Physics 4E — Quiz 1 Solutions — Spring 2015

Problem 1

$$f(v_x) = \left(\frac{m}{2\pi k T}\right)^{1/2} e^{-\frac{1}{2}\frac{m v_x^2}{k_B T}} \quad ; \quad v_x = 100\,m/s$$

$$\frac{f(v_x=0)}{f(v_x)} = e^{\frac{1}{2}\frac{m v_x^2}{k_B T}} = e^{\frac{v_x^2}{60{,}000\,m^2/s^2}} = e^{\frac{1}{6}} \Rightarrow f(v_x=0) = e^{1/6} f(v_x = 100\tfrac{m}{s})$$

$$\langle v^2 \rangle = \frac{3kT}{m} = \left(300\,\frac{m}{s}\right)^2 = 90{,}000\,\frac{m^2}{s^2} \Rightarrow \boxed{\frac{kT}{m} = 30{,}000\,\frac{m^2}{s^2}}$$

$$\Rightarrow N(v_x=0) = e^{1/6} N(v_x = 100\,\tfrac{m}{s}) = \boxed{5907 \text{ molecules}}$$

(b)
$$\frac{f(2v_x)}{f(v_x)} = e^{-\frac{3}{2}\frac{m v_x^2}{k_B T}} = e^{-\frac{v_x^2}{20{,}000\,m^2/s^2}} = e^{-\frac{1}{2}}$$

$$\Rightarrow N(200\,m/s) = e^{-1/2} N(100\,m/s) = \boxed{3033 \text{ molecules}}$$

(c) Distribution of speeds

$$g(v) = A v^2 e^{-\frac{1}{2} m v^2 / k_B T} \Rightarrow$$

$$\frac{g(2v)}{g(v)} = 4 e^{-\frac{3}{2}\frac{m v^2}{k_B T}} = 4 e^{-1/2}$$

$$\Rightarrow N(v = 200\,m/s) = N(v = 100\,m/s) \cdot 4e^{-1/2} = \boxed{12{,}131 \text{ molecules}}$$

Problem 2

$n(\lambda) = \frac{8\pi}{\lambda^4}$ = density of modes per unit volume per unit wavelength

$$N_\lambda = \frac{8\pi}{\lambda^4} \cdot V \cdot \Delta\lambda = 10^{12} \Rightarrow V = \frac{10^{12} \cdot (10^4 \text{Å})^4}{8\pi \cdot 5 \text{Å}} = \frac{10^{28} \text{Å}^3}{40\pi} = \frac{10^4 \text{cm}^3}{40\pi}$$

$$\boxed{V = \frac{1000}{4\pi} \text{cm}^3 = 79.6 \text{ cm}^3}$$

(b) $$\bar{E}_\lambda = \frac{hc/\lambda}{e^{hc/\lambda kT} - 1} = 1.24 \times 10^{-6} \text{eV} \; ; \quad \frac{hc}{\lambda} = \frac{12{,}400 \text{ eVÅ}}{10{,}000 \text{ Å}} = 1.24 \text{eV}$$

$$\Rightarrow e^{hc/\lambda k_B T} = 10^6 \Rightarrow \frac{hc}{\lambda k_B T} = \ln 10^6 \Rightarrow T = \left(\frac{hc}{\lambda}\right) \frac{1}{k_B \ln 10^6} \Rightarrow$$

$$\Rightarrow T = \frac{1.24 \text{eV} \cdot 11{,}600}{\text{eV} \cdot \ln 10^6} \text{K} \Rightarrow \boxed{T = 1041 \text{ K}}$$

(c) av # of photons in mode = 10 ⇒

$$\frac{1}{e^{hc/\lambda kT} - 1} = 10 \Rightarrow e^{hc/\lambda kT} - 1 = 0.1 \Rightarrow e^{\frac{hc}{\lambda kT}} = 1.1 \Rightarrow$$

$$\frac{hc}{\lambda kT} = \ln 1.1 = 0.0953 \Rightarrow \boxed{T = \frac{hc}{\lambda k \cdot \ln 1.1} = 150{,}918 \text{ K}}$$

What fraction of kT is the average energy in this mode?

$$\bar{E}_\lambda = \frac{hc/\lambda}{e^{hc/\lambda kT} - 1} = \frac{hc/\lambda kT}{e^{hc/\lambda kT} - 1} \cdot kT = \ln(1.1) \cdot 10 \, kT = \boxed{0.953 \, kT}$$

Problem 3

(a) At same T, right bulb would emit 80 W ⇒ it is at lower T

⇒ right bulb looks more yellowish

(b) $R = \sigma T^4$ = power emitted per unit area

we have for the areas $A_r = 2 A_e$ ⇒ for total powers emitted:

$40\,W = A_e \cdot \sigma T_e^4$; $60\,W = 2A_e \cdot \sigma \cdot T_r^4 \Rightarrow 30\,W = A_e \cdot \sigma \cdot T_r^4 \Rightarrow$

$$\frac{T_r^4}{T_e^4} = \frac{3}{4} \Rightarrow T_r = \left(\frac{3}{4}\right)^{1/4} T_e \Rightarrow \text{for } T_e = 4000\,K, \boxed{T_r = 3722\,K}$$

(c) # of photons emitted $\sim \frac{1}{e^{hc/\lambda kT} - 1} \times$ area ⇒ for right

to emit fewer than left, $\frac{2}{e^{hc/\lambda kT_r} - 1} < \frac{1}{e^{hc/\lambda kT_e} - 1} \Rightarrow \boxed{\frac{e^{hc/\lambda kT_r} - 1}{e^{hc/\lambda kT_e} - 1} > 2}$

will happen for small λ. Assume we can ignore the 1's (check validity later)

$$\frac{e^{hc/\lambda kT_r}}{e^{hc/\lambda kT_e}} > 2 \Rightarrow e^{\frac{hc}{\lambda k}\left(\frac{1}{T_r} - \frac{1}{T_e}\right)} > 2 \Rightarrow \frac{hc}{\lambda k}\left(\frac{1}{T_r} - \frac{1}{T_e}\right) > \ln 2 \Rightarrow$$

$$\Rightarrow \lambda < \frac{hc}{k \ln 2\, T_e}\left(\frac{T_e}{T_r} - 1\right) = 51{,}879\,Å \left(\frac{T_e}{T_r} - 1\right) = \boxed{3875\,Å}$$

Check validity: $e^{\frac{hc}{\lambda k T_e}} = e^{9.28} = 10{,}721 >> 1$ ⇒ it was o.k. to ignore the 1's

So: right emits fewer than left for $\lambda < 3875\,Å$
right emits more than left for $\lambda > 3875\,Å$