Physics 4A
March 5, 2015

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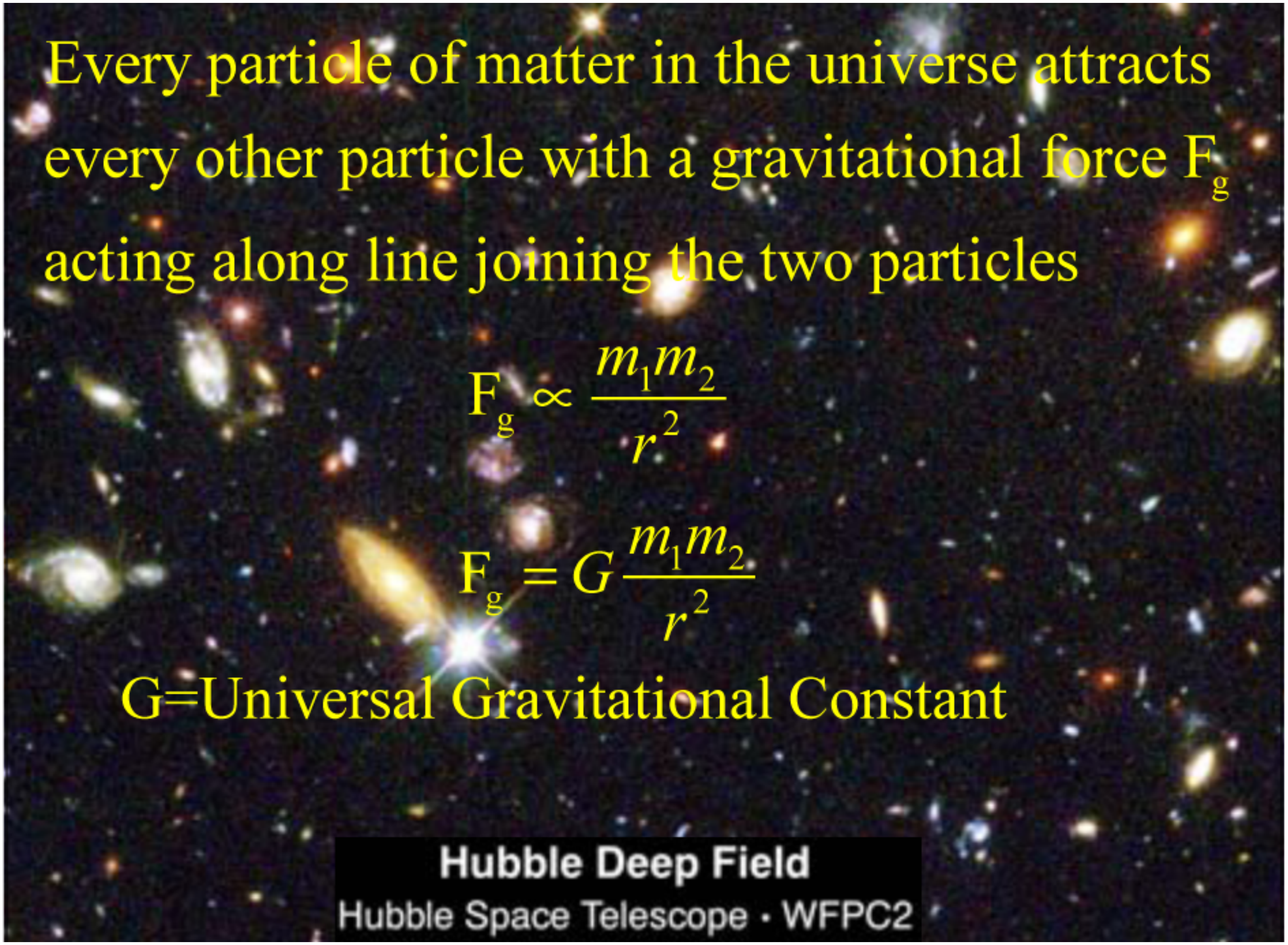
UCSD Physics

Every particle of matter in the universe attracts every other particle with a gravitational force F_g acting along line joining the two particles

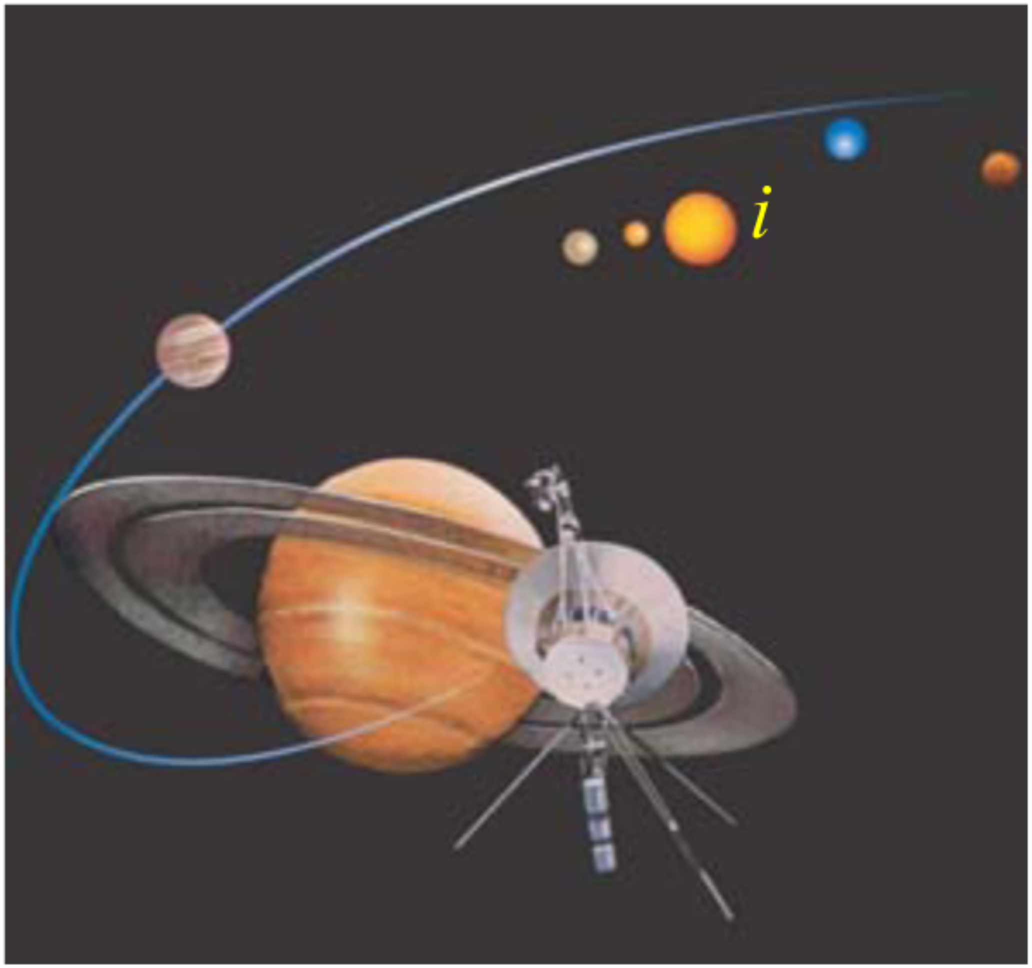
$$F_g \propto \frac{m_1 m_2}{r^2}$$

$$F_g = G\frac{m_1 m_2}{r^2}$$

G=Universal Gravitational Constant



Superposition Of Gravitational Forces



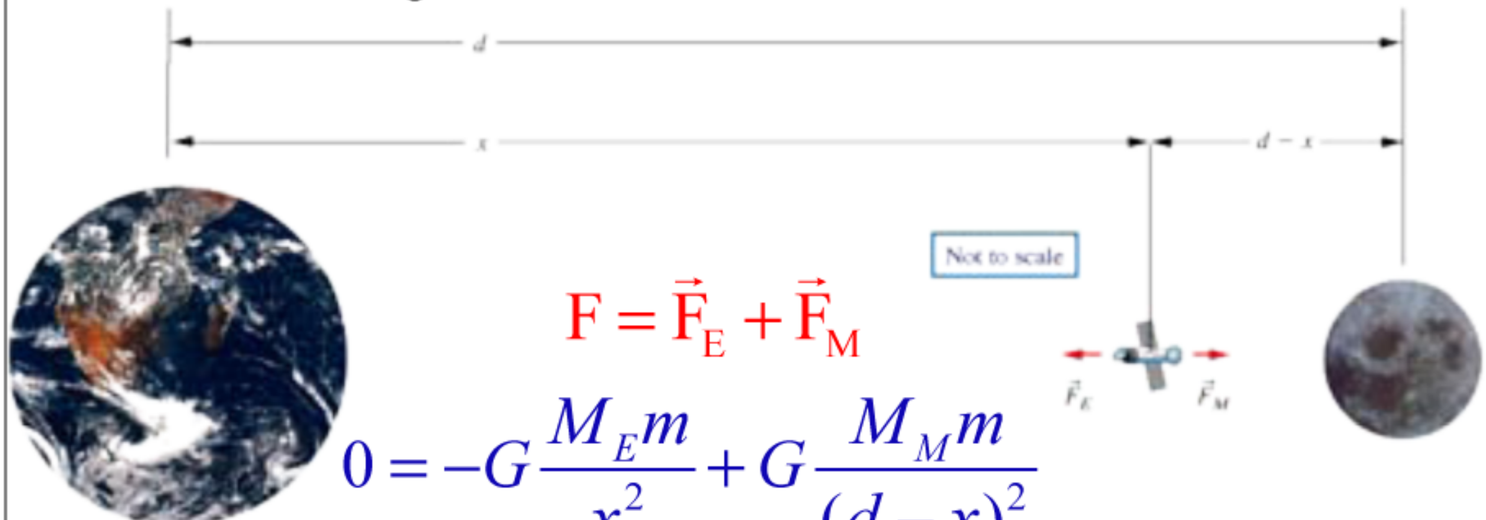
$$\vec{F}_{sat} = \sum_i \vec{F}_{i \to sat}$$

$$\vec{F}_{i \to sat} = \frac{G M_i m_{sat}}{r_i^2}$$

Superposition Of Gravitational Forces

Gravitational forces combine vectorially

A satellite is to be sent to position x between earth & moon where there is no NET gravitational force due to these bodies. Find x.



$$F = \vec{F}_E + \vec{F}_M$$

$$0 = -G\frac{M_E m}{x^2} + G\frac{M_M m}{(d-x)^2}$$

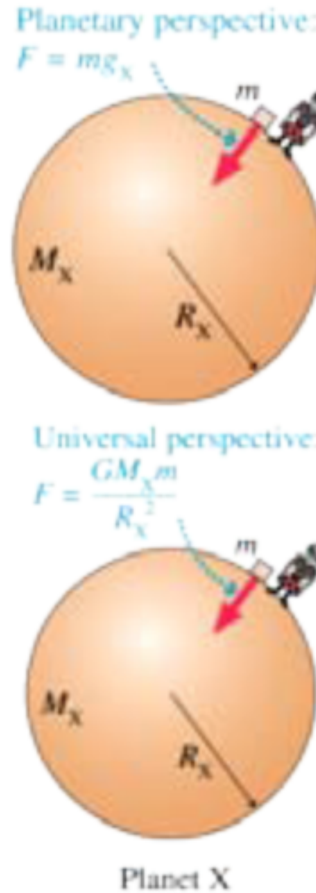
$$\Rightarrow (d-x)^2 = x^2(M_M / M_E)$$

$$\Rightarrow x = 0.9\text{d} = \text{equilibrium point}$$

Relating Little g and Big G

Revised Definition: Weight of a body is the total gravitational force exerted on that body by all other bodies in the universe !

When body is near earth, influence of all other objects is negligible (far far away)
⇒ Weight = Earth's grav. attraction

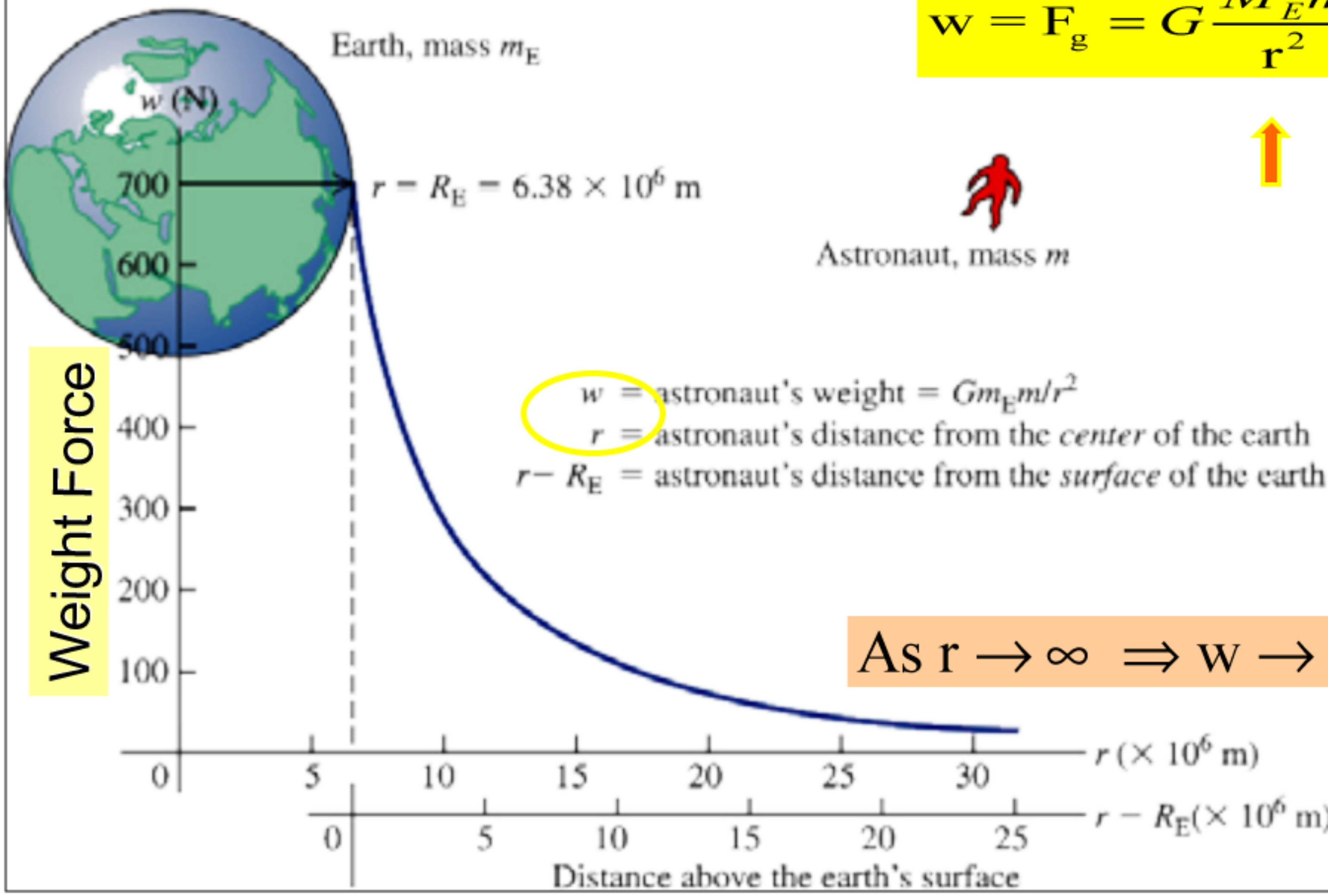


Weight force on a body of mass m at earth's surface

$$w = mg = F_g = G\frac{M_E m}{R_E^2} \Rightarrow \boxed{g = \frac{GM_E}{R_E^2}}$$

Weight Force On A Body Near Earth

$$w = F_g = G\frac{M_E m}{r^2}$$

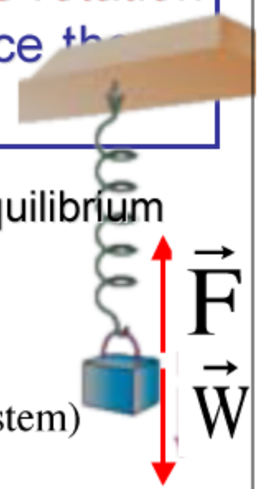


$$\text{As } r \to \infty \;\Rightarrow\; w \to 0$$

Measuring Weight Force

Gravity is not a force that one measures directly. If you hold a spring scale with some mass m hanging on it

Spring scale applies tension force **F** to hanging body and reading on the scale is |**F**|. If you are unaware of earth's rotation you would think that scale reading = weight of body since th spring is in equilibrium $\vec{F} = -\vec{w}$; w = **apparent** weight



But if bodies are rotating with Earth then they are not <u>exactly</u> in equilibrium
Apparent weight $w \neq w_0$ (true weight)

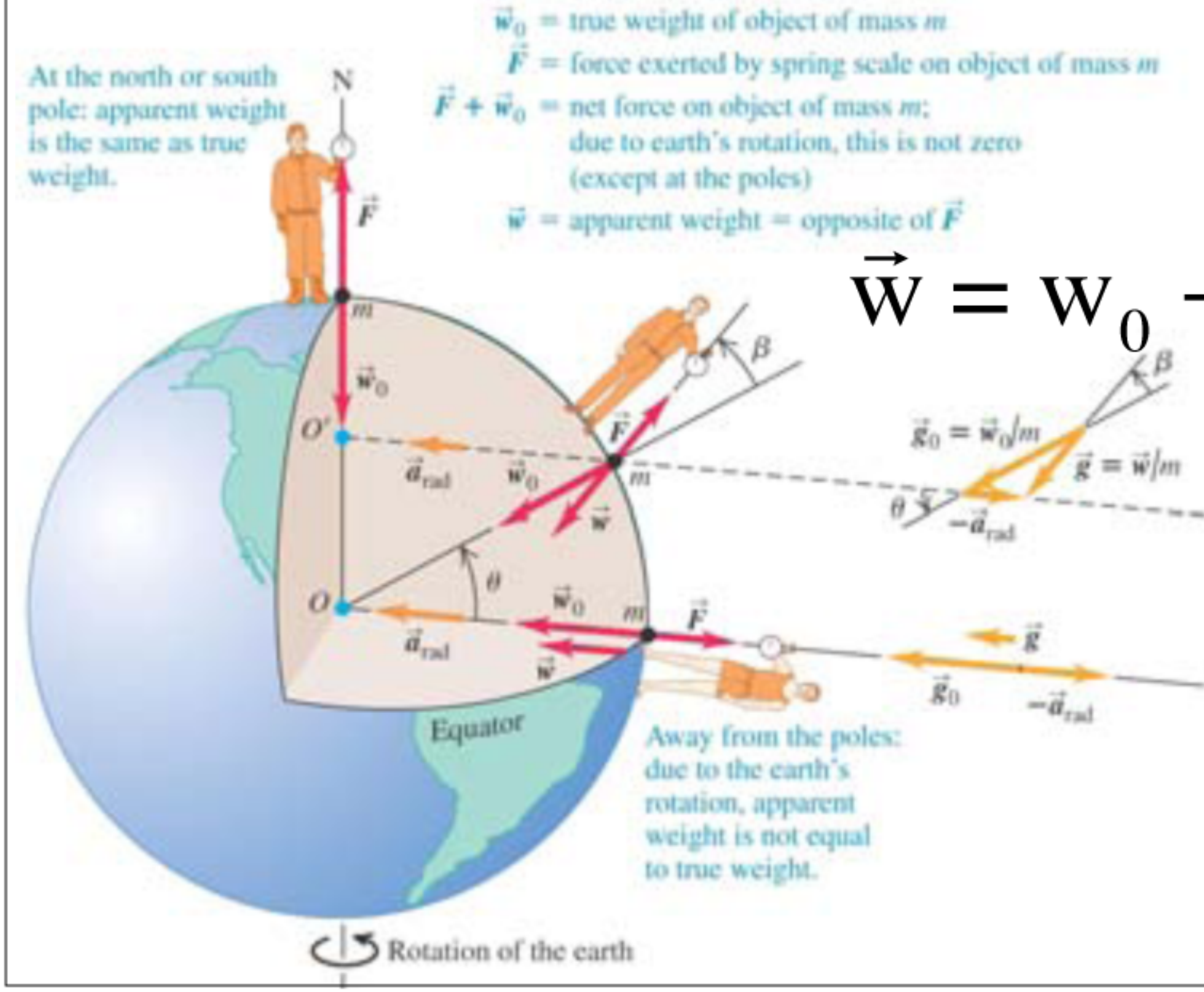
$w_0 = \frac{GM_E m}{R_E^2}$ and is measured at the North pole (no rotation $\Rightarrow$ Inertial system)

At Equator, body moving in circle of radius $R_E \Rightarrow w_0 - F = mv^2 / R_E$ (Centripetal Force)

So apparent weight which is = magnitude of F $\Rightarrow w = w_0 - (mv^2 / R_E)$

$\Rightarrow g_{equator} = g_0 - (v^2 / R_E)$ and $\Delta g = \frac{v^2}{R_E} = \frac{(468\,m/s)^2}{6.38 \times 10^6\,m} = 0.0337\,m/s^2$

True Weight Vs Apparent Weight: Redux



$$\vec{w} = w_0 - m\vec{a}_{rad}$$

(Apparent) Weightlessness In Space

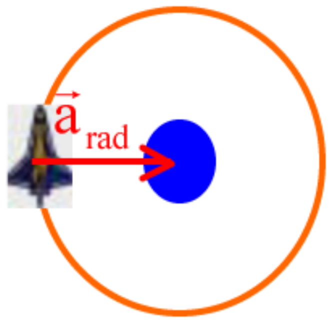
Bodies in orbiting spacecraft are not weightless ! Earth's gravity continues to attract them as though they were at rest w.r.t earth.

Apparent weight of a body in orbiting craft: $\vec{w}' = \vec{w}_0 - m\vec{a}_{rad} = m(\vec{g}_0 - \vec{a}_{rad})$

Only force acting on craft is the Earth's gravity

$\Rightarrow \vec{a}_{rad}$ towards earth's center

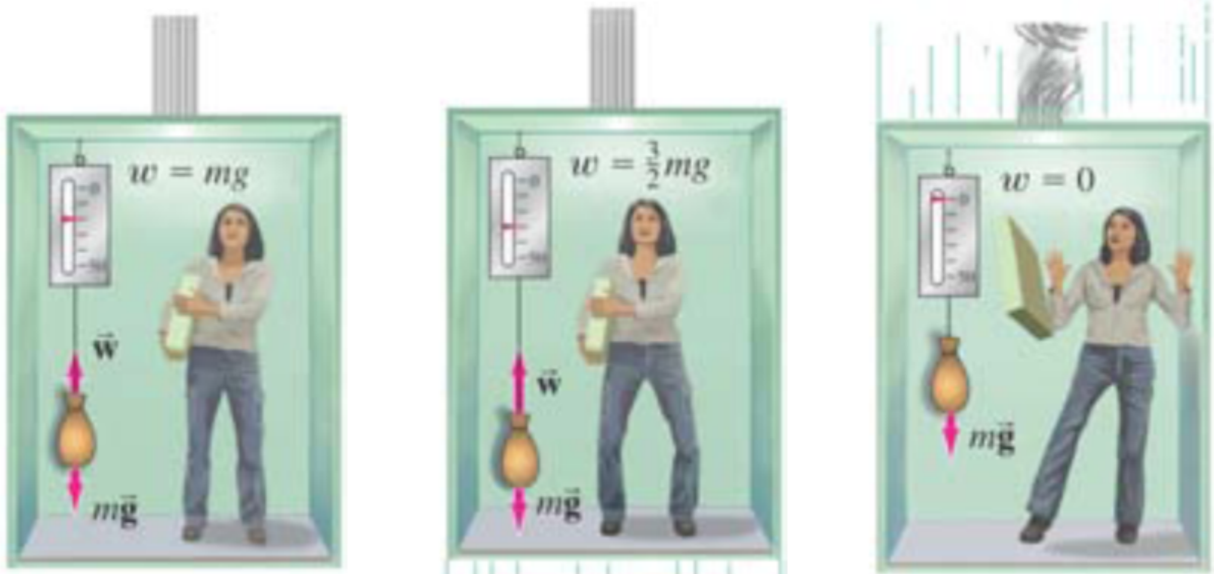
= value of acc. due to gravity at that point $\Rightarrow \vec{g}_0 = \vec{a}_{rad} \Rightarrow$ app. weight $w' = 0$!



$\Rightarrow$ Apparent weightlessness of satellites

Apparent Weight in Elevator

$$w - mg = ma$$



(a) $\vec{a} = 0$ (b) $\vec{a} = \frac{1}{2}\vec{g}$ (up) (c) $\vec{a} = \vec{g}$ (down)

Gravitational Potential Energy

V_{grav} done by grav. force when body moves directly

wards or away from center of earth $r = r_1 \rightarrow r = r_2$

$$V_{grav} = \int_{r_1}^{r_2} F_r \, dr = -GM_E m \int_{r_1}^{r_2} \frac{dr}{r^2} = GM_E m \left(\frac{1}{r_2} - \frac{1}{r_1} \right)$$

hange in grav. potential energy $U_1 - U_2 = W_{grav}$

body moves away from earth, r increases &

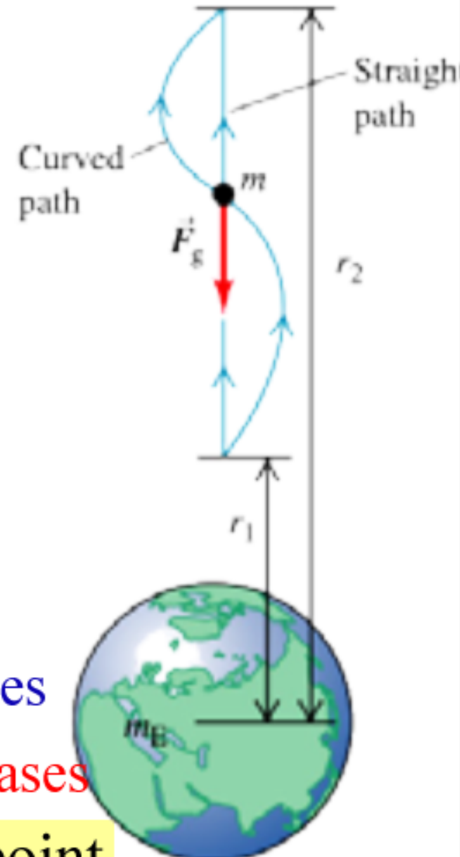
ravity does negative work on object $\Rightarrow$ U increases

body falls towards earth, r decreases $\Rightarrow$ U decreases

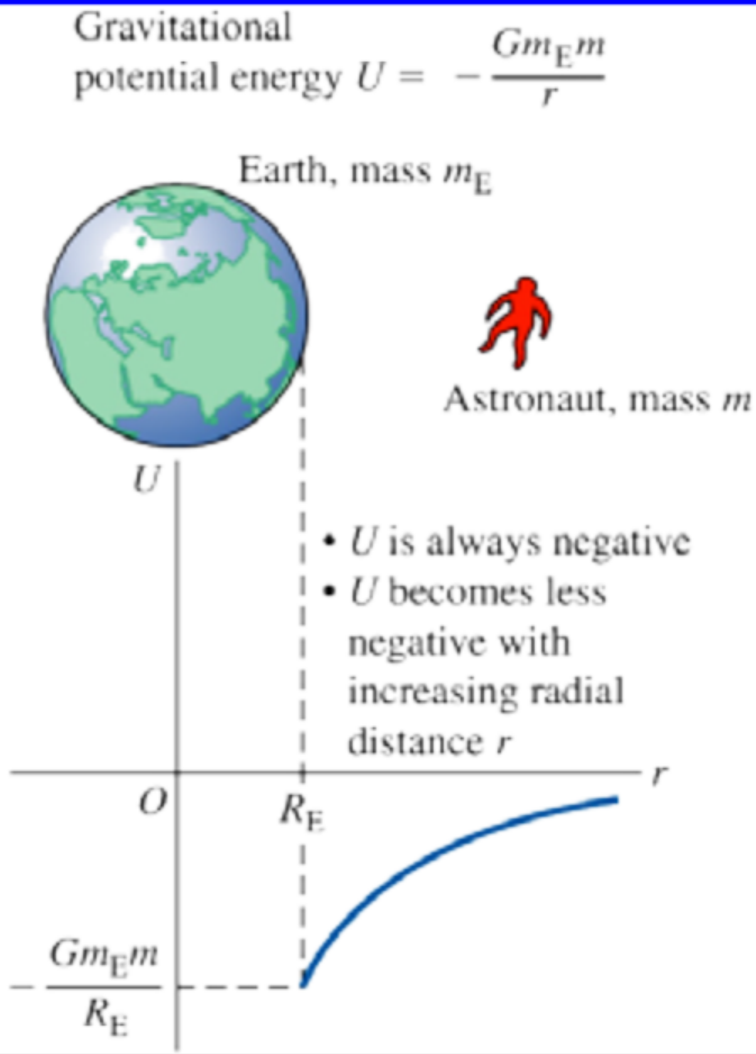
otential energy is measured relative to a ref. point

Vork done in moving an object from $r = \infty$ to $r = r$

$$V_{grav} = \frac{GM_E m}{r} \Rightarrow \Delta U = \boxed{U = -\frac{GM_E m}{r}}$$



Gravitational Potential Energy



$$U = -\frac{GM_E m}{r}$$

As $r \to \infty \Rightarrow U \to 0$

Getting Back Good Old U_{grav} = mgh Relation

Above earth but close to it, work done in moving from $r_1 \rightarrow r_2$

$$W_{grav} = +GM_E m\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = GM_E m\frac{r_1 - r_2}{r_1 r_2}$$

If object stays close to earth $\Rightarrow r_1 \simeq R_E \; , \; r_2 \simeq R_E$

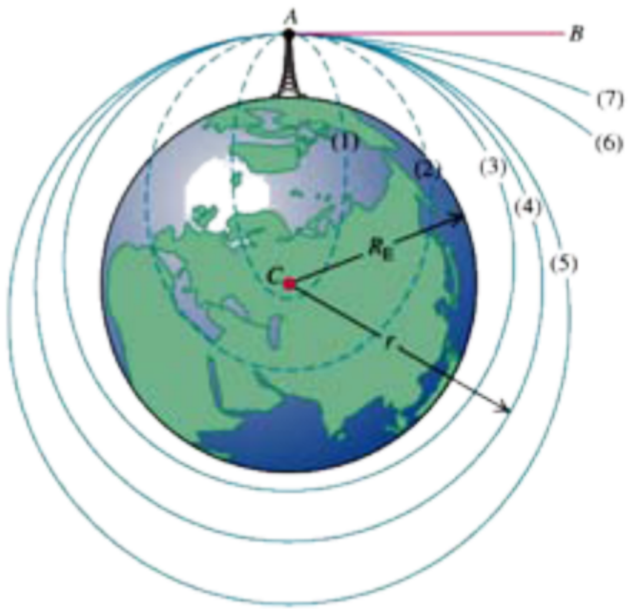
Rewrite $W_{grav} = GM_E m\left(\frac{r_1 - r_2}{R_E^2}\right)$ but since $g = \frac{GM_E}{R_E^2}$

$$\Rightarrow W_{grav} = mg(r_1 - r_2)$$

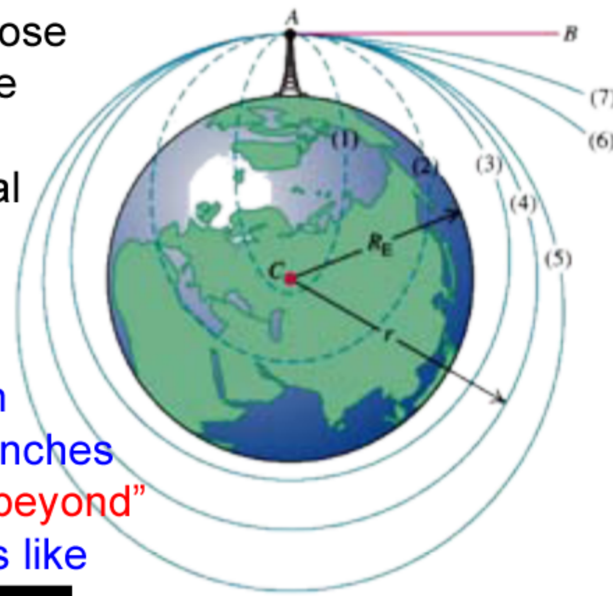
work done under constant acceleration due to gravity

Satellite Takes A Fall

A cannonball shot horizontally from mountaintop will fall to ground on a parabolic path. If shot with much higher speed it will go far enough that surface of spherical earth falls away beneath it. Ball will never catch up with the earth's surface falling away and ball's motion will be a **circular orbit of "constant fall"**



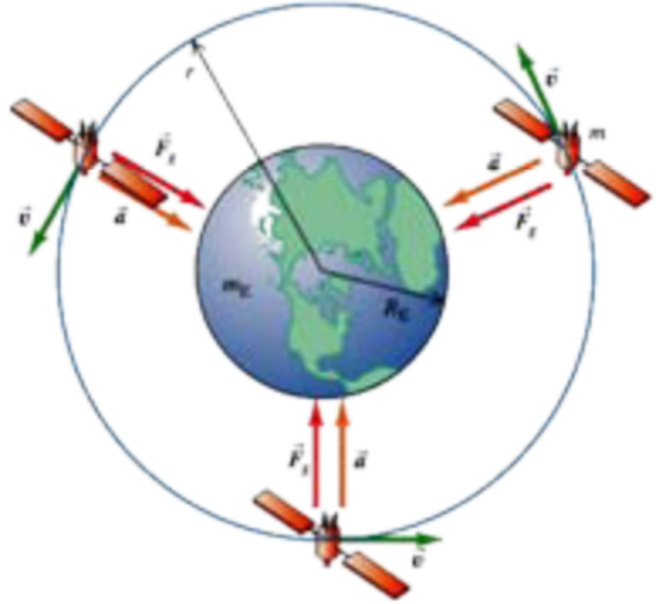
Satellite Motion: Closed & Open Orbits



Closed Orbit: Orbits (1) thru (5) close on themselves. All closed orbits are ellipses or segment of an ellipse. Trajectory (4) is a circular, a special case of an elliptical orbit.

Open Orbit: trajectories (6) & (7) are open orbits. For these paths the projectile never returns to earth but travels further away. NASA launches such probes to travel to "infinity & beyond" to probe properties of other planets like Jupiter & Saturn

Circular Orbit Of Artificial Satellites



For satellite in circular orbit,

(assume $\approx$ vaccum @ such heights)

only force acting on it

is the gravitational attaction

of earth, directed towards

center of earth

$\Rightarrow$ center of satellite orbit

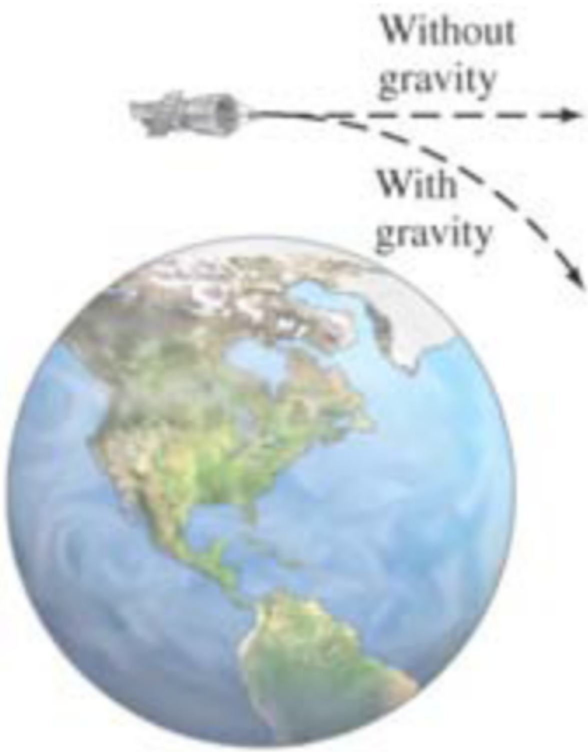
$$\text{Second law} \Rightarrow \frac{GM_E m}{r^2} = \frac{m v_{sat}^2}{r} \Rightarrow v_{sat} = \sqrt{\frac{GM_E}{r}}$$

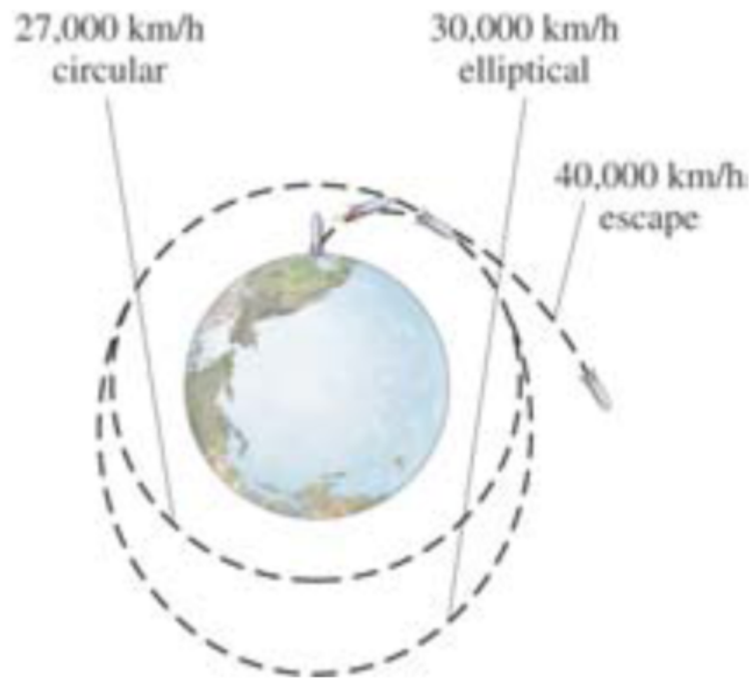
v_{sat} does not depend on m_{sat} & v_{sat} is fixed

for a given orbit radius, making "catching up"

with a satellite an interesting maneuver!

Fate Of Artificial Satellites





Q: What keeps a satellite rotating?

A: Its speed !

Too low and it crashes down

Too high and it escapes earth's grip !

$$\frac{GM_E m}{r^2} = \frac{m v_{sat}^2}{r}$$

Orbit Of Satellites

$$v_{sat} = \sqrt{\frac{GM_E}{r}}$$

In circular motion

$$v = \frac{2\pi r}{T} \Rightarrow T = \frac{2\pi r}{v}$$

$$T_{sat} = 2\pi r\sqrt{\frac{r}{GM_E}} = \frac{2\pi r^{3/2}}{\sqrt{GM_E}}$$

$$E_{sat} = K_{sat} + U_{sat} = \frac{1}{2}m_{sat}v_{sat}^2 + \left(-\frac{GM_E m_{sat}}{r}\right)$$

$$\Rightarrow E_{sat} = \frac{1}{2}m_{sat}\left(\frac{GM_E}{r}\right) - \frac{GM_E m_{sat}}{r} = -\frac{GM_E m_{sat}}{2r}$$

Total mech. energy is negative ! $\Rightarrow$ system is a bound state

smaller the r, lesser is the energy of the satellite-earth system

If r gets small enough, dissipative (drag) forces bring satellite down

Projectile Velocity Needed To Escape Earth's Gravity

Gravity is a conservative force. Total energy of body of mass m, speed v is $E=K+U=\frac{1}{2}mv^2-\frac{GM_E m}{r}$

Escape speed v_{esc} of a body launched from earth surface ($r=R_E$) is the minimum launch speed in order to escape earth's gravity $\Rightarrow$ travel to $r=\infty$

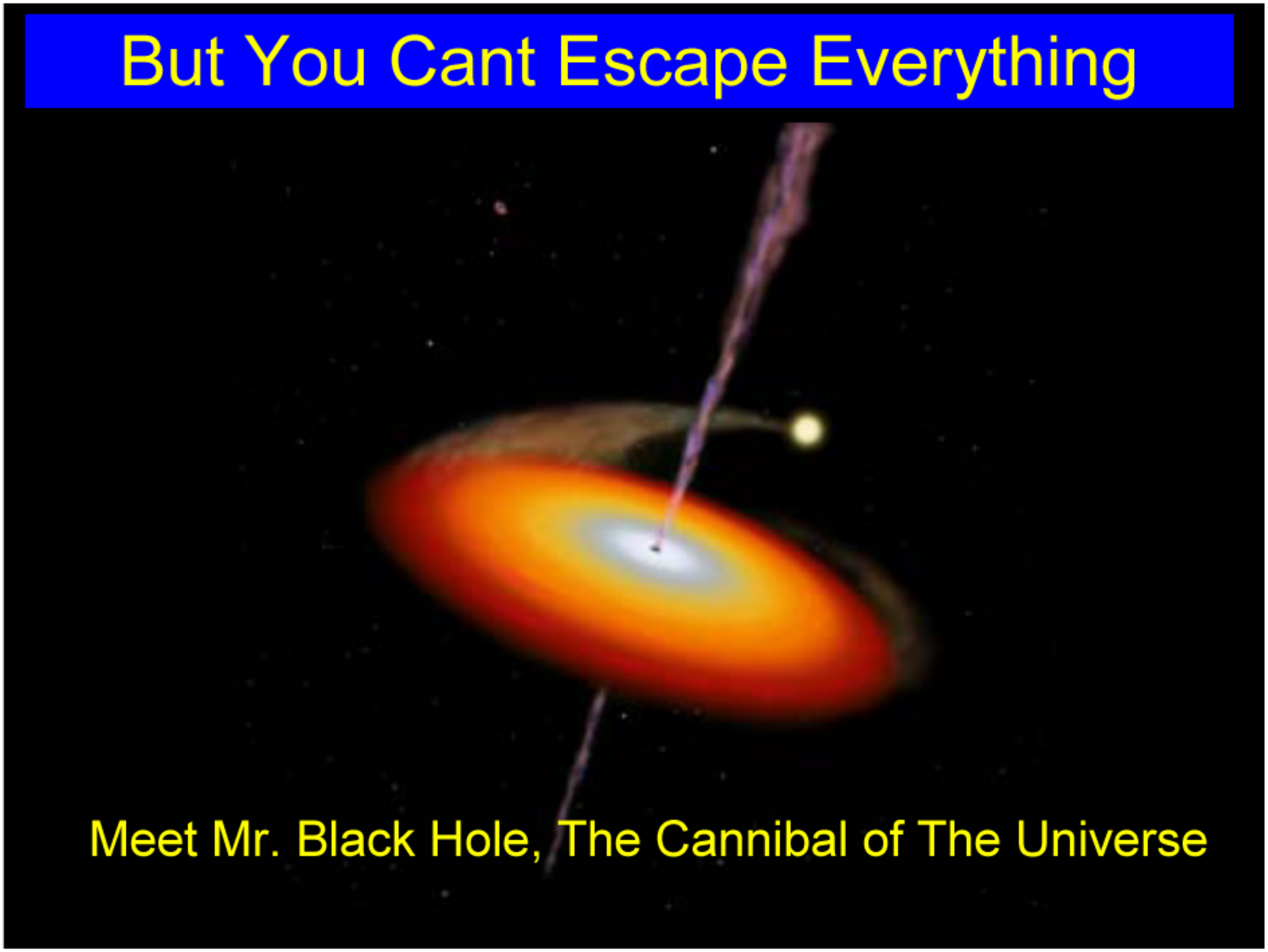
Body with escape velocity will have v=0 at $r=\infty$

$\Rightarrow K=0$ & $U=0 \Rightarrow \boxed{E=0}$

Energy Conservation $\Rightarrow$ E=0 at launch & all points after

At launch, $E=\frac{1}{2}mv_{esc}^2-\frac{GM_E m}{R_E}=0 \Rightarrow v_{esc}=\sqrt{\frac{2GM_E}{R_E}}$;

$\Rightarrow v_{esc}=11200$m/s !! Hence those booster rockets on shuttle


But You Cant Escape Everything
Meet Mr. Black Hole, The Cannibal of The Universe