Physics 4E Quiz 2 Solutions Spring 2015

Energy conservation: $\frac{hc}{\lambda} - \frac{hc}{\lambda'} = E_{kin}^{electron} \Rightarrow$

$$\frac{hc}{\lambda} = \frac{12{,}400 eV\mathring{A}}{1.24\mathring{A}} + 26.32 eV = 10{,}026.32 eV \Rightarrow$$

$$\boxed{\lambda = 1.237\mathring{A}}$$

(b) $\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1-\cos\theta) \Rightarrow 1-\cos\theta = \frac{\lambda'-\lambda}{h/mc} \Rightarrow$

$$\cos\theta = 1 - \frac{\lambda'-\lambda}{h/mc} = 1 - \frac{0.00326}{0.0243} = 0.866 \Rightarrow \boxed{\theta = 30°}$$

(c) Momentum conservation:

$$p_e \sin\phi = p'\sin\theta$$
$$p_e \cos\phi = p - p'\cos\theta$$

$\Rightarrow \tan\phi = \frac{p'\sin\theta}{p - p'\cos\theta} \Rightarrow$ (use $p = \frac{h}{\lambda}$)

$$\tan\phi = \frac{\frac{1}{\lambda'}\sin\theta}{\frac{1}{\lambda} - \frac{1}{\lambda'}\cos\theta} = \frac{\sin\theta}{1 - \frac{\lambda}{\lambda'}\cos\theta} = \frac{1/2}{1 - \frac{\lambda}{\lambda'}\frac{\sqrt{3}}{2}} = 3.675$$

$$\Rightarrow \boxed{\phi = 74.8°}$$

or, approximately: $\tan\phi = \frac{\sin\theta}{1 - \frac{\lambda}{\lambda'}\cos\theta} \simeq \frac{\sin\theta}{1-\cos\theta} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} =$

$$= \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} \Rightarrow \boxed{\phi = 90° - \frac{\theta}{2} = 75°}$$

Problem 2

(a) Since Rutherford's law is satisfied for all angles $\Rightarrow$ even in a head-on collision, α particle does not penetrate nucleus.

Distance of closest approach: r_d

$$E_{\alpha in} = \frac{k q_\alpha Q}{r_d} = \frac{k e^2 \cdot 2 \cdot 47}{r_d} = \frac{94 k e^2}{r_d} \Rightarrow$$

$$\Rightarrow r_d = \frac{94 k e^2}{E_{\alpha in}} = \frac{94 \cdot 14.4 \, \cancel{eV} \cdot \mathring{A}}{20 \cdot 10^6 \, \cancel{eV}} = 6.77 \times 10^{-5} \, \mathring{A}$$

$\Rightarrow$ $\boxed{\text{radius of Ag nucleus is smaller than } 6.77\times10^{-5}\,\mathring{A}}$ (a)

(b) $f = \pi b^2 n t$ = fraction of particles scattered at angle $> \theta$

with $b = C \cdot \cot\frac{\theta}{2}$ $\Rightarrow$ $b(60°) = b(120°) \cdot \frac{\cot 30°}{\cot 60°}$

$$\frac{\cot 30°}{\cot 60°} = \frac{\cos 30° \; \sin 60°}{\sin 30° \cdot \cos 60°} = \frac{\sin^2 60°}{\sin^2 30°} = \left(\frac{\sqrt{3}}{2}\right)^2 \cdot \frac{1}{\left(\frac{1}{2}\right)^2} = 3$$

$\Rightarrow$ $b(60°) = 3 b(120°) \Rightarrow$ fraction scattered at angle $> 60° = \frac{9}{18{,}000} = \frac{1}{2{,}000}$

(c) When energy is increased to 25 MeV, if ratio is different $\Rightarrow$

$\boxed{\#\text{ at } \measuredangle > 120° / \#\text{ at angle} > 60° \text{ is } \underline{\text{smaller}}}$ because large $\measuredangle$ scattering is less if α-particle penetrates in nucleus.

$$r_d = \frac{94 k e^2}{25 \text{ MeV}} = 5.41 \times 10^{-5} \, \mathring{A} \Rightarrow$$

$\Rightarrow$ $\boxed{\text{radius of Ag nucleus is } \underline{\text{larger}} \text{ than } 5.41\times10^{-5}\,\mathring{A}}$

Problem 3

$$\frac{hc}{\lambda} = E_0 Z^2\left(1-\frac{1}{n^2}\right) \; ; \; \lambda = 60.78\,\text{Å}, \; E_0 = 13.6\,\text{eV} \Rightarrow$$

$$Z^2 = \frac{hc/\lambda}{E_0\left(1-\frac{1}{n^2}\right)} = \frac{12,400}{60.78\times 13.6\times\left(1-\frac{1}{n^2}\right)} = \frac{15}{1-\frac{1}{n^2}}$$

If $n=2$, $\frac{15}{1-\frac{1}{4}} = \frac{15}{3}\cdot 4 = 20 = Z^2$ not possible, Z has to be integer.

If $n=3$, $1-\frac{1}{9} = \frac{8}{9} \Rightarrow \frac{15}{8}\cdot 9$ not integer $\Rightarrow$ not possible

If $n=4$, $1-\frac{1}{16} = \frac{15}{16} \Rightarrow Z^2 = 16 \Rightarrow \boxed{Z=4}$ (a)

If $n>4$, $15 < Z^2 < 16$ not possible. So $Z=4$.

(b) The initial state was $\boxed{n=4}$

(c) If it is in state $n=4$ and absorbs a photon, the lowest energy photon it can absorb brings it to $n=5$, that corresponds to the largest wavelength

$$\frac{hc}{\lambda} = E_0 Z^2\left(\frac{1}{4^2}-\frac{1}{5^2}\right) = E_0 Z^2 \cdot \frac{9}{400} \Rightarrow$$

$$\Rightarrow \lambda = \frac{hc}{E_0 Z^2}\,\frac{400}{9} = \frac{12,400}{13.6\times 16}\cdot\frac{400}{9} = \frac{12,400}{13.6}\cdot\frac{25}{9} = 2532.7\,\text{Å}$$

$$\boxed{\lambda = 2532.7\,\text{Å}} \text{ (c)}$$