Magnetic Moment

If a system of charged particles is rotating, it has a *magnetic moment* proportional to its angular momentum, a result from classical electrodynamics sometimes known as the *Larmor theorem.* Consider a particle of mass M and charge q moving in a circle of radius r with speed v and frequency $f = v/2\pi r$; this constitutes a current loop. The angular momentum of the particle is $L = Mvr$. The magnetic moment of the current loop is the product of the current and the area of the loop. For a circulating charge, the current is the charge times the frequency,

$$i = qf = \frac{qv}{2\pi r} \qquad \textbf{MM-1}$$

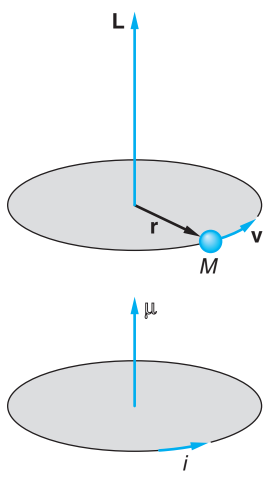
and the magnetic moment μ is[8]

$$\mu = iA = q\left(\frac{v}{2\pi r}\right)(\pi r^2) = \frac{1}{2}q\left(\frac{L}{M}\right) \qquad \textbf{MM-2}$$

From Figure MM-1 we see that, if q is positive, the magnetic moment is in the same direction as the angular momentum. If q is negative, $\boldsymbol{\mu}$ and $\mathbf{L}$ point in opposite directions, that is, they are antiparallel. This enables us to write Equation MM-2 as a vector equation:

$$\boldsymbol{\mu} = \frac{q}{2M}\mathbf{L} \qquad \textbf{MM-3}$$

Equation MM-3, which we have derived for a single particle moving in a circle, also holds for a system of particles in any type of motion if the charge-to-mass ratio q/M is the same for each particle in the system.



MM-1 A particle moving in a circle has angular momentum **L**. If the particle has a positive charge, the magnetic moment due to the current is parallel to **L**.

The behavior of a system with a magnetic moment $\boldsymbol{\mu}$ in an external (to the system) magnetic field $\mathbf{B}$ can be visualized by considering a small bar magnet (Figure MM-2). When placed in an external magnetic field $\mathbf{B}$, the bar magnet's magnetic moment $\boldsymbol{\mu}$ experiences a torque $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$ that tends to align the magnet with the field $\mathbf{B}$. If the magnet is spinning about its axis, the effect of the torque is to make the spin axis precess about the direction of the external field, just as a spinning top or gyroscope precesses about the direction of the gravitational field.

To change the orientation of the magnet relative to the applied field direction (whether or not it is spinning), work must be done on it. If it is moved through angle $d\theta$, the work required is

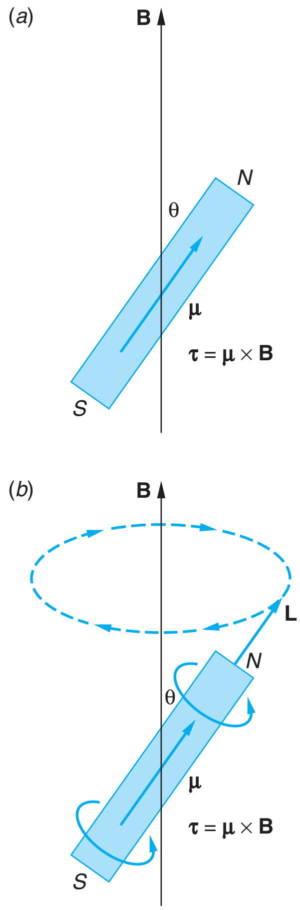
$$dW = \tau\, d\theta = \mu B \sin\theta\, d\theta = d(-\mu B \cos) = d(-\boldsymbol{\mu} \cdot \mathbf{B})$$

The potential energy of the magnetic moment $\boldsymbol{\mu}$ in the magnetic field $\mathbf{B}$ can thus be written

$$U = -\boldsymbol{\mu} \cdot \mathbf{B} \qquad \textbf{MM-4}$$

If **B** is in the z direction, the potential energy is

$$U = -\mu_z B \qquad \textbf{MM-5}$$



MM-2 Bar-magnet model of magnetic moment. (*a*) In an external magnetic field, the moment experiences a torque that tends to align it with the field. If the magnet is spinning (*b*), the torque causes the system to precess around the external field.