6-44. (a) For $x > 0$, $\hbar^2 k_2^2 / 2m + V_0 = E = \hbar^2 k_1^2 / 2m = 2V_0$

So, $k_2 = \; 2mV_0 \;^{1/2} / \hbar$. Because $k_1 = \; 4mV_0 \;^{1/2} / \hbar$, then $k_2 = k_1 / \sqrt{2}$

(b) $R = \; k_1 - k_2 \;^2 \big/ \; k_1 + k_2 \;^2$ (Equation 6-68)

$= \; 1 - 1/\sqrt{2} \;^2 \Big/ \; 1 + 1/\sqrt{2} \;^2 = 0.0294$, or 2.94% of the incident particles are reflected.

(c) $T = 1 - R = 1 - 0.0294 = 0.971$

(d) 97.1% of the particles, or $0.971 \times 10^6 = 9.71 \times 10^5$, continue past the step in the $+x$ direction. Classically, 100% would continue on.

6-45 (a) Equation 6-76: $T \approx 16 \dfrac{E}{V_0}\left(1 - \dfrac{E}{V_0}\right) e^{-2\alpha a}$ where $\alpha = 2\sqrt{2m_p(V_0 - E)} / \hbar$

and $a =$ barrier width.

$$-2\alpha a = -2\left[\sqrt{2(938\,\text{MeV/c}^2)(50-44)\text{MeV}} / 6.58 \times 10^{-22}\,\text{MeV}\cdot\text{s}\right] \times 10^{-15} = -1.075$$

$$T \approx 16 \frac{44\,\text{MeV}}{50\,\text{MeV}}\left(1 - \frac{44\,\text{MeV}}{50\,\text{MeV}}\right) e^{-1.075}$$

$$T \approx 0.577$$

(b) decay rate $\approx N \times T$ where

$$N = \frac{v_{proton}}{2R} = \left[\frac{2 \times 44\,\text{MeV} \times 1.60 \times 10^{-13}\,\text{J/MeV}}{1.67 \times 10^{-27}\,\text{kg}}\right]^{1/2} \times \frac{1}{2 \times 10^{-15}\,\text{m}} = 4.59 \times 10^{22}\,\text{s}^{-1}$$

$$\text{decay rate} \approx 0.577 \times 4.59 \times 10^{22}\,\text{s}^{-1} = 2.65 \times 10^{22}\,\text{s}^{-1}$$

(c) In the expression for T, $e^{-1.075} \Rightarrow e^{-2.150}$, and so $T \approx 0.577 \Rightarrow T \approx 0.197$. The decay rate then becomes $9.05 \times 10^{21}\,\text{s}^{-1}$, a factor of 0.34× the original value.

6-46. (a) For $x > 0$, $\hbar^2 k_2^2 / 2m - V_0 = E = \hbar^2 k_1^2 / 2m = 2V_0$

So, $k_2 = \; 6mV_0 \;^{1/2} / \hbar$. Because $k_1 = \; 4mV_0 \;^{1/2} / \hbar$, then $k_2 = \sqrt{3/2}\, k_1$

(b) $R = (k_1 - k_2)^2 / (k_1 + k_2)^2$

$$R = (k_1 - k_2)^2 / (k_1 + k_2)^2 = \left(1 - \sqrt{3/2}\right)^2 \Big/ \left(1 + \sqrt{3/2}\right)^2 = 0.0102$$

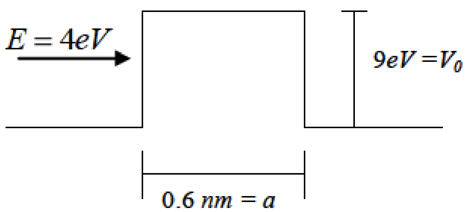
Or 1.02% are reflected at $x = 0$.

(c) $T = 1 - R = 1 - 0.0102 = 0.99$

(d) 99% of the particles, or $0.99 \times 10^6 = 9.9 \times 10^5$, continue in the $+x$ direction.

Classically, 100% would continue on.

6-47. (a)



$$\alpha = \sqrt{2m(V_0 - E)}/\hbar$$

$$= \sqrt{2(0.511 \times 10^6 eV/c^2)(5eV)}/\hbar$$

$$= \sqrt{5.11 \times 10^6 eV}\,\frac{eV}{c}/\hbar$$

$$= \frac{2260eV}{197.3eV \bullet nm} = 11.46nm^{-1}$$

and $\alpha a = 0.6nm \times 11.46nm^{-1} = 6.87$

Since αa is not $\ll 1$, use Equation 6-75:

The transmitted fraction

$$T = \left[1 + \frac{\sinh^2 \alpha a}{4(E/V_0)(1 - E/V_0)}\right]^{-1} = \left[1 + \left(\frac{81}{80}\right)\sinh^2(6.87)\right]^{-1}$$

Recall that $\sinh x = (e^x - e^{-x})/2$,

$$T = \left[1 + \frac{81}{80}\left(\frac{e^{6.87} - e^{-6.87}}{2}\right)^2\right]^{-1} = 4.3 \times 10^{-6} \text{ is the transmitted fraction.}$$

(b) Noting that the size of T is controlled by αa through the $\sinh^2 \alpha a$ and increasing T implies increasing E. Trying a few values, selecting $E = 4.5eV$ yields $T = 8.7 \times 10^{-6}$ or approximately twice the value in part (a).

6-50. Using Equation 6-76,

$$T \approx 16\frac{E}{V_0}\left(1-\frac{E}{V_0}\right)e^{-2\alpha a} \text{ where } E = 2.0eV,\ V_0 = 6.5eV, \text{ and } a = 0.5nm.$$

$$T \approx 16\left(\frac{2.0}{6.5}\right)\left(1-\frac{2.0}{6.5}\right)e^{-2\ 10.87\ \ 0.5} \approx 6.5\times10^{-5} \text{ (Equation 6-75 yields } T = 6.6\times10^{-5}.)$$

6-51. $R = \frac{k_1 - k_2\ ^2}{k_1 + k_2\ ^2}$ and $T = 1 - R$ (Equations 6-68 and 6-70)

(a) For protons:

$$k_1 = \sqrt{2mc^2E}/\hbar c = \sqrt{2\ \ 938MeV\ \ \ 40MeV}/197.3MeV\bullet fm = 1.388$$

$$k_2 = \sqrt{2mc^2\ \ E - V_0}/\hbar c = \sqrt{2\ \ 938MeV\ \ \ 10MeV}/197.3MeV\bullet fm = 0.694$$

$$R = \left(\frac{1.388 - 0.694}{1.388 + 0.694}\right)^2 = \left(\frac{0.694}{2.082}\right)^2 = 0.111 \quad \text{And } T = 1 - R = 0.889$$

(b) For electrons:

$$k_1 = 1.388\left(\frac{0.511}{938}\right)^{1/2} = 0.0324 \qquad k_2 = 0.694\left(\frac{0.511}{938}\right)^{1/2} = 0.0162$$

$$R = \left(\frac{0.0324 - 0.0162}{0.0324 + 0.0162}\right)^2 = 0.111 \quad \text{And } T = 1 - R = 0.889$$

No, the mass of the particle is not a factor. (We might have noticed that $\sqrt{m}$ could be canceled from each term.

6-56. (a) The requirement is that $\psi^2(x) = \psi^2(-x) = \psi(-x)\psi(-x)$. This can only be true if:

$\psi(-x) = \psi(x)$ or $\psi(-x) = -\psi(x)$.

(b) Writing the Schrödinger equation in the form $\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi$, the general solutions of this 2nd order differential equation are: $\psi(x) = A\sin kx$ and $\psi(\mathrm{x}) = A\cos kx$ where $k = \sqrt{2mE}/\hbar$. Because the boundaries of the box are at $x = \pm L/2$, both solutions are allowed (unlike the treatment in the text where one boundary was at $x = 0$). Still, the solutions are all zero at $x = \pm L/2$ provided that an integral number of half wavelengths fit between $x = -L/2$ and $x = +L/2$. This will occur for:

$\psi_n(x) = (2/L)^{1/2}\cos(n\pi x/L)$ when $n = 1, 3, 5, \cdots$. And for

$\psi_n(x) = (2/L)^{1/2}\sin(n\pi x/L)$ when $n = 2, 4, 6, \cdots$.

The solutions are alternately even and odd.

(c) The allowed energies are: $E = \hbar^2 k^2/2m = \hbar^2(n\pi/L)^2/2m = n^2h^2/8mL^2$.

6-58. $\langle x^2 \rangle = \int_0^L \frac{2}{L} x^2 \sin^2 \frac{n\pi x}{L} dx$ Letting $u = n\pi x / L$, $du = n\pi / L \; dx$

$$\langle x^2 \rangle = \frac{2}{L}\left(\frac{L}{n\pi}\right)^2 \left(\frac{L}{n\pi}\right)\int_0^{n\pi} u^2 \sin^2 u du$$

$$= \frac{2}{L}\left(\frac{L}{n\pi}\right)^3 \left[\frac{u^3}{6} - \left(\frac{u^2}{4} - \frac{1}{8}\right)\sin 2u - \frac{u\cos 2u}{4}\right]\Bigg|_0^{n\pi}$$

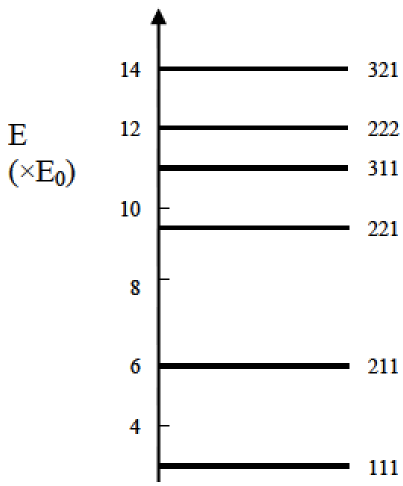
$$= \frac{2}{L}\left(\frac{L}{n\pi}\right)^3 \left[\frac{n\pi^{\;3}}{6} - 0 - \frac{n\pi}{4} - 0\right] = \frac{L^2}{3} - \frac{L^2}{2n^2\pi^2}$$

7-1. $E_{n_1 n_2 n_3} = \frac{\hbar^2\pi^2}{2mL^2}\left(n_1^2 + n_2^2 + n_3^2\right)$ (Equation 7-4)

$E_{311} = \frac{\hbar^2\pi^2}{2mL^2}\left(3^2 + 1^2 + 1^2\right) = 11E_0$ where $E_0 = \frac{\hbar^2\pi^2}{2mL^2}$

$E_{222} = E_0\left(2^2 + 2^2 + 2^2\right) = 12E_0$ and $E_{321} = E_0\left(3^2 + 2^2 + 1^2\right) = 14E_0$

The 1st, 2nd, 3rd, and 5th excited states are degenerate.



7-2. $E_{n_1 n_2 n_3} = \frac{\hbar^2\pi^2}{2m}\left(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2} + \frac{n_3^2}{L_3^2}\right) = \frac{\hbar^2\pi^2}{2mL_1^2}\left(n_1^2 + \frac{n_2^2}{4} + \frac{n_3^2}{9}\right)$ (Equation 7-5)

$n_1 = n_2 = n_3 = 1$ is the lowest energy level.

$E_{111} = E_0\left(1 + 1/4 + 1/9\right) = 1.361E_0$ where $E_0 = \frac{\hbar^2\pi^2}{2mL_1^2}$

The next nine levels are, increasing order,

n_1	n_2	n_3	$E\left(\times E_0\right)$
1	1	2	1.694
1	2	1	2.111
1	1	3	2.250
1	2	2	2.444
1	2	3	3.000
1	1	4	3.028
1	3	1	3.360
1	3	2	3.472
1	2	4	3.778

7-3. (a) $\psi_{n_1 n_2 n_3}(x,y,z) = A\cos\frac{n_1\pi x}{L}\sin\frac{n_2\pi y}{L}\sin\frac{n_3\pi z}{L}$

(b) They are identical. The location of the coordinate origin does not affect the energy level structure.

7-4. $\psi_{111}(x,y,z) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{2L_1}\sin\frac{\pi z}{3L_1}$ $\quad\psi_{112}(x,y,z) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{2L_1}\sin\frac{2\pi z}{3L_1}$

$\psi_{121}(x,y,z) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{L_1}\sin\frac{\pi z}{3L_1}$ $\quad\psi_{122}(x,y,z) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{L_1}\sin\frac{2\pi z}{3L_1}$

$\psi_{113}(x,y,z) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{2L_1}\sin\frac{\pi z}{L_1}$

7-7. $$E_0 = \frac{\hbar^2\pi^2}{2mL^2} = \frac{\left(1.055\times10^{-34}J\bullet s\right)^2\pi^2}{2\left(9.11\times10^{-31}kg\right)\left(0.10\times10^{-9}m\right)^2\left(1.609\times10^{-19}J/eV\right)} = 37.68eV$$

$$E_{311} - E_{111} = \Delta E = 11E_0 - 3E_0 = 8E_0 = 301eV$$

$$E_{222} - E_{111} = \Delta E = 12E_0 - 3E_0 = 9E_0 = 339eV$$

$$E_{321} - E_{111} = \Delta E = 14E_0 - 3E_0 = 11E_0 = 415eV$$

7-8. (a) Adapting Equation 7-3 to two dimensions (i.e., setting $k_3 = 0$), we have

$$\psi_{n_1 n_2} = A\sin\frac{n_1\pi x}{L}\sin\frac{n_2\pi y}{L}$$

(b) From Equation 7-5, $E_{n_1 n_2} = \frac{\hbar^2 \pi^2}{2mL^2}\left(n_1^2 + n_2^2\right)$

(c) The lowest energy degenerate states have quantum numbers $n_1 = 1$, $n_2 = 2$, and $n_1 = 2$, $n_2 = 1$.

7-5. $E_{n_1 n_2 n_3} = \frac{\hbar^2\pi^2}{2m}\left(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{(2L_1)^2} + \frac{n_3^2}{(4L_1)^2}\right) = \frac{\hbar^2\pi^2}{2mL_1^2}\left(n_1^2 + \frac{n_2^2}{4} + \frac{n_3^2}{16}\right)$ (from Equation 7-5)

$E_{n_1 n_2 n_3} = \left(n_1^2 + \frac{n_2^2}{4} + \frac{n_3^2}{16}\right)$ where $E_0 = \frac{\hbar^2\pi^2}{2mL_1^2}$

(Problem 7-5 continued)

(a)

n_1	n_2	n_3	$E\,(\times E_0)$
1	1	1	1.313
1	1	2	1.500
1	1	3	1.813
1	2	1	2.063
1	1	4	2.250
1	2	2	2.250
1	2	3	2.563
1	1	5	2.813
1	2	4	3.000
1	3	1	3.313

(b) 1,1,4 and 1,2,2