

# Experiment 4, Physics 2BL

Build and tune a voltmeter.  
Use it to calibrate a High-Voltage Source.

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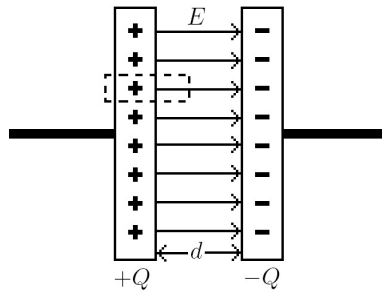
## Preparation

Before this experiment, we recommend you review or familiarize yourself with the following:

- Chapters 8 and 12 in Taylor
- Capacitors
- Torque

## 1. PHYSICS

### 1.1. Capacitors: Derivation of Force Between Plates in Terms of Dimensions and Voltage



The figure above shows a capacitor that has two plates of area  $A$  and charged with  $+Q$  on the left and  $-Q$  on the right. We can find the electric field  $E$ , represented by the arrows, by using Gauss's law. Suppose we make a pill box that is the size of the dotted line. The cross sectional area of the pill box is  $a$ . Gauss's law is as follows.

$$\mathbf{E} \cdot \mathbf{a} = \frac{Q_{enc}}{\epsilon_0}$$

Where  $Q_{enc}$  is the total charge enclosed by the pill box and  $\epsilon_0 = 8.85 \times 10^{-12} \frac{F}{m}$  is the permittivity of a vacuum. We will actually be conducting our experiment in air, not vacuum, but these permittivities are very close. Since the charge is evenly distributed over the plate we know that  $Q_{enc} = Q \frac{a}{A}$  and

$$E = \frac{Q}{A\epsilon_0} \quad (1)$$

Next we want to calculate the voltage between the plates. To do that all we need is to integrate the electric

field on a path from one plate to the other.  $V = -\int \mathbf{E} \cdot d\mathbf{r}$  gives us:

$$V = Ed \quad (2)$$

To calculate the force between the two plates we use Coulomb's force law which states that  $F = QE$ . However, since half the electric field creates a force on one plate and half the electric field creates a force on the other plate, the total force between them is only

$$F = \frac{1}{2}QE \quad (3)$$

If we combine the last three equations we can solve for  $F$  and eliminate  $E$  and  $Q$ . What we end up with is an equation for force in terms of the dimensions of the capacitor and the voltage.

$$F = \frac{1}{2} \frac{A\epsilon_0}{d^2} V^2 \quad (4)$$

## Questions

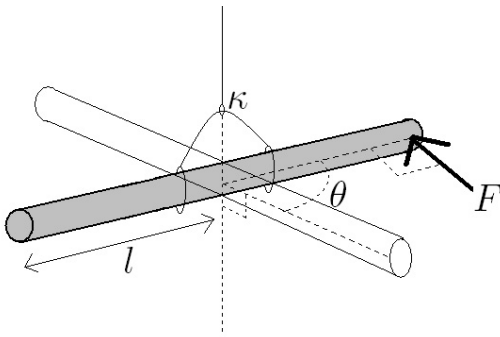
1. Use equations (1), (2), and (3) to eliminate  $E$  and  $Q$  and derive equation (4).
2. Use equations (1) and (2) and  $V = \frac{Q}{C}$  to derive an expression for the capacitance,  $C$ , of a parallel plate capacitor. What are two ways of improving the capacitance by changing the dimensions of the capacitor,  $A$  and  $d$ ?

### 1.2. Using Torque to Measure a Force

In this experiment we will be constructing a torsional pendulum, which has a period

$$T = 2\pi \sqrt{\frac{I}{\kappa}} \quad (5)$$

where  $I$  is the moment of inertia of whatever is being rotated and  $\kappa$  is a torsional constant similar to a spring constant. See the Appendix 1 for more information on torsional pendulums. Torsional pendulums have a relationship between torque and displaced angle that resembles Hooke's Law for regular springs. Recall that for a



regular spring the force is  $F = -kx$ , where  $x$  is the displacement of the spring from equilibrium. For a torsional pendulum we have that the torque is  $\tau = -\kappa\theta$ , where  $\theta$  is the angular displacement from equilibrium.

Suppose we have a torsional pendulum like the one shown in the diagram above. If we construct a parallel plate capacitor on one end of this torsional pendulum's rod and apply a voltage across it, then there will be a force on the end of the moment arm given by equation (4) (This electrostatic force is perpendicular to the moment arm). You now have a torque on this torsional pendulum given by  $\tau = \mathbf{l} \times \mathbf{F} = lF$ , where  $F$  is given by equation (4). Now, to separate the plates from each other, we must rotate the torsional pendulum in a direction opposite than the direction the capacitor forces the pendulum. In doing this, you are applying a torque (now in the opposite direction as the torque due to the capacitor) given by  $\tau = -\kappa\theta$ . When the system is in equilibrium, the sum of the torques should be zero. The result is

$$lF = \kappa\theta \quad (6)$$

Note that the system is in equilibrium the moment just before the capacitor plates separate. This is when the torques are equal.

### Questions

3. If the moment arm is increased do we need more or less of an angle to torque the plates apart from each other? Is the torsion constant,  $\kappa$ , altered if you change the moment of inertia of the hanger?

We can now determine the voltage  $V$  applied across this capacitor (of area  $A$ , thickness  $d$ , and a distance  $l$  from the pendulum axis) by combining equations (4) and (6).

$$V = d\sqrt{\frac{2\kappa\theta}{lA\epsilon_0}} \quad (7)$$

Where  $\theta$  here is the rotation angle that puts the torsional pendulum in a state of equilibrium (i.e. when the torques are equal).

### Questions

4. In order to calculate the voltage in terms of Volts, what must be the appropriate units of  $d$ ,  $\kappa$ ,  $\theta$ ,  $l$ , and  $A$  in equation (7)?
5. Use equations (4) and (6) to derive equation (7).
6. Given that  $I = 2ml^2$ , use equation (5) to derive an expression for  $\sigma_\kappa$  if the fractional error in the measurement of  $T$  is much bigger than the fractional errors for  $m$  and  $l$

## 2. METHODS FOR STATISTICAL ANALYSIS

### 2.1. Curve Fitting

Suppose you have a series of  $N$  points that have coordinates  $(x_i, y_i)$ . In order to find the equation of a straight line,  $y = A + Bx$ , that is a least-squares fit to your series of  $N$  points, use the following formulae.

$$A = \frac{\sum x^2 \sum y - \sum x \sum xy}{\Delta}$$

$$B = \frac{N \sum xy - \sum x \sum y}{\Delta}$$

$$\Delta = N \sum x^2 - (\sum x)^2$$

Note that  $\sum x^2 = x_1^2 + x_2^2 + \dots$  whereas  $(\sum x)^2 = (x_1 + x_2 + \dots)^2$ .

For extra practice see Taylor #8.6 and #8.10

### 2.2. Chi Squared

In order to get a measure of how good your fit is, first use the following formula:

$$\chi^2 = \sum \frac{(y_i - f(x_i))^2}{\sigma_{y_i}^2}$$

where  $f(x_i)$  is a function that represents your fitted curve evaluated at  $x_i$  and where  $\sigma_{y_i}$  is the uncertainty on the data point  $y_i$ .

It is then necessary to find the reduced Chi Squared,  $\tilde{\chi}^2$  given by:

$$\tilde{\chi}^2 = \frac{\chi^2}{d}$$

where  $d$  is the number of degrees of freedom of the system and is given by  $d = N - c$  where  $N$  is the number of data points/bins and  $c$  is the number of parameters calculated from the data. The probability of obtaining a  $\tilde{\chi}^2$  value greater than or equal to your calculated chi-squared value,  $\tilde{\chi}_0^2$ , is given by the following chart. (See Taylor for an expanded version of this chart.)

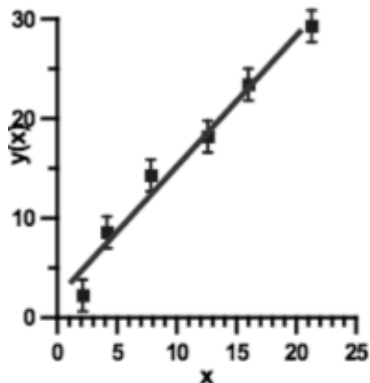
$d$	$\tilde{\chi}_0^2$												
	0	0.25	0.5	0.75	1.0	1.25	1.5	1.75	2	3	4	5	6
1	100	62	48	39	32	26	22	19	16	8	5	3	1
2	100	78	61	47	37	29	22	17	14	5	2	0.7	0.2
3	100	86	68	52	39	29	21	15	11	3	0.7	0.2	—
5	100	94	78	59	42	28	19	12	8	1	0.1	—	—
10	100	99	89	68	44	25	13	6	3	0.1	—	—	—
15	100	100	94	73	45	23	10	4	1	—	—	—	—

For extra practice see Taylor #12.3 and #12.14

### Question

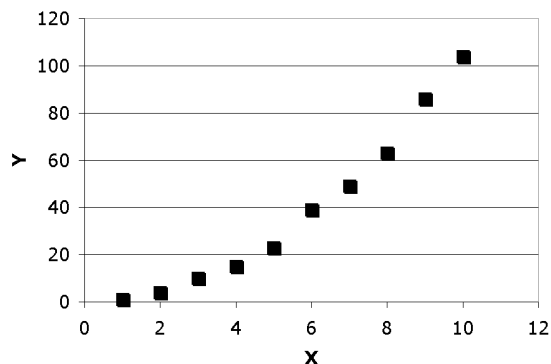
7.  $\chi^2$  is formulaically similar to the  $t$ -score. How is  $t$ -score different from  $\chi^2$ ?

8. See the plot below. A set of data has been plotted and fitted by a straight line. If you were calculating  $\tilde{\chi}^2$ , what would be the number of degrees of freedom  $d$ ? Don't just state the number, explain how you determined the number.



### Question

9. A set of data has been plotted in the next column. Suppose you decided to fit a second order polynomial to this data (i.e.  $y = Ax^2 + Bx + C$ ). If you were calculating  $\tilde{\chi}^2$ , what would be the number of degrees of freedom  $d$ ? Don't just state the number, explain how you determined the number.



## 3. PROCEDURE

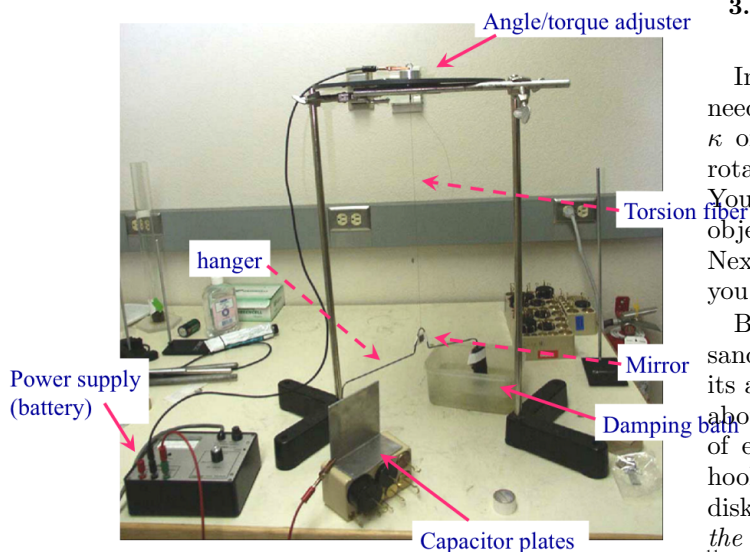
This experiment is very sensitive and time consuming. You will be making precision measurements of forces on the order of micro-Newtons, so be as careful as possible with your setup. Avoid disturbing the table (by leaning on it, writing on it, etc.) and avoid abrupt movements in the vicinity of your apparatus.

Come well prepared and work as a team in order to complete the experiment in a timely fashion.

### 3.1. Setup Description

This section is meant to familiarize you with the setup. There are further details about the setup in the next few sections.

A large grey disk is first raised a couple feet from the table and clamped in a horizontal position with the attached protractor facing upward. One of the metal pistons (with eye hooks screwed into both its ends) used in experiment 3 should be placed in the hole of the large grey disk. Also, obtain a small rod and slide it through the small hole on the side of the piston. Next, a torsional pendulum should hang from the eye hook that points down. This torsional pendulum is composed of a copper wire with a straight metal rod (with a small notch in the center) tied to its end. The notched metal rod should have a copper disk taped to the ends of each of its arms. One of the copper disks should have a rubber stopper attached to its center. This disk should be taped such



that the rubber stopper points away from the notched metal rod. Also, a small mirror should be taped to the center of the notched metal rod. See details in picture of example setup.

### 3.2. Moment of Inertia of Notched Metal Rod+Disks

Measure the mass and dimensions of the notched metal rod+disks before assembling it: This includes the mass of the metal rod,  $m$ , the masses of the copper disks,  $m_1$  and  $m_2$  (One of these disks should have a rubber stopper attached to it.), and the radius of one of the disks,  $R$ . Also, using calipers, measure the thickness  $d$  of the rubber stopper. (This last measurement is not needed to calculate the moment of inertia. It is needed in the voltage calculation.) The measurement of the length of the moment arm,  $l$  (see diagram on page 1) should be done only AFTER you hang the pendulum and make sure it is balanced and horizontal.

Calculate the moment of inertia of the notched metal rod+disks and its error  $\sigma_I$ .

$$I = \frac{1}{3}ml^2 + (m_1 + m_2)l^2 + (m_1 + m_2)\frac{R^2}{4}$$

This expression for the moment of inertia is a reasonable approximation of the moment of inertia of the notched rod+disks you will be using. Describe any inconsistencies of your setup with the approximations stated above, and associate (systematic) uncertainties with them.

### 3.3. Measuring the Torsion Constant $\kappa$

In order to calculate the voltage using equation (7), you need to know  $\kappa$  of the copper wire. You can determine  $\kappa$  of the wire if you know the moment of inertia and rotational period for an object that hangs from the wire. You've already determined the moment of inertia of your object (notched metal rod+disks) in the previous section. Next you need to construct this torsional pendulum so you can measure the rotational period of the object.

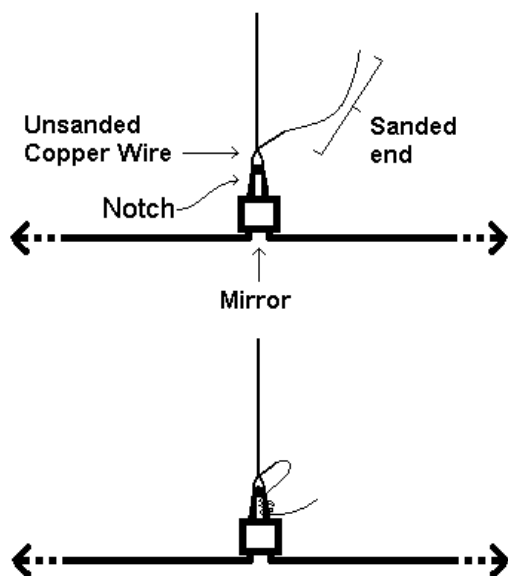
Before constructing the pendulum, using sand paper, sand the notch of the metal rod and the end of one of its arms. Get a piece of copper wire, gauge 26AWG and about 60cm long. Then sand the last few centimeters of each end of the copper wire and sand the two eye hooks attached to small piston. Next, tape the copper disks on either end of the metal rod's arms. *Be sure that the disk with the rubber stopper is taped to the end of the metal rod that has been sanded and that the rubber stopper points away from the metal rod.* Because you won't be able to deconstruct your setup later to resand things you may have sanded poorly, you now need to check that everything is conducting well. *Before moving on, ask one of your TAs to check that everything conducts well (This will be done using a multimeter).*

Next, construct the torsional pendulum according to the setup description in section 3.1 BUT also consider the following details. The ends of the copper wire have been sanded and are now weak, so it would be a good idea to tie the copper wire to the notch of the metal rod using the unsanded portion of the copper wire that comes just before the sanded end. Then wrap the small piece of sanded copper wire to the notch. See the diagram below for clarification. Do the same when tying the copper wire to the eye hook.

NOTES:

- If you tied the sanded portion of the copper wire to the notch, the copper wire may break during your experiment and then you will have to start over from the beginning.
- Make sure the rod is balanced and sits horizontally when undisturbed. Ask your TA to check that your current setup conducts well.

To measure the period of oscillations line up the laser so that it reflects off of the mirror on the metal rod. Give the pendulum a small perturbation by gently twisting the wire from the top using the lever (a few degrees are enough). It should start moving in a smooth horizontal path. Watch the motion of the laser across the room. Mark off a point near the center of the path with a piece of tape. Measure the time of 5 oscillations, where one oscillation is defined as the laser proceeding from the point you marked off to its extreme point, back through the marked point and off to the other extreme, and finally back to the starting point. This full oscillation should take less than half a minute (depending on your wire length). Measure about 5 of these times (in other words: five measurements of five oscillations each). You can then determine the time of a single period and its uncertainty



by following a similar procedure to that used in experiment 1 when determining the period of a pendulum. NOTE: The advantage of using a point near the center of the path is that the laser moves relatively quickly through that point, as opposed to the extreme points where the laser can linger for several seconds without moving very much.

Calculate  $\kappa = \frac{4\pi^2 I}{T^2}$  and its error  $\sigma_\kappa$ .

### 3.4. Setting up the Capacitor

Wait for the metal rod to reach an equilibrium position. Place an aluminum plate up against the copper disk with the rubber stopper. Orient the aluminum plate such that it is PARALLEL to the copper disk. Obtain a small bucket of water and submerge the lower part of the other disk in it to minimize vibrations from air currents. Be sure to allow water currents to dissipate before making measurements. Obtain a voltage source, some banana cables, and some cables with alligator clip ends. Attach one cable to the aluminum plate and plug it into the ground terminal on the voltage source. Then, plug a cable into the positive terminal and connect the other end to the eye hook pointing up from the piston (You might have to connect a couple cables together in order to have a cable long enough to reach the top of the setup.). There may be some charge build up on the plates, so you should discharge the plates. Finally, turn the dial on the voltage source so that it clicks off (at 0V). Plug the source into an outlet and flip the black switch about the terminals to the +1000V setting.

FROM HERE ON, IT IS VERY IMPORTANT NOT TO DISTURB THE TABLE, SO AS NOT TO STIR UP THE WATER IN THE BUCKET. DO NOT LEAN ON THE TABLE AND MINIMIZE ANY VIBRATIONS

THAT CAN TRANSFER TO THE WATER.

#### Question

10. In this experiment, why must you first wait for the metal rod to reach an equilibrium position before placing the aluminum plate next to the copper plate?

### 3.5. Measuring the Voltage

Properly setting and measuring the equilibrium is crucial for accurate results in this experiment. Allow the pendulum to come to equilibrium, making sure that the damping disk is not resting on the edge of the container. Discharge any static charge on the plates by short circuiting the leads attached to the terminals. With the leads still shorted, bring the aluminum plate up against the rubber stopper.

Record the equilibrium angle of the pendulum using the protractor on the top of the gray disk.

Next, you will calibrate the voltage source by measuring the force between the plates of the capacitor for different settings of applied voltage, specifically for 600, 700, 800, 900, and 1000V. You will do this by measuring the torsion required to counteract the attractive force between the plates. Remove the short circuit between the leads at the voltage source, making sure that one of them is connected to the ground terminal and one of them is connected to the +1000V positive terminal.

- Set the voltage source to 600V. The aluminum plate and copper disk should attract.
- Rotate the piston slowly and carefully, in 1 degree increments, until the capacitor plates barely separate, and record this angle. Do it with two people: one observing the plates and one rotating the piston.
- Observe the system and determine whether you think you overshot the exact angle required to balance the force between the plate and disk, and if so, by how much. Discuss the consequences of this (systematic) error in your conclusion.
- Rotate the pendulum back to the initial equilibrium angle, and increase the voltage to 700V. It is not necessary to discharge the plates at this point. Repeat the procedure to record the angle required to separate the disk and plate. Rotate the pendulum back to equilibrium and repeat the angle measurements for 800, 900, and 1000V.

At this stage, you should have angle measurements for 600, 700, 800, 900, and 1000V. Remove the aluminum plate, discharge the aluminum plate and copper disk, and check how close to your original equilibrium angle is the

current equilibrium state. In other words, if the pendulum is no longer in the old equilibrium, then rotate the piston until the pendulum is at equilibrium and record this angle as the new equilibrium angle for your next set of data.

Repeat the above steps until you have at least five angle measurements for each voltage setting. Remember to discharge the capacitors and verify your equilibrium after each set of voltages. Calculate the average angle and its standard deviation at each voltage source setting, and calculate the voltage using Eq. (7).

## 4. CAUTIONS

### 4.1. Minimizing Vibrations

Make sure that no one is leaning on the table while the measurements are being taken. Try to block out air currents by surrounding your setup with boxes or boards.

### 4.2. Risk of Shock

Do not touch the electrical connections when a voltage is being applied. Keep the desk and your hands dry.

## 5. ERROR ANALYSIS

Use error propagation to find the error of your measurements,  $\sigma_V$ .

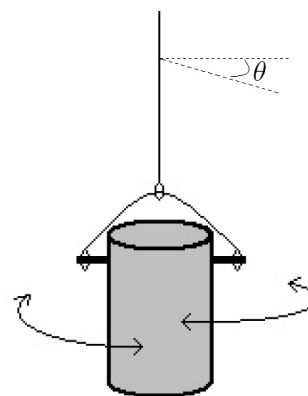
Make a graph of your data where the  $x$  axis is the values of the voltage supplied to the capacitor by the voltage source (600-1000 V) and the  $y$  axis is the calculated values obtained from the torsional pendulum. Make sure to include vertical error bars on your points. Use the procedure outlined in section 2.1 of this lab guide to calculate a line of best fit and plot that line on your graph. Then calculate  $\tilde{\chi}^2$  for this fit and find the probability of obtaining a chi-squared value greater than or equal to the value you calculated by using the chart in the back of Taylor (or the one included in these guidelines).

## Appendix 1: Torsional Pendulum

Instead of gravity, a torsional pendulum begins its motion when it is given an angular displacement. The equations for torque are as follows, where the dot signifies a derivative in time and  $\kappa$  is the torsional constant of the fiber.

$$\tau = -\kappa\theta = I\ddot{\theta}$$

where  $I$  is the moment of inertia of a cylinder about its longitudinal axis. Rearranging gives a differential equation for  $\theta$ .



$$\ddot{\theta} + \frac{\kappa}{I}\theta = 0$$

The solution is periodic motion where the frequency of oscillations,  $\omega = \sqrt{\frac{\kappa}{I}}$ . The period of this pendulum is easily measured and is given by the formula:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{\kappa}}$$