HW 9

PHYS 4A

WINTER '15

4.) $g = \frac{GM}{R^2}$

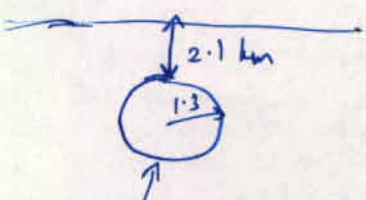
a) $g_M = \frac{6.67 \times 10^{-11} \times M_M}{R_M^2} = 3.7 \ m/s^2$

b) $g_T = \frac{G M_T}{R_T^2} = 1.4 \ m/s^2$

where $M_M = 0.33 \times 10^{24}$ kg ; $R_M = 2.44 \times 10^6$ m

& $M_T = 0.135 \times 10^{24}$ kg ; $R_T = 2.58 \times 10^6$ m

10.)



$r = 1.3$ km

Denser Rock

$\rho_{Rock} = \rho_E + 0.3\rho_E = 1.3\rho_E$

$\Rightarrow \rho_{Rock} = 1.3\rho_E = 2700 \ kg/m^3$ — ①

$\therefore$ "Extra" Mass of the denser Rock $= 0.3\rho_E \times \frac{4}{3}\pi r^3$

(M extra) $= 0.3 \times \frac{2700}{1.3} \times \frac{4}{3}\pi \times (1.3 \times 10^3)^3$

$$a \text{ (due to this extra mass)} = \frac{G\, m_{extra}}{[(2.1+1.3)\times 10^3]^2}$$

$$= \frac{G\,\overbrace{(0.3\rho_E)}^{\text{Density}} \times \overbrace{\tfrac{4}{3}\pi(1.3\times10^3)^3}^{\text{Vol.}}}{(3.4\times10^3)^2}$$

Also, $\dfrac{G\rho_E \times \frac{4}{3}\pi R_E^3}{R_E^2} = \dfrac{GM_E}{R_E^2} = g$

$$\Rightarrow g = G\rho_E \tfrac{4}{3}\pi R_E$$

$$\therefore a = \frac{G(0.3\rho_E)\times\frac{4}{3}\pi(1.3\times10^3)^3}{(3.4\times10^3)^2} \times \frac{\frac{4}{3}\pi R_E}{\frac{4}{3}\pi R_E}$$

$$\Rightarrow a = \frac{0.3\times\frac{4}{3}\pi(1.3\times10^3)^3}{(3.4\times10^3)^2} \times \frac{g}{\frac{4}{3}\pi R_E}$$

$$\Rightarrow \frac{a}{g} = \frac{0.3\times1.3^3}{(3.4)^2\times(6370)}$$

$$\Rightarrow a/g = 8.95\times10^{-6}$$

$\therefore$ 0.000895 % increase in g due to extra mass of the rock.

18.) a) For orbital speed,

$$\frac{m v^2}{r} = \frac{G M m}{r^2}$$

$$\Rightarrow v = \sqrt{\frac{GM}{r}}$$

$$\Rightarrow v = \sqrt{\frac{G M_s}{R_E}}$$

$$\Rightarrow v = \sqrt{\frac{6.67 \times 10^{-11} \times 1.99 \times 10^{6} \times 10^{24}}{6.37 \times 10^{6}}}$$

$$\Rightarrow v = 4.56 \times 10^{6} \text{ m/s}$$

b) $$T = \left(\frac{4\pi^2 r^3}{GM}\right)^{\frac{1}{2}} = \left(\frac{4\pi^2 R_E^3}{GM_s}\right)^{\frac{1}{2}}$$

$$\Rightarrow T = \left(4\pi^2 R_E^2\right)^{\frac{1}{2}} \left(\frac{R_E}{GM_s}\right)^{\frac{1}{2}}$$

$$\Rightarrow T = \frac{2\pi \times 6.37 \times 10^{6} \text{ m}}{4.56 \times 10^{6} \text{ m/s}}$$

$$\Rightarrow T = 8.77 \text{ s}$$

28.) $\Delta E = \Delta K + \Delta U$

For circular orbit, we know that

$$U = -2K$$

$$\therefore E = U + K$$
$$= U - U/2$$
$$= U/2$$

$$\therefore \Delta E = \frac{\Delta U}{2}.$$
$$= \frac{1}{2}\left(-U_{perihelion} + U_{aphelion}\right)$$
$$= \frac{1}{2}\left[-\frac{G M_E M_S}{1.52 \times 10^{11}} - \left(-\frac{G M_E M_S}{1.47 \times 10^{11}}\right)\right]$$
$$= +8.72 \times 10^{42} \text{ J}$$

48.) $T = \frac{2\pi R_{orbit}}{V_{orbital}}$

where $R_{orbit} = R_E + h$
$= 6370 + 5500$ km
$= 11870$ km

a) $V_{orbit} = \sqrt{\frac{GM}{R_{orbit}}}$ $\left\{ \frac{mv^2}{r} = \frac{GMm}{r} \right\}$

New $V'_{orbit} = 1.1\, V_{orbit}$

$$\Rightarrow \sqrt{\frac{GM}{R'_{orbit}}} = 1.1 \sqrt{\frac{GM}{R_{orbit}}}$$

$$\Rightarrow R'_{orbit} = \frac{R_{orbit}}{(1.1)^2}$$

$$\therefore \Delta R_{orbit} = R_{orbit} - R'_{orbit}$$

$$= R_{orbit} \left(1 - \frac{1}{(1.1)^2}\right)$$

$$= R_{orbit} \times \frac{(0.1 \times 1.2)}{(1.1)^2}$$

$$= 0.099 \times 11,870 \text{ km}$$

$$\simeq 1175 \text{ km}$$

b) $$T = \frac{2\pi R_{orbit}}{V_{orbit}} = \left(\frac{4\pi^2 R_{orbit}^3}{GM}\right)^{1/2}$$

$$\therefore T' = 0.9\, T$$

$$\Rightarrow \left(R'^3_{orbit}\right)^{1/2} = 0.9 \left(R^3_{orbit}\right)^{1/2}$$

$$\Rightarrow R'_{orbit} = (0.9)^{2/3} R_{orbit}$$

$$\simeq 11064 \text{ km}$$

$$\therefore \Delta R_{orbit} = 11870 - 11064$$

$$= 806 \text{ km}$$

60.)

$$V_{esc} = \sqrt{\frac{2GM}{r}}$$

$$\Rightarrow 0.13 \ m/s = \sqrt{\frac{2GM}{(14 \times 10^3)}}$$

$$\Rightarrow M = \frac{14 \times 10^3 \times (0.13)^2}{2 \times 6.67 \times 10^{-11}}$$

$$\Rightarrow M = 1.77 \times 10^{12} \ kg$$