HW #8

PHYS 4A

WINTER '15

2.) a) $\vec{F}_{Net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$

$$= (2\hat{i} + 2\hat{j}) + (-2\hat{i} - 3\hat{j}) + 1\hat{j}$$

$$= 0$$

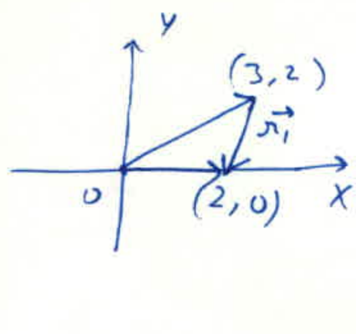
b) $\vec{\tau}_{Net} = \sum_{i=1}^{3} \vec{r}_i \times \vec{F}_i$

$$= 2\hat{i} \times (2\hat{i} + 2\hat{j}) + (-1\hat{i}) \times (-2\hat{i} - 3\hat{j}) + (-7\hat{i} + \hat{j}) \times (1\hat{j})$$

$$= 4\hat{k} + 3\hat{k} - 7\hat{k}$$

$$= 0$$

c) When calculating torque about any other point, eg: (3m, 2m), $\vec{r}_i$ will change.



New $\vec{r}_1 = 2\hat{i} - (3\hat{i} + 2\hat{j})$

$$= -\hat{i} - 2\hat{j}$$

$$\vec{r}_2 = -1\hat{i} - (3\hat{i} + 2\hat{j})$$

$$= -4\hat{i} - 2\hat{j}$$

$$\vec{r}_3 = (-7\hat{i} + 1\hat{j}) - (3\hat{i} + 2\hat{j})$$

$$= -10\hat{i} - 1\hat{j}$$

$$\therefore \underset{\text{about}(3,2)}{\vec{\tau}_{Net}} = \sum_{i=1}^{3} \vec{r}_i \times \vec{F}_i$$

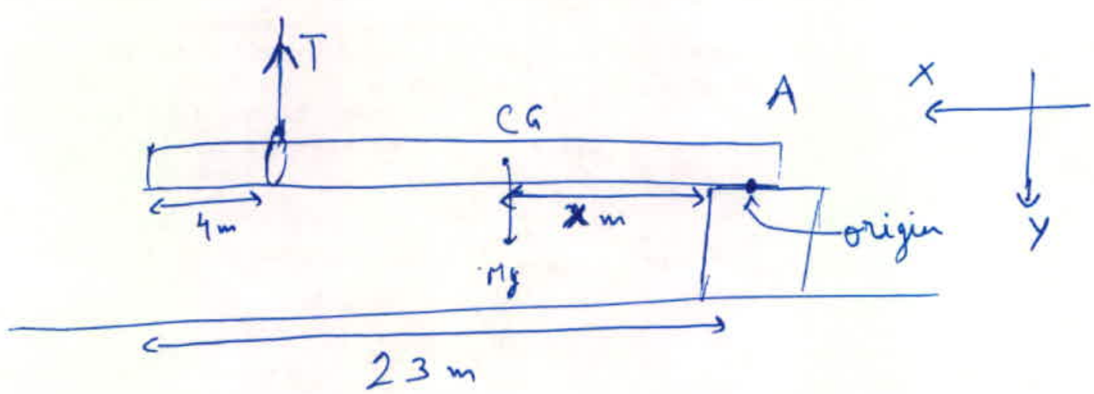
$$= (-1\hat{i}-2\hat{j}) \times (2\hat{i}+2\hat{j})$$
$$+ (-4\hat{i}-2\hat{j}) \times (-2\hat{i}-3\hat{j})$$
$$+ (-10\hat{i}-1\hat{j}) \times (1\hat{j})$$

$$= -2\hat{k} + 4\hat{k} + 12\hat{k} - 4\hat{k} - 10\hat{k}$$

$$= 0$$

Similarly, for the point (−7 m, 1 m) and any other point.

16.)



Let the centre of gravity (CG) be x m from the wall.

Balancing Torque about point A (where the log is supported by the wall)

$$\vec{\tau}_{Net} = 0$$

$$\Rightarrow \vec{r}_{CG} \times \vec{F}_{gravity} + \vec{r}_T \times \vec{F}_T = 0$$

$\Rightarrow (x\,\hat{i}) \times (7.5\times10^3\,\hat{j}) + (23-4)\hat{i} \times (-6.2\times10^3\,\hat{j})$

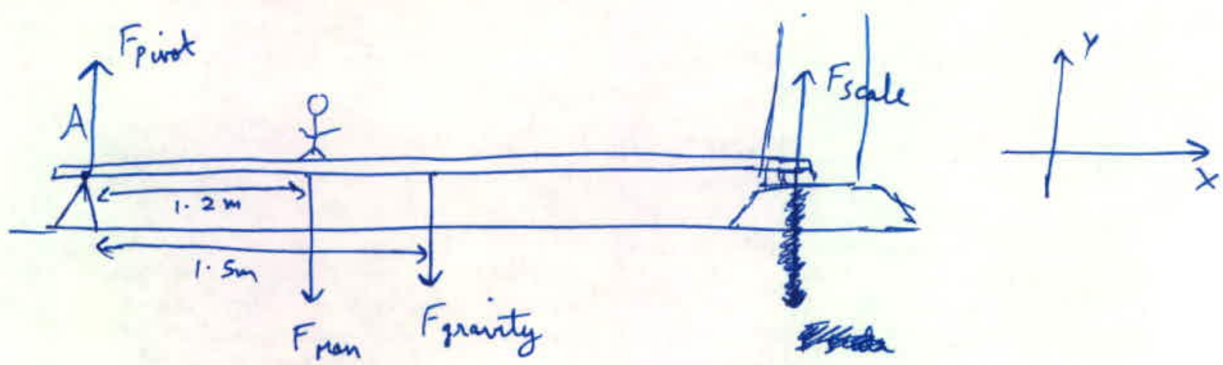
$= 0$

$\Rightarrow (7.5\times10^3)x\,\hat{k} - 19\times6.2\times10^3\,\hat{k} = 0$

$$\Rightarrow x = \frac{19\times6.2\times10^3}{7.5\times10^3}$$

$$\Rightarrow x = 15.71\ \underline{\underline{m}}$$

18.)



Free Body Diagram of the board.

Balancing Torque about point A,

$$\vec{\tau}_{Net} = 0.$$

$$\Rightarrow (\vec{r}_{pivot}\times\vec{F}_{pivot}) + (\vec{r}_{Man}\times\vec{F}_{Man}) + (\vec{r}_{CG}\times\vec{F}_{gravity}) + (\vec{r}_{scale}\times\vec{F}_{scale}) = 0$$

$$\Rightarrow \left(0 \times \vec{F}_{Pivot}\right) + 1.2\hat{i} \times (-F_{Man})\hat{j} + 1.5\hat{i} \times (-3.4g)\hat{j}$$
$$+ 3\hat{i} \times 210\hat{j} = 0.$$

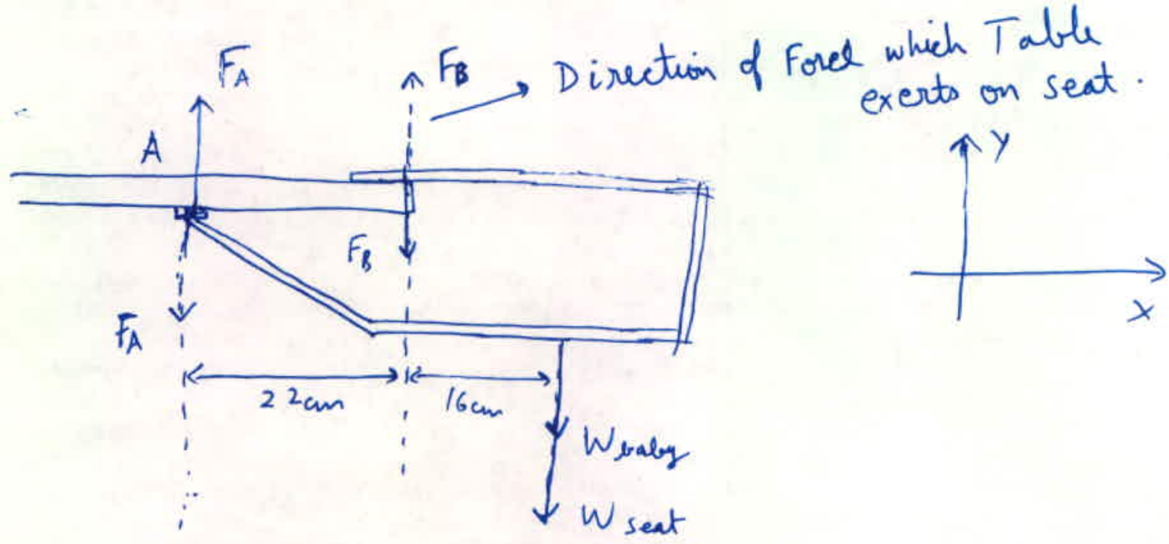
$$\Rightarrow -1.2 F_{man}\hat{k} - 5.1g\hat{k} + 630\hat{k} = 0$$

$$\therefore 1.2F_{man} + 5.1g = 630$$

$$\Rightarrow F_{man} = 483.35 \text{ N}$$

$\therefore$ Weight of Man = 483.35 N

20.)



Balancing Torque about point A (where F_A is acting)

$$\vec{\tau}_{Net} = 0.$$

$$\Rightarrow \left(\vec{r}_A \times \vec{F}_A\right) + \left(\vec{r}_B \times \vec{F}_B\right) + \left(\vec{r}_{cm} \times \vec{W}_{baby}\right)$$
$$+ \left(\vec{r}_{cm} \times \vec{W}_{seat}\right) = 0.$$

$$\Rightarrow (0 \times \vec{F_A}) + 22\hat{i} \times (+F_B)\hat{j} + (22+16)\hat{i} \times (-12g)\hat{j}$$

$$+ (22+16)\hat{i} \times (-1.5g)\hat{j} = 0$$

$$\Rightarrow 22 F_B \hat{k} + (-38 \times 12g)\hat{k} - 38 \times 1.5g\,\hat{k} = 0$$

$$\Rightarrow F_B = \frac{38g\ (12+1.5)}{22}$$

$$\Rightarrow F_B = 228.52\ N$$

Now, $\vec{F}_{Net}$ should also be 0.

$$\therefore \vec{F}_A + \vec{F}_B + \vec{W}_{baby} + \vec{W}_{seat} = 0$$

$$\Rightarrow 228.52\hat{j} + \vec{F}_A + (-12g)\hat{j} - (1.5g)\hat{j} = 0$$

$$\Rightarrow \vec{F}_A = -96.22\ \hat{j}\ N$$

36.) $h = 0.94x - 0.01x^2$

a) $U = mgh$

For equilibrium, $\vec{F}_{Net} = 0$ or $\frac{dU}{dx} = 0$.

$$\frac{dU}{dx} = 0$$

$$\Rightarrow mg\frac{dh}{dx} = 0$$

$$\Rightarrow \frac{d}{dx}(0.94x - 0.01x^2) = 0$$

$$\Rightarrow 0.94 - 2 \times 0.01x = 0$$

$$\Rightarrow \underline{\underline{x = 47 \text{ m}}}.$$

b) (at $x=47$)

$$\left.\frac{d^2U}{dx^2}\right|_{x=47} = mg\left.\frac{d^2h}{dx^2}\right|_{x=47}$$

$$= mg(-2 \times 0.01)\Big|_{x=47}$$

$$= -0.02\,mg$$

$$\frac{d^2U}{dx^2} < 0$$

$\therefore$ $\underline{\underline{\text{Unstable equilibrium}}}$.

c) $$h\Big|_{x=47} = 0.94(47) - 0.01(47)^2$$

$$= \underline{\underline{22.09 \text{ m}}}.$$