Use: $\hbar^2/2m_e = 3.81 eVA^2$, m_e=electron mass. **Justify all your answers to all problems.**

Problem 1 (10 pts)
A light bulb emits 40W of power. The filament is 2cm long and 0.2mm in diameter.
(a) Find the temperature of the filament, in K.
(b) Find the wavelength for which the emitted power per unit wavelength is largest, in A (Angstrom).
(c) How many times more power per unit wavelength does this light bulb emit at the wavelength found in (b) compared to 1/2 that wavelength?

Problem 2 (10 pts)
An electron has a de Broglie wavelength equal to its Compton wavelength.
(a) What is the kinetic energy of this electron assuming classical mechanics, in eV?
(b) What is the kinetic energy of this electron assuming relativistic mechanics, in eV?
(c) Explain why the answers of (a) and (b) disagree and which one is right.

Problem 3 (10 pts)
An electron in the hydrogen Bohr atom is orbiting at speed 219 km/s.
(a) What is the quantum number n?
(b) What is the radius of the orbit in Angstrom?
(c) What is the frequency of revolution in Hz?
(d) If this electron makes a transition to the orbit with quantum number n-1, what is the frequency of the photon emitted, in Hz?
(e) Explain why the answers in (c) and (d) agree or don't agree.

Problem 4 (10 pts)
An electron is in a stationary state of a one-dimensional potential V(x) that satisfies V(x=0)=0. It is described by the wavefunction
$$\psi(x) = C(1 - x^2)^2 \qquad -1 \le x \le 1$$
$$\psi(x) = 0 \qquad\qquad |x| > 1$$
where C is a constant. Assume distances are measured in A (Angstrom), i.e. x=1 means x=1A.
(a) Find the energy of this electron, in eV.
(b) Find the value of V(x) at x=0.5, in eV.
(c) Find the classical turning points for this electron, i.e. the limits of the range in x that a classical electron would reach with this energy in this potential.
(d) Make a qualitative graph of V(x) versus x in the range (-1,1) and indicate in the graph the energy of the electron and the classical turning points.

Problem 5 (10 pts)

100eV

50eV

left   well   right

0

d_1 d_2 d_3
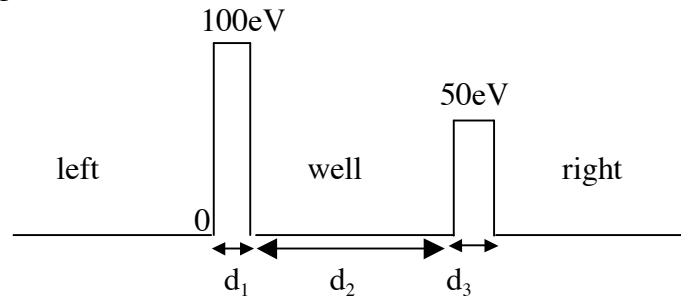
When an electron is in the ground state of the potential well shown in the figure, after a while it is found either in the region labeled "left" or in the region labeled "right" in the figure, twice as often in the left region as in the right region.
If d_1=0.1A and d_2=6A, what is d_3?

Problem 6 (10 pts)
There are 5 electrons in a two-dimensional infinite square well of sides L_1=4A, L_2=8A. Find the ground state energy of the system, in eV. Take into account that electrons have spin ½ and the Pauli exclusion principle.

Problem 7 (10 pts)
The radial wavefunction for an electron in a hydrogen-like ion of nuclear charge Ze is

$$R(r) = Cr(1 - \frac{r}{a_0})e^{-r/a_0}$$

with C a constant.
(a) Find the value of C in terms of a_0.
(b) Give the values of Z, n and ℓ for this wavefunction.
(c) Calculate $< \frac{1}{r} >$ for this electron in terms of a_0.
(d) Give the average potential energy for this electron, in eV.
(e) Find $< \frac{p_{op}^2}{2m} >$ for this wavefunction, with m the reduced mass, give the answer in eV.

Hint: it is not necessary to do integrals to find the answer to (e).

Problem 8 (10 pts)
The distance between protons in the molecule H_2 is 0.74A. The distance between protons in the molecular ion H_2^+ is 1.06A. Assume the frequencies of vibration for both molecules is the same, $\omega = 0.2eV/\hbar$.
(a) Assuming these molecules are in their lowest vibrational and rotational energy state, find the wavelengths of the photons they can absorb in a transition that does not change the electronic state. Hint: only 1 transition is allowed for each.
(b) At the temperature where the molar heat capacity of H_2 is 2R, is the molar heat capacity of H_2^+ larger or smaller? Explain why.
(c) Estimate the temperature below which the rotational energy does not contribute to the heat capacity of H_2^+. For H_2, that temperature is 74K.