5-3. $E_k = eV_o = \frac{p^2}{2m\lambda^2} = \frac{(hc)^2}{2mc^2\lambda^2}$ $\qquad V_o = \frac{1}{e}\cdot\frac{(1240eV\cdot nm)^2}{2(5.11\times10^5 eV)(0.04nm)^2} = 940V$

5-4. $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_k}} = \frac{hc}{\sqrt{2mc^2E_k}}$ (from Equation 5-2)

(a) For an electron: $\lambda = \frac{1240eV\cdot nm}{\left[(2)(0.511\times10^6 eV)(4.5\times10^3 eV)\right]^{1/2}} = 0.0183nm$

(b) For a proton: $\lambda = \frac{1240eV\cdot nm}{\left[(2)(983.3\times10^6 eV)(4.5\times10^3 eV)\right]^{1/2}} = 4.27\times10^{-4}\,nm$

(c) For an alpha particle:

$$\lambda = \frac{1240eV\cdot nm}{\left[(2)(3.728\times10^9 eV)(4.5\times10^3 eV)\right]^{1/2}} = 2.14\times10^{-4}\,nm$$

5-5. $\lambda = h/p = h/\sqrt{2mE_k} = hc/\left[2mc^2(1.5kT)\right]^{1/2}$ (from Equation 5-2)

Mass of N_2 molecule =

$$2\times14.0031u(931.5MeV/uc^2) = 2.609\times10^4\,MeV/c^2 = 2.609\times10^{10}\,eV/c^2$$

$$\lambda = \frac{1240eV\cdot nm}{\left[(2)(2.609\times10^{10} eV)(1.5)(8.617\times10^{-5} eV/K)(300K)\right]^{1/2}} = 0.0276nm$$

5-11. $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_pE_k}} = 0.25nm$

Squaring and rearranging,

$$E_k = \frac{h^2}{2m_p\lambda^2} = \frac{(hc)^2}{2(m_pc^2)\lambda^2} = \frac{(1240eV\cdot nm)^2}{2(938\times10^6 eV)(0.25nm)^2} = 0.013eV$$

$n\lambda = D\sin\phi \quad\rightarrow\quad \sin\phi = n\lambda/D = (1)(0.25nm)/(0.304nm)$

$\sin\phi = 0.822 \quad\rightarrow\quad \phi = 55°$

5-6. $$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_k}} = \frac{hc}{\sqrt{2mc^2E_k}} = \frac{1240eV \bullet nm}{\left[2\left(939.57\times10^6 eV\right)\left(0.02eV\right)\right]^{1/2}} = 0.202nm$$

5-12. (a) $n\lambda = D\sin\phi \quad \therefore \quad D = \dfrac{n\lambda}{\sin\phi} = \dfrac{nhc}{\sin\phi\sqrt{2mc^2E_k}}$

$$= \frac{(1)(1240eV\cdot nm)}{(\sin 55.6°)\left[2\left(5.11\times10^5 eV\right)(50eV)\right]^{1/2}} = 0.210nm$$

(b) $\sin\phi = \dfrac{n\lambda}{D} = \dfrac{(1)(1240eV\cdot nm)}{(0.210nm)\left[2\left(5.11\times10^5 eV\right)(100eV)\right]^{1/2}} = 0.584$

$\phi = \sin^{-1}(0.584) = 35.7°$

5-17. (a) $y = y_1 + y_2$

$$= 0.002m\cos(8.0x/m - 400t/s) + 0.002m\cos(7.6x/m - 380t/s)$$

$$= 2(0.002m)\cos\left[\frac{1}{2}(8.0x/m - 7.6x/m) - \frac{1}{2}(400t/s - 380t/s)\right]$$

$$\times\cos\left[\frac{1}{2}(8.0x/m + 7.6x/m) - \frac{1}{2}(400t/s + 380t/s)\right]$$

$$= 0.004m\cos(0.2x/m - 10t/s)\times\cos(7.8x/m - 390t/s)$$

(b) $v = \dfrac{\overline{\omega}}{\overline{k}} = \dfrac{390/s}{7.8/m} = 50m/s$

(c) $v_s = \dfrac{\Delta\omega}{\Delta k} = \dfrac{20/s}{0.4/m} = 50m/s$

(d) Successive zeros of the envelope requires that $0.2\Delta x/m = \pi$, thus

$\Delta x = \dfrac{\pi}{0.2} = 5\pi m$ with $\Delta k = k_1 - k_2 = 0.4m^{-1}$ and $\Delta x = \dfrac{2\pi}{\Delta k} = 5\pi m.$

5-22. (a) $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE_k}} = \dfrac{hc}{\sqrt{2mc^2E_k}} = \dfrac{1240eV\cdot nm}{\left[2\left(0.511\times10^6 eV\right)(5eV)\right]^{1/2}} = 0.549nm$

$d\sin\theta = \lambda/2$ For first minimum (see Figure 5-17).

$d = \dfrac{\lambda}{2\sin\theta} = \dfrac{0.549nm}{2\sin 5°} = 3.15nm$ slit separation

(b) $\sin 5° = 0.5cm/L$ where L = distance to detector plane $L = \dfrac{0.5cm}{2\sin 5°} = 5.74cm$

5-23. (a) The particle is found with equal probability in any interval in a force-free region.

Therefore, the probability of finding the particle in any interval Δx is proportional to

Δx. Thus, the probability of finding the sphere *exactly* in the middle, i.e., with

$\Delta x = 0$ is zero.

(b) The probability of finding the sphere somewhere within 24.9*cm* to 25.1*cm* is

proportional to $\Delta x = 0.2cm$. Because there is a force free length $L = 48cm$ available

to the sphere and the probability of finding it somewhere in L is unity, then the

probability that it will be found in $\Delta x = 0.2cm$ between 24.9*cm* and 25.1*cm* (or any

interval of equal size) is: $P\Delta x = (1/48)(0.2cm) = 0.00417$.

5-24. Because the particle must be in the box $\int_0^L \psi^* \psi\, dx = 1 = \int_0^L A^2 \sin^2(\pi x/L)dx = 1$

Let $u = \pi x/L$; $x = 0 \to u = 0$; $x = L \to u = \pi$ and $dx = (L/\pi)du$, so we have

$$\int_0^\pi A^2 (L/\pi)\sin^2 u\,du = A^2 (L/\pi)\int_0^\pi \sin^2 u\,du = 1$$

$$(L/\pi)A^2\int_0^\pi \sin^2 u\,du = (L/\pi)A^2\left[\frac{u}{2} - \frac{\sin 2u}{4}\right]_0^\pi = (L/\pi)A^2(\pi/2) = (LA^2)/2 = 1$$

$$\therefore \quad A^2 = 2/L \to A = (2/L)^{1/2}$$

5-25. (a) At $x = 0$: $Pdx = |\psi(0,0)|^2\, dx = |Ae^0|^2\, dx = A^2 dx$

(b) At $x = \sigma$: $Pdx = \left|Ae^{-\sigma^2/4\sigma^2}\right|^2 dx = \left|Ae^{-1/4}\right|^2 dx = 0.61A^2 dx$

(c) At $x = 2\sigma$: $Pdx = \left|Ae^{-4\sigma^2/4\sigma^2}\right|^2 dx = \left|Ae^{-1}\right|^2 dx = 0.14A^2 dx$

(d) The electron will most likely be found at $x = 0$, where Pdx is largest.

5-32. Because $c = f\lambda$ for photon, $\lambda = c/f = hc/hf = hc/E$, so

$$E = \frac{hc}{\lambda} = \frac{1240eV\bullet nm}{5.0\times10^{-3}nm} = 2.48\times10^{5}eV$$

and $p = \frac{E}{c} = \frac{2.48\times10^{5}eV}{3\times10^{8}m/s} = 8.3\times10^{-7}eV\bullet s/m$

For electron:

$$\Delta p = \frac{h}{\Delta x} = \frac{4.14\times10^{-15}eV\bullet s}{5.0\times10^{-12}m} = 8.3\times10^{-4}eV\bullet s/m$$

Notice that Δp for the electron is 1000 times larger than p for the photon.

5-27. $\Delta E \Delta t \approx \hbar \;\rightarrow\; \Delta E \approx \hbar / \Delta t = \dfrac{1.055 \times 10^{-34} J \cdot s}{10^{-7} s \left(1.609 \times 10^{-19} J / eV\right)} \approx 6.6 \times 10^{-9} eV$

5-35. The size of the object needs to be of the order of the wavelength of the $10 MeV$ neutron.

$\lambda = h / p = h / \gamma m u$. γ and u are found from:

$E_k = m_n c^2 (\gamma - 1)$ or $\gamma - 1 = 10 MeV / 939 MeV$

$\gamma = 1 + 10/939 = 1.0106 = 1/\left(1 - u^2/c^2\right)^{1/2}$ or $u = 0.14c$

Then, $\lambda = \dfrac{h}{\gamma m u} = \dfrac{hc}{\left[\gamma m c^2 (u/c)\right]} = \dfrac{1240 eV \cdot nm}{\left[(1.0106)(939 \times 10^6 eV)(0.14)\right]} = 9.33 fm$

Nuclei are of this order of size and could be used to show the wave character of $10 MeV$ neutrons.

5-39. $\Delta E \Delta t \approx \dfrac{\hbar}{2} \;\rightarrow\; \Delta t = \dfrac{\hbar}{2 \Delta E}$

$\Delta t = \dfrac{6.58 \times 10^{-16} eV \cdot s}{2 \times 250 \times 10^6 eV} = 1.32 \times 10^{-24} s$

5-4 (a) For a proton or neutron:

$\Delta x \Delta p \approx \dfrac{\hbar}{2}$ and $\Delta p = m \Delta v$ assuming the particle speed to be non-relativistic.

$\Delta v = \dfrac{\hbar}{2 m \Delta x} = \dfrac{1.055 \times 10^{-34} J \cdot s}{2\left(1.67 \times 10^{-27} kg\right)\left(10^{-15} m\right)} = 3.16 \times 10^7 m/s \approx 0.1c$ (non-relativistic)

(b) $E_k \approx \dfrac{1}{2} m v^2 = \dfrac{\left(1.67 \times 10^{-27} kg\right)\left(3.16 \times 10^7 m/s\right)^2}{2} = 8.34 \times 10^{-13} J = 5.21 MeV$

(c) Given the proton or neutron velocity in (a), we expect the electron to be relativistic,

in which case, $E_k = mc^2 (\gamma - 1)$ and

$$\Delta p = \frac{\hbar}{2\Delta x} \approx \gamma m v \quad \rightarrow \quad \gamma v \approx \frac{\hbar}{2m\Delta x}$$

For the relativistic electron we assume $v \approx c$

$$\gamma \approx \frac{\hbar}{2mc\Delta x} = \frac{1.055\times 10^{-34}\, J\cdot s}{2\left(9.11\times 10^{-31}\, kg\right)\left(3.00\times 10^{8}\, m/s\right)\left(10^{-15}\, m\right)} = 193$$

$$E_k = mc^2\left(\gamma - 1\right) = \left(9.11\times 10^{-31}\, kg\right)\left(3.00\times 10^{8}\, m/s\right)^2\left(192\right) = 1.58\times 10^{-11}\, J = 98 MeV$$