \frac{d^2x}{dt^2}\,\hat{i} + \frac{d^2y}{dt^2}\,\hat{j} + \frac{d^2z}{dt^2}\,\hat{k}$$

$$= a_x\,\hat{i} + a_y\,\hat{j} + a_z\,\hat{k}$$

When computing components use the **correct** def. of angle



3D motion with constant acceleration

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

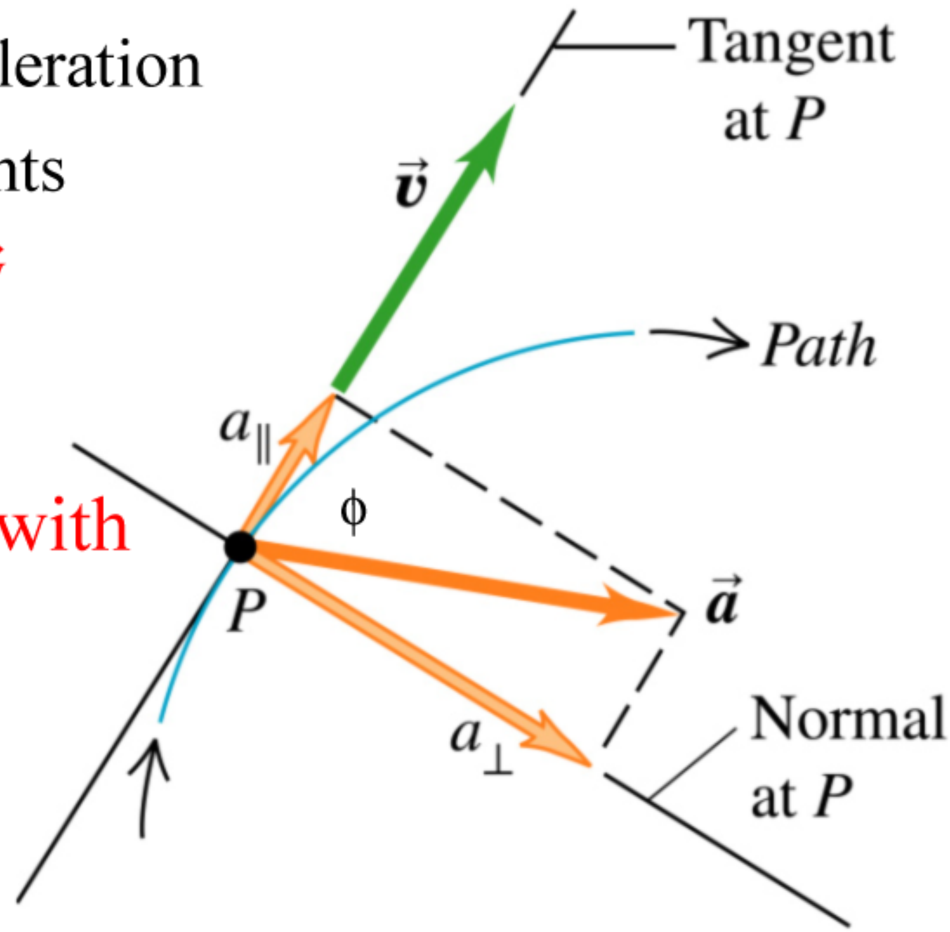
$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

$$v^2 = v_0^2 + 2\vec{a}.(\vec{r} - \vec{r}_0)$$

$$\vec{r} = \vec{r}_0 + \frac{(\vec{v}_0 + \vec{v}_f)}{2}t$$

|| and ⊥ components of Acceleration $\vec{a}$

When moving in curved path useful to describe acceleration $\vec{a}$ in terms of components which are || & ⊥ to $\vec{v}$

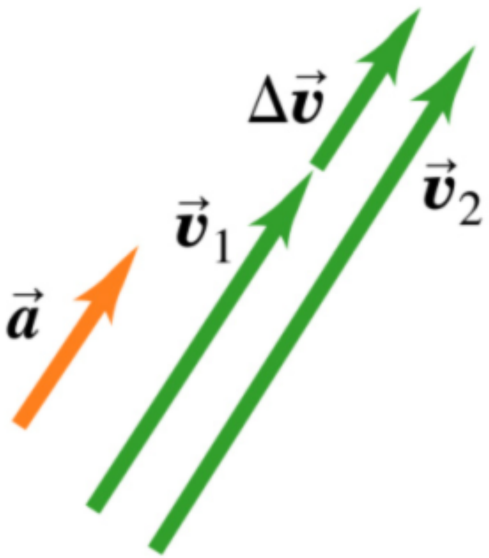


Write $\vec{a} = \vec{a}_{\parallel} + \vec{a}_{\perp}$ with

$$\vec{a}_{\parallel} = |a| \cos\varphi$$

$$\vec{a}_{\perp} = |a| \sin\varphi$$

$\parallel$ and $\perp$ components of Vector $\vec{a}$

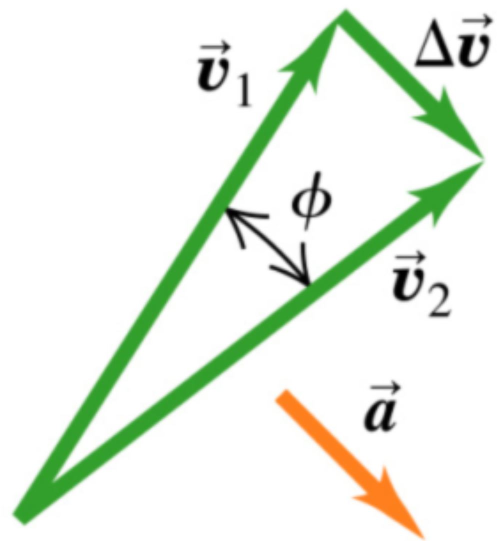


When $\vec{a} \parallel \vec{v}$ or anti-$\parallel$

vector addition $\Rightarrow$

change in magnitude of $\vec{v}$

but not its direction



When $\vec{a} \perp \vec{v}$

vector addition $\Rightarrow$

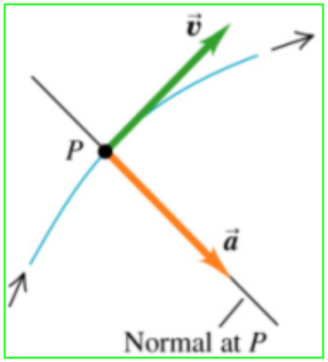
change the direction of $\vec{v}$

but not its magnitude

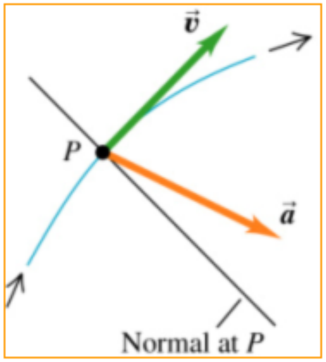
(speed remains unchanged!)

Some Scenarios

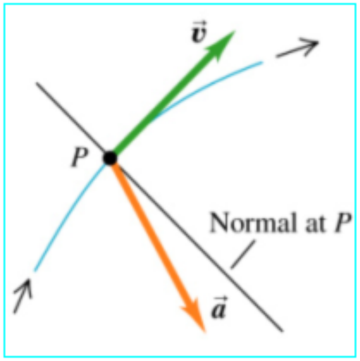
When particle travels along curved path with constant speed, $\vec{a}$ is $\perp$ to the path & $\perp$ to $\vec{v}$



When particle travels along curved path with increasing speed, $\vec{a}$ has components $\perp$ & $\parallel$ to $\vec{v}$ & points ahead of the *normal* to the path



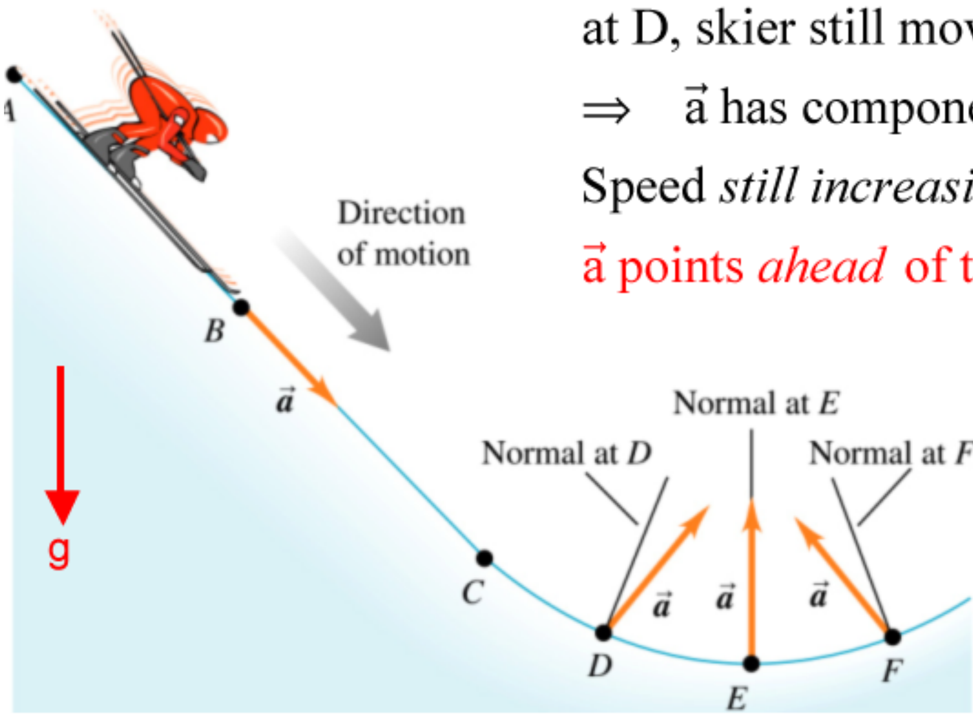
When particle travels along curved path with decreasing speed, $\vec{a}$ has components $\perp$ & anti-$\parallel$ to $\vec{v}$ & points behind the *normal* to the path



Pop quiz:

- Under a constant acceleration **a** in 3 Dimensions, with initial velocity **Vo** the particle speed after a time t is given by
- (A) Vo + at
- (B) the square root of (Vo^2 + 2.a. distance travelled)
- (C)the square root of [(Vo$_x$ +a$_x$t)^2+(Vo$_y$ + a$_y$t)^2 +(Vo$_z$+a$_z$t)^2]
- (D) Vo + (a$_x$ +a$_y$+a$_z$)t

Skiing: Curved Path In Snow !



at D, skier still moving along curved path:

$\Rightarrow$ $\vec{a}$ has component $\perp$ to path

Speed *still increasing* $\Rightarrow$ $\vec{a}$ has component $\parallel$ to path

$\vec{a}$ points *ahead* of the normal to her path at point D

at point E, velocity is max.

$$\vec{a}_{\parallel} = \frac{d\vec{v}_{\parallel}}{dt} = 0$$

$\vec{a} = \vec{a}_{\perp}$ and points towards normal

at point F, skier moving along curved path:

$\Rightarrow$ $\vec{a}$ has component $\perp$ to path

Speed now *decreasing* $\Rightarrow$ $\vec{a}$ has component anti-$\parallel$ to path

$\vec{a}$ points *behind* the normal to her path at point F


Relative Velocity makes mid-air refueling possible !

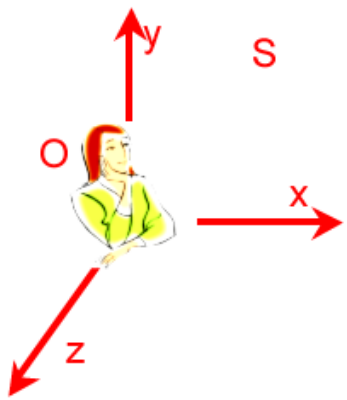
Relative Velocity

Blue Angel pilots must keep track of their velocity w.r.t air so as to maintain enough airflow over their wings to sustain the “lift” & not crash

They must also be aware of relative velocity of their aircraft w.r.t another !

Frames of Reference, Observers & Motion

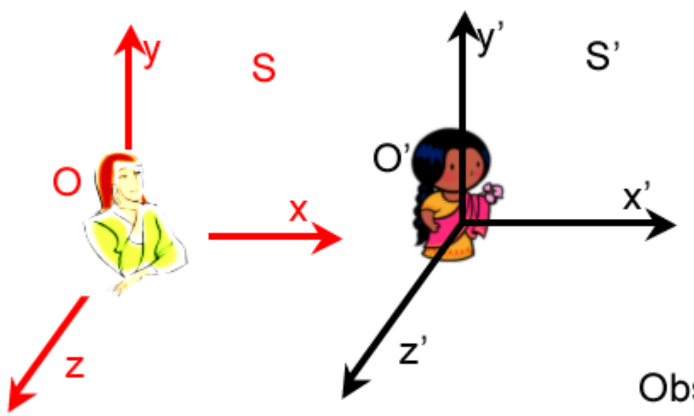
Event: Some thing happening, some where at some time



Frame of reference S = a coordinate system + clock

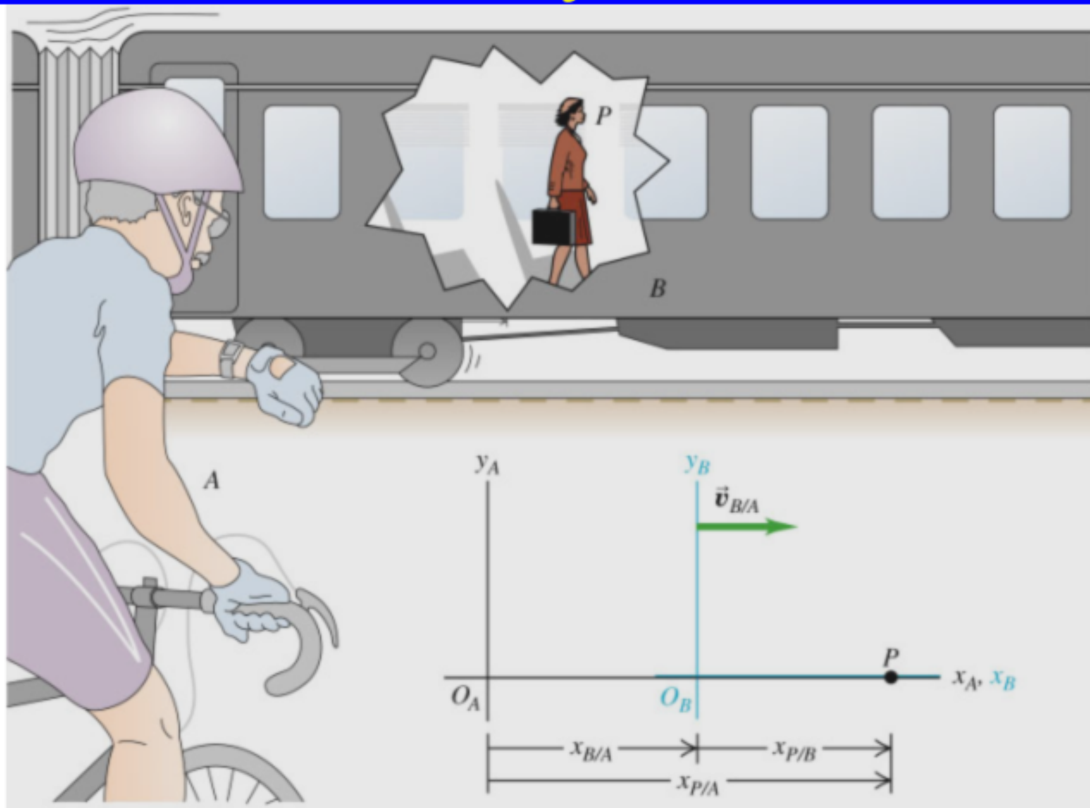
Observer O : sits in S, measures events with ruler, clock

Observers in diff frames of refs, depending on relative location may measure different positions for an event but measure same time. Their clocks are synchronized !



Observers can move w.r.t each other

Relative Velocity in 1 Dimension



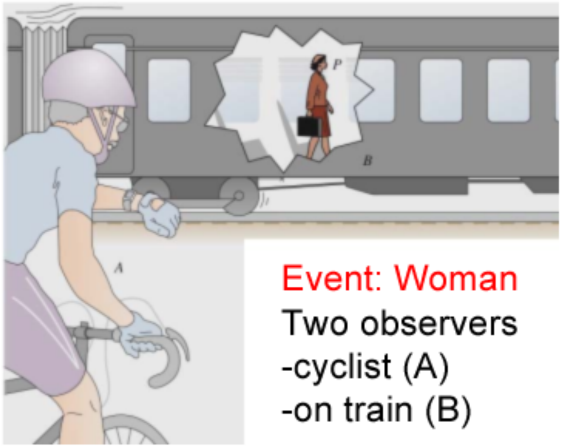
Woman walks with velocity of 1.0 m/s along train's aisle

Train is moving with velocity of 3.0 m/s to the right

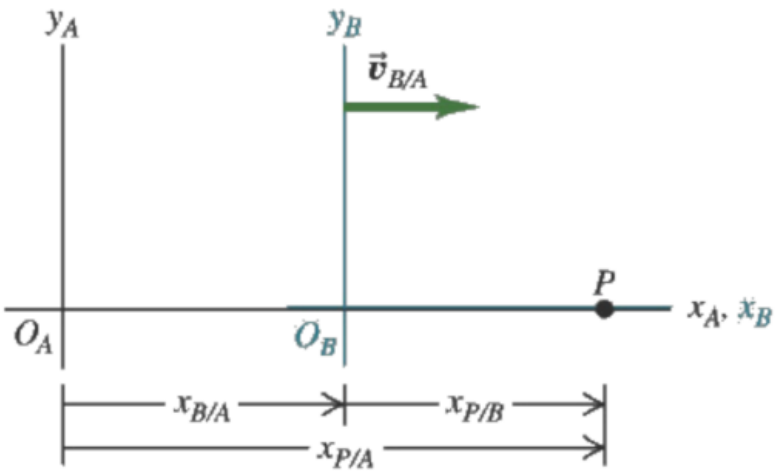
What is the woman's velocity ?

According to which Observer ???

Relative Velocity in 1 Dimension



Event: Woman
Two observers
-cyclist (A)
-on train (B)



At any instant (take a snapshot)

$x_{P/A}$ = pos. of P rel. to frame A ; $x_{P/B}$ = pos. of P rel. to frame B (train)

$x_{B/A}$ = distance from origin of A to origin of B

Clearly $\boxed{x_{P/A} = x_{P/B} + x_{B/A}}$ & $\frac{dx_{P/A}}{dt} = \frac{dx_{P/B}}{dt} + \frac{dx_{B/A}}{dt} \Rightarrow \boxed{v_{P/A} = v_{P/B} + v_{B/A}}$

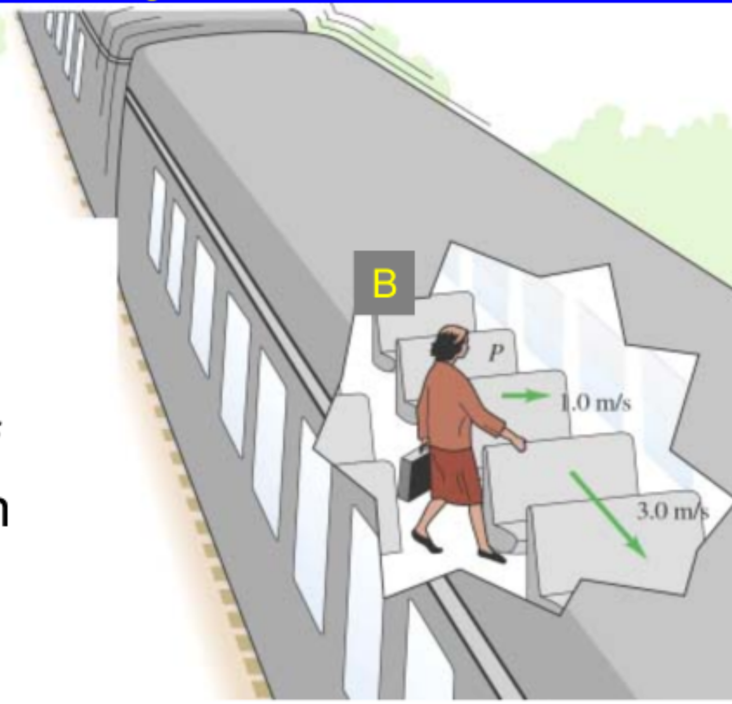
So woman's velocity as seen by cyclist (A)

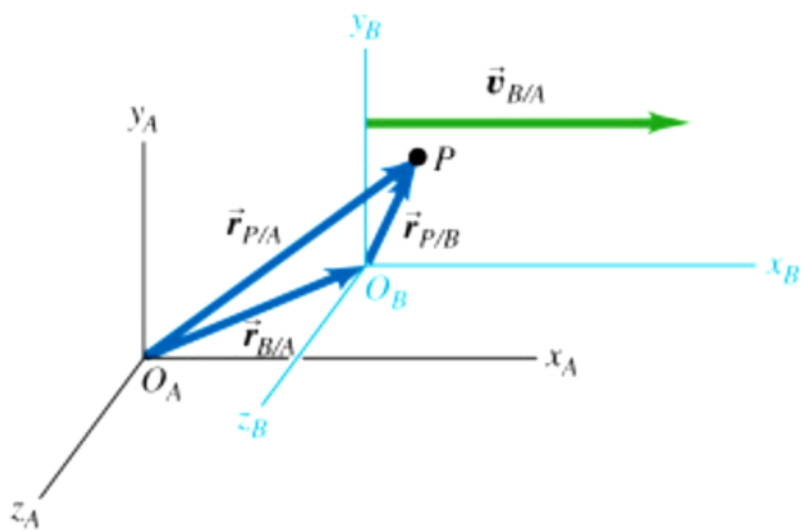
$v_{P/A} = 1.0\,m/s + 3.0\,m/s = 4.0\,m/s$ but If woman was walking in opp. dir. in train

$v_{P/A} = -1.0\,m/s + 3.0\,m/s = 2.0\,m/s$

Relative Velocity in 3D

Suppose woman walks from one side of car to another with speed of 1.0m/s . Her motion is perpendicular to direction of the aisle and the train's motion

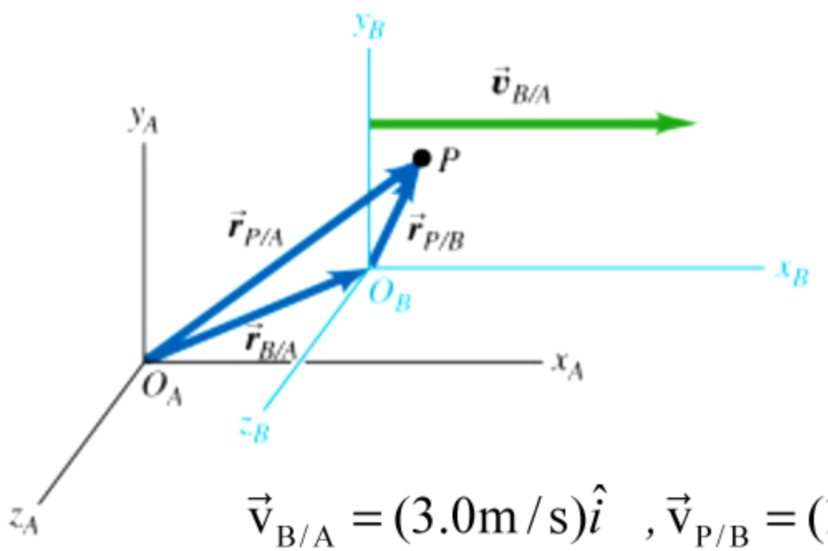


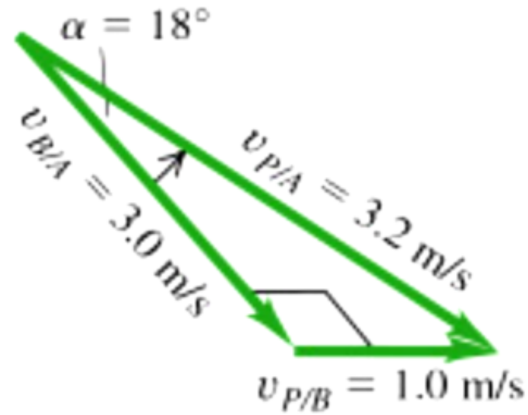


$$\vec{r}_{P/A} = \vec{r}_{P/B} + \vec{r}_{B/A}$$

$$\Rightarrow \vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$$

Relative Velocity in 3D





$$\vec{v}_{B/A} = (3.0\text{m/s})\hat{i} \quad , \vec{v}_{P/B} = (1.0\text{m/s})\hat{j}$$

$$\Rightarrow \boxed{\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A} = (3.0\hat{i} + 1.0\hat{j})\text{m/s}}$$

speed of P seen by A= $|\vec{v}_{P/A}| = \sqrt{(3.0\text{m/s})^2 + (1.0\text{m/s})^2}$

Cyclist on ground sees woman moving at angle α w.r.t.

train's motion: $\tan\alpha = \dfrac{v_{P/B}}{v_{B/A}} = \dfrac{1.0\text{m/s}}{3.0\text{m/s}} \Rightarrow \alpha = 18^o$

Flying In Crosswind

compass indicates, plane heading due North
airspeed indicator shows it moving thru air at 240km/h
If there is wind of 100km/h west to east,
What is velocity of plane relative to earth?

Two observers: In air (A), on earth (E)

Watch plane (P)'s motion

$\vec{v}_{P/A} = (240 km/h)\hat{j}$; $\vec{v}_{A/E} = (100 km/h)\hat{i}$

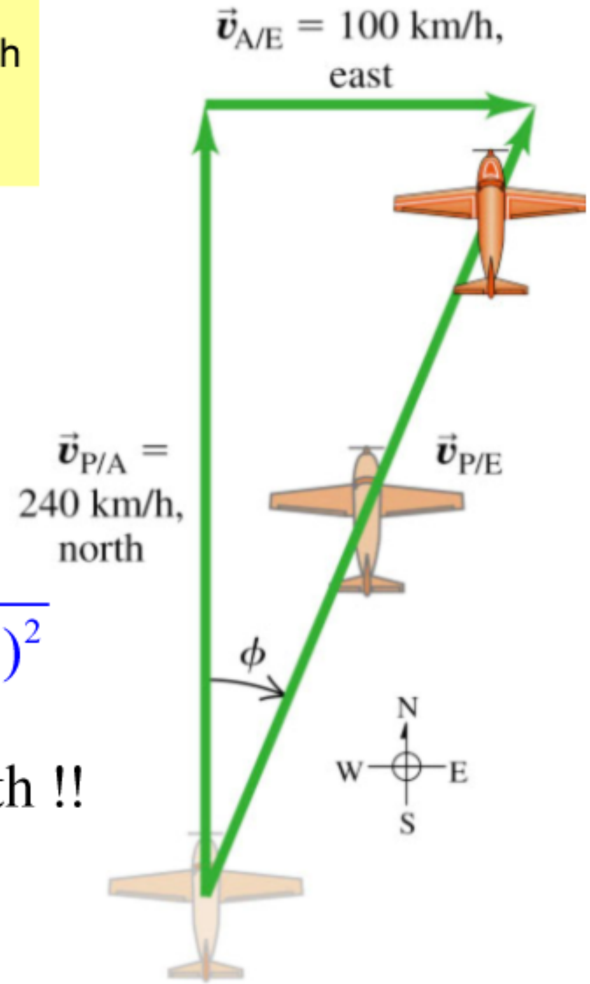
$\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E} = (100\hat{i} + 240\hat{j}) km/h$

speed $|\vec{v}_{P/E}| = \sqrt{(240 km/h)^2 + (100 km/h)^2}$

$\phi = \tan^{-1}\left(\frac{100 km/h}{240 km/h}\right) = 23^0$ East of North !!

Must make course correction if

he wants to land on an airport due north !



Course Correction: How Much, Which Way ?

Because of crosswind what direction should the pilot be headed to travel **due North**. What will be his velocity relative to earth?

$$\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$$

pilot must point plane's nose at angle ϕ w.r.t wind to makeup.

Angle ϕ tells direction of $\vec{v}_{P/A}$

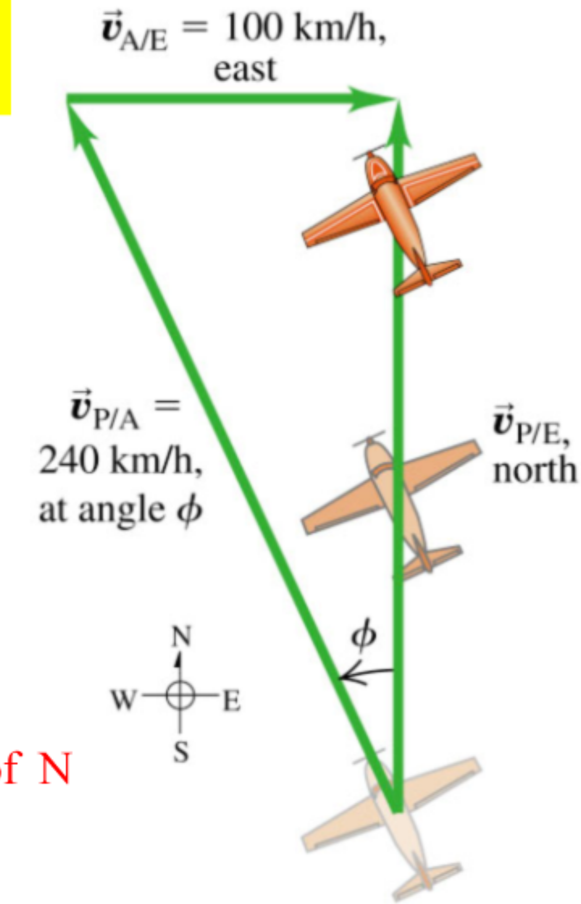
$\vec{v}_{P/E}$: speed ? but due N

$\vec{v}_{P/A}$: speed = 240km/h, dir?

$\vec{v}_{A/E}$: speed = 100km/h, due E

$$\phi = \sin^{-1}\left(\frac{v_{A/E}}{v_{P/A}}\right) = \sin^{-1}\left(\frac{100\text{km/h}}{240\text{km/h}}\right) = 25^{\circ} \text{ W of N}$$

$$v_{P/E} = \sqrt{(v_{P/A})^2 - (v_{A/E})^2} = 218\text{km/h}$$

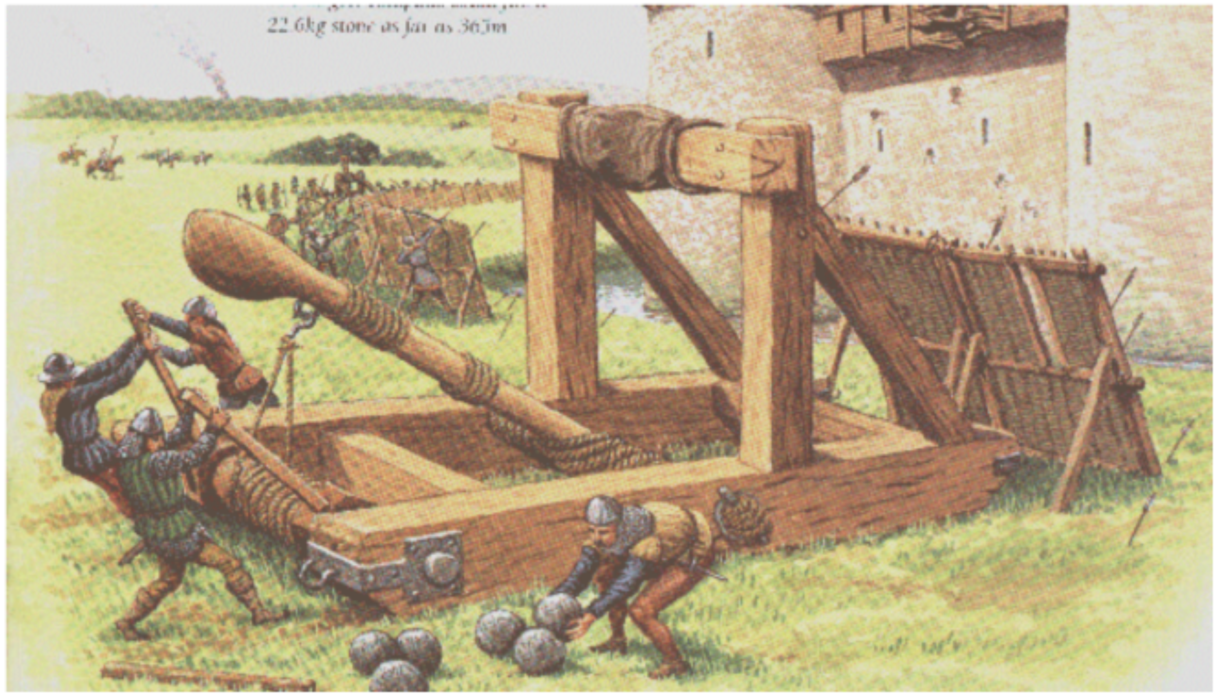


Archimedes & Weapon of Mass Destruction

Epic account of Syracusan's defense of their city from Roman invaders
Thanks to Archimedes (and his eureka moment !)

The Catapult As A War Machine

Catapults were invented in many civilizations. Earliest known record is from 9^{th} century BC in Nimrud (modern Day Iraq !). But Archimedes's catapults were fantastic. They could throw 100kg boulder +200m away → sank roman ships

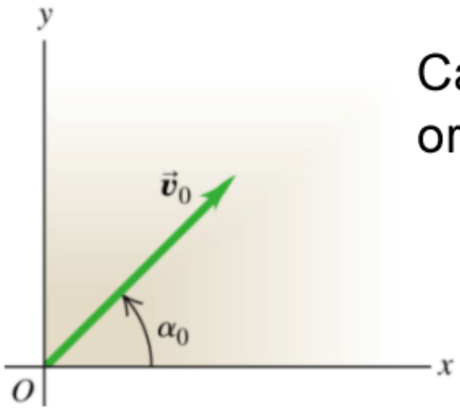


Archimedes genius was in his exquisite control of projectile trajectories !

Projectile Motion

Projectile: An object launched with some initial velocity that follows a path determined entirely by effects of gravitational acceleration $g = -9.8\ m/s^2$ (and air resistance)

Trajectory: path of a projectile (such as from a catapult)



All projectile motion occurs in the vertical plane containing initial $\vec{v}_0$ vector

Can decompose any projectile motion into two orthogonal and independent components

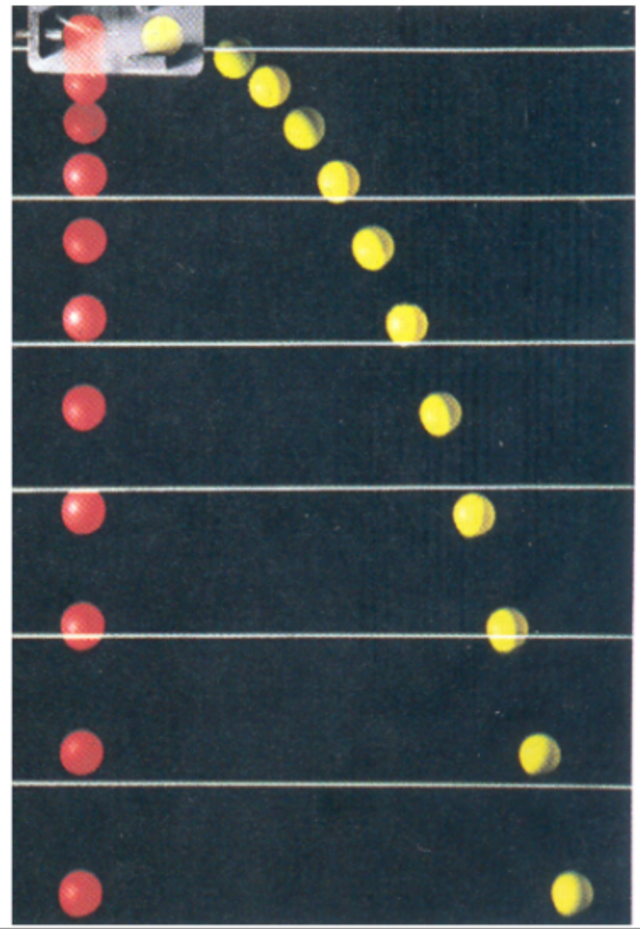
- along x axis with constant velocity, $a_x=0$
- along y axis with constant accel., $a_y= g$

Can express all relations for $\vec{r}$, $\vec{v}$ and $\vec{a}$ in terms of *seperate* components along x, y axis.

Projectile motion is superposition of these seperate and independent motions.

Independence of Motion in X & Y

Stroboscopic pictures:



Red ball dropped from rest

Yellow ball simultaneously projected horizontally

At any time both balls have same y position, velocity & acceleration

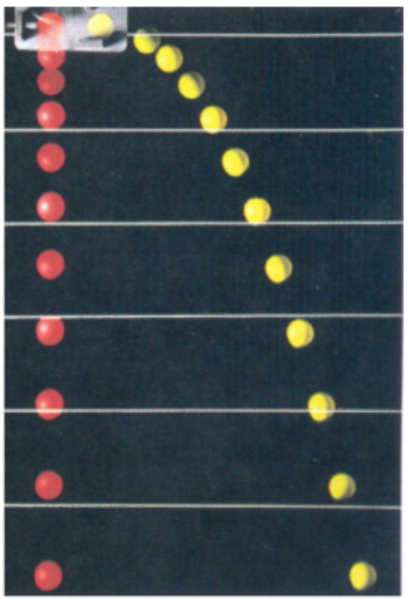
But diff. x position and velocity

Motion in 1D with Constant Acceleration

Reminder

$$v_x = v_{0x} + a_x t \; ; \; x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$

Now use $\vec{a}_x = 0, \;\; \vec{a}_y = -g = -9.80 \text{m/s}^2$



$\Rightarrow$ $v_x = v_{0x} t \; ; \; x = x_0 + v_{0x} t$ and

$$v_y = v_{0y} - gt \; ; \; y = y_0 + v_{0y} t - \frac{1}{2} g t^2$$

Thus

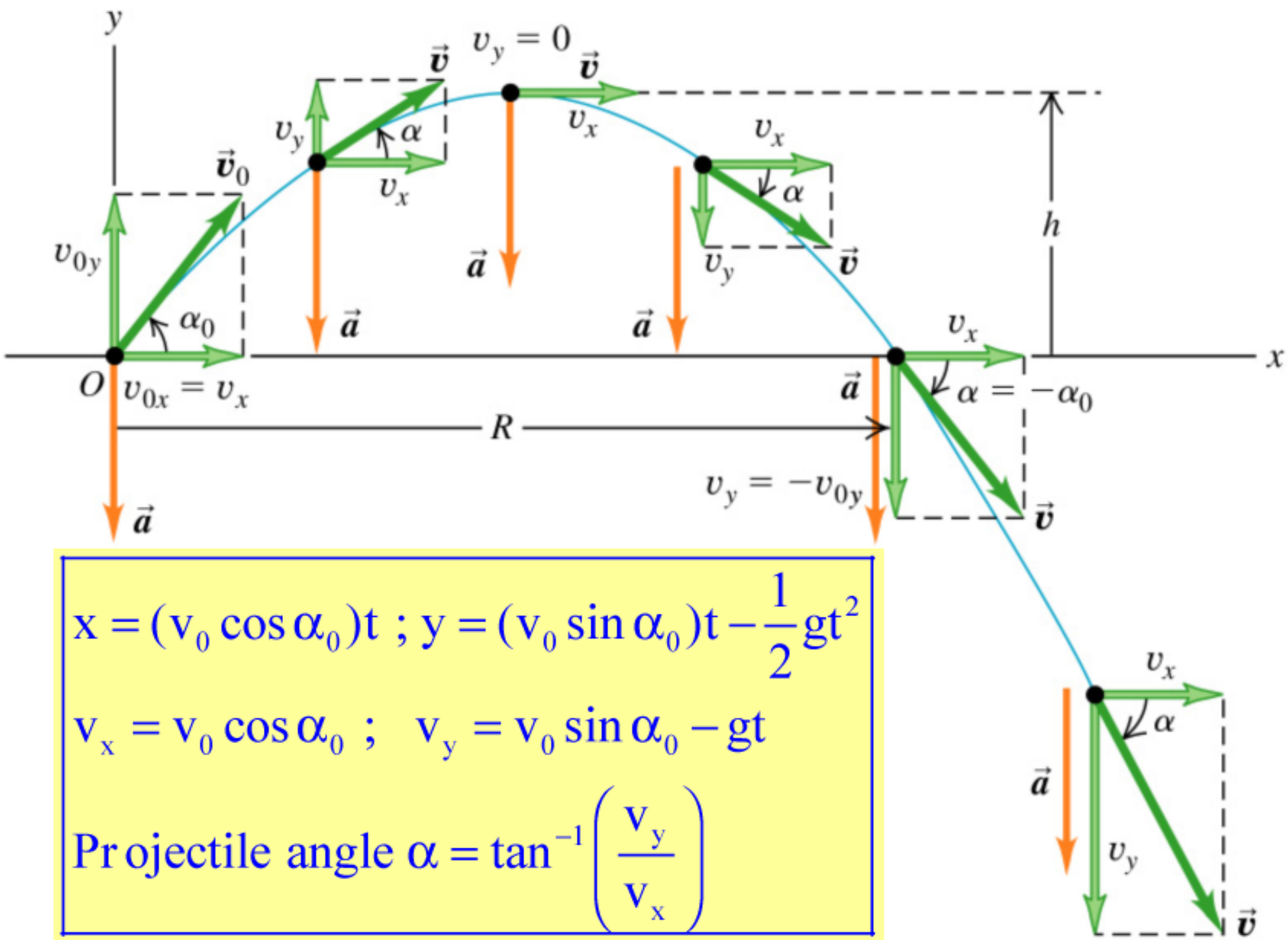
Displacement : $\vec{r} = x\hat{i} + y\hat{j}$

Velocity : $\vec{v} = v_x \hat{i} + v_y \hat{j}$

Quick Question

- If I flick a ball A off a table top with a velocity 2 m/s and simultaneously simply drop another ball B from the same table top:
- (A) they will hit the ground (assumed) horizontal) together
- (B) A will hit the ground first
- (C) B will hit the ground first

Trajectory of projectile with velocity $\vec{v}_0$ at t=0



$$x = (v_0 \cos\alpha_0)t \ ; \ y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$$

$$v_x = v_0 \cos\alpha_0 \ ; \quad v_y = v_0 \sin\alpha_0 - gt$$

$$\text{Projectile angle } \alpha = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Projectile Trajectory is Parabolic

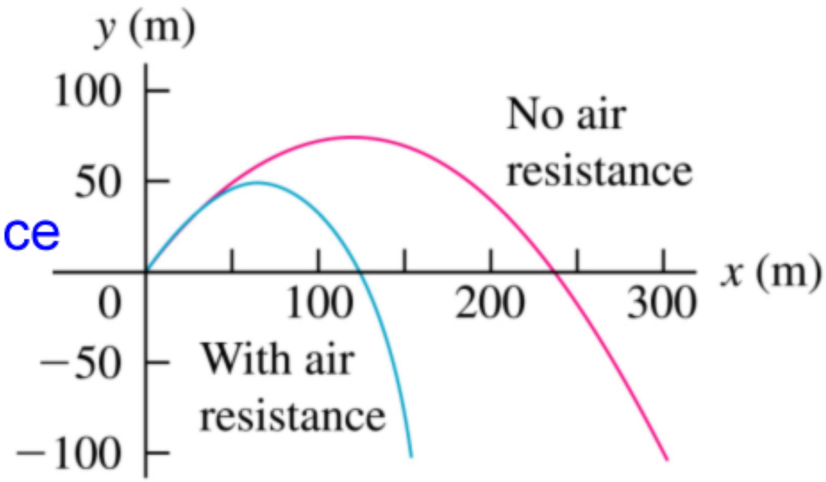
Equation for trajectory along y axis

Use $t = x/(v_0 \cos\alpha_0)$

$$\Rightarrow \boxed{y = (\tan\alpha_0)x - \frac{g}{2v_0\cos^2\alpha_0}x^2}$$

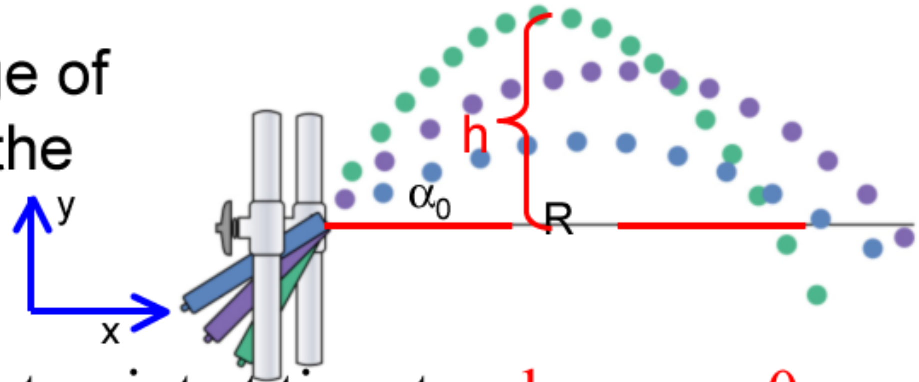
Trajectory is always *parabolic* in x

Effect of (neglected) air resistance



Height & Range of Projectiles

Max. height & the range of projectile depends on the firing angle α_0



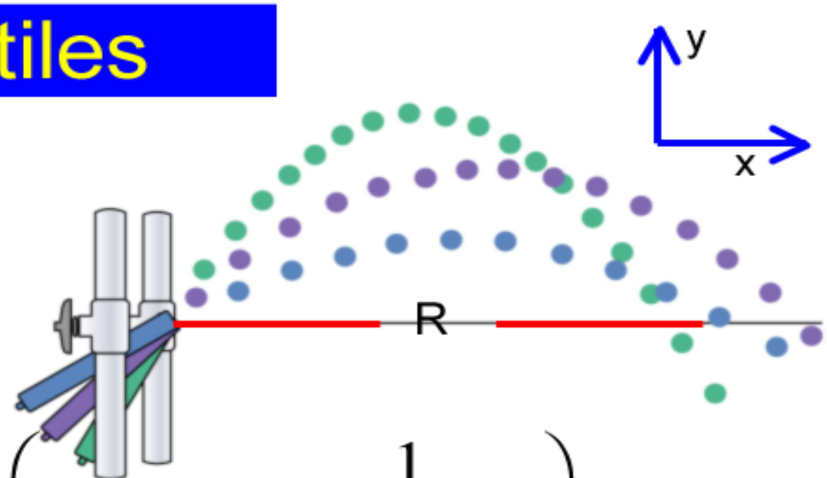
$$\text{Projectile at its highest point at time } t_1 \text{ when } v_y = 0$$

$$\Rightarrow v_y = v_0 \sin\alpha_0 - g t_1 = 0 \Rightarrow t_1 = \frac{v_0 \sin\alpha_0}{g}$$

$$\text{at this time, } y = h = v_0 \sin\alpha_0 \frac{v_0 \sin\alpha_0}{g} - \frac{1}{2} g \left(\frac{v_0 \sin\alpha_0}{g} \right)^2$$

$$\Rightarrow \boxed{h = \frac{v_0^2 \sin^2 \alpha_0}{2g}}; \quad \text{largest at } \alpha_0 = 90^0 \text{ (vertical launch)}$$

Range of Projectiles



R is the projectile's x location at some $t=t_2$ when $y=0$

$$0=(v_0 \sin\alpha_0)\, t_2 - \frac{1}{2} g t_2^2 = t_2\left(v_0 \sin\alpha_0 - \frac{1}{2} g t_2\right) = 0$$

Two solutions for t_2 : $\boxed{t_2 = 0}$ & $\boxed{t_2 = \frac{2 v_0 \sin\alpha_0}{g}}$

$$\text{Range } R = v_0 \cos\alpha_0 . \frac{2 v_0 \sin\alpha_0}{g} = \boxed{\frac{v_0^2 \sin 2\alpha_0}{g} = R}$$

(Using trig. identity: 2sinθcosθ=sin2θ)

$R = R_{max}$ when $\alpha_0 = 45^o$

Trajectory Of a Batted Baseball

Height and Range of a Baseball: Visually