

Use: $\hbar^2/2m_e = (\hbar c)^2/2m_e c^2 = (1973\text{eV}\text{\AA})^2/(2 \times 511,000\text{eV}) = 3.81\text{eV}\text{\AA}^2$, m_e =electron mass

Problem 1 (10 pts)

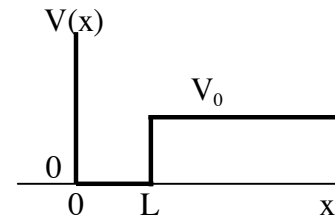
An electron is in a stationary state of an infinite square well that extends from $x=0$ to $x=12\text{\AA}$. It is equally likely to be found at $x=2\text{\AA}$ as at $x=6\text{\AA}$.

- Find the energy of this electron (in eV), assuming it is in the state of lowest energy that has this property. Justify your answer.
- How much more or less likely is it to find this electron at $x=2\text{\AA}$ than at $x=1\text{\AA}$?
- Find the wavelengths (in \AA) of all the photons that this electron can emit by making transitions from this state to other stationary states in this well. Ignore selection rules.

Problem 2 (10 pts)

An electron is in the lowest energy state of the potential $V(x)$

shown in the figure: $V(x)=\infty$ for $x<0$
 $V(x)=0$ for $0<x<L$
 $V(x)=V_0$ for $x>L$.



In the region $0<x<L$ the wavefunction of this electron is $\psi(x) = A \sin kx$, with $k = 0.95\pi/L$. Its energy is 5eV .

- Explain clearly how you can tell from this information that V_0 is smaller than 10^{20}eV .
- Find V_0 , in eV. Explain all the steps.

Hint: use the Schrodinger equation and the conditions of continuity of ψ and ψ' .

Problem 3 (10 pts)

An electron is in the ground state of a harmonic oscillator potential and has energy 3eV . Using the fact that for a particle in a harmonic oscillator potential the average potential and kinetic energies are equal:

- Find the uncertainty in the position of this electron, Δx , in \AA .
- Find the uncertainty in the momentum of this electron divided by \hbar , i.e. $(\Delta p/\hbar)$, in units \AA^{-1} .
- Verify that the uncertainty principle is satisfied.

Hint: Use that ΔO , the uncertainty of an operator O , is defined as

$$\Delta O = \sqrt{\langle O^2 \rangle - \langle O \rangle^2}$$

Justify all steps. Don't use formulas that are not in the textbook unless you derive them.

Justify all your answers to all problems. You may not use formulas that are not in the textbook without deriving them.