4-2. $\frac{1}{\lambda_{mn}} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$ where $m = 2$ for Balmer series (Equation 4-2)

$$\frac{1}{379.1nm} = \frac{1.097\times10^7 m^{-1}}{10^9 nm/m}\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

$$\frac{1}{4} - \frac{1}{n^2} = \frac{10^9 nm/m}{379.1nm\left(1.097\times10^7 m^{-1}\right)} = 0.2405$$

$$\frac{1}{n^2} = 0.2500 - 0.2405 = 0.0095$$

$$n^2 = \frac{1}{0.0095} \quad\rightarrow\quad n = (1/0.0095)^{1/2} = 10.3 \quad\rightarrow\quad n = 10$$

$$n = 10 \rightarrow n = 2$$

4-3. $\frac{1}{\lambda_{mn}} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$ where $m = 1$ for Lyman series (Equation 4-2)

$$\frac{1}{164.1nm} = \frac{1.097\times10^7 m^{-1}}{10^9 nm/m}\left(1 - \frac{1}{n^2}\right)$$

$$\frac{1}{n^2} = 1 - \frac{10^9 nm/m}{164.1nm\left(1.097\times10^7 m^{-1}\right)} = 1 - 0.5555 = 0.4445$$

$$n = \left(1/0.4445\right)^{1/2} = 1.5$$

No, this is not a hydrogen Lyman series transition because n is not an integer.

4-4. $\frac{1}{\lambda_{mn}} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$ (Equation 4-2)

For the Brackett series $m = 4$ and the first four (i.e., longest wavelength lines have $n = 5$, 6, 7, and 8.

$$\frac{1}{\lambda_{45}} = 1.097\times10^7 m^{-1}\left(\frac{1}{4^2} - \frac{1}{5^2}\right) = 2.468\times10^5 m^{-1}$$

$\lambda_{45} = \frac{1}{2.68\times10^5 m^{-1}} = 4.052\times10^{-6} m = 4052nm.$ Similarly,

$$\lambda_{46} = \frac{1}{3.809\times10^5 m^{-1}} = 2.625\times10^{-6} m = 2625nm$$

$$\lambda_{47} = \frac{1}{4.617\times10^5 m^{-1}} = 2.166\times10^{-6} m = 2166nm$$

$$\lambda_{48} = \frac{1}{5.142\times10^5 m^{-1}} = 1.945\times10^{-6} m = 1945nm$$

These lines are all in the infrared.

4-7. $\Delta N \propto \dfrac{1}{\sin^4(\theta/2)} = \dfrac{A}{\sin^4(\theta/2)}$ (From Equation 4-6), where A is the product of the two quantities in parentheses in Equation 4-6.

(a) $\dfrac{\Delta N(10°)}{\Delta N(1°)} = \dfrac{A/\sin^4(10°/2)}{A/\sin^4(1°/2)} = \dfrac{\sin^4(0.5°)}{\sin^4(5°)} = 1.01\times10^{-4}$

(b) $\dfrac{\Delta N(30°)}{\Delta N(1°)} = \dfrac{\sin^4(0.5°)}{\sin^4(15°)} = 1.29\times10^{-6}$

4-9. $r_d = \dfrac{kq_\alpha Q}{(1/2)m_\alpha v^2} = \dfrac{ke^2 \bullet 2 \bullet 79}{E_{k\alpha}}$ (Equation 4-11)

For $E_{k\alpha} = 5.0 MeV$: $r_d = \dfrac{(1.44 MeV \bullet fm)(2)(79)}{5.0 MeV} = 45.5 fm$

For $E_{k\alpha} = 7.7 MeV$: $r_d = 29.5 fm$

For $E_{k\alpha} = 12 MeV$: $r_d = 19.0 fm$

4-10. $r_d = \dfrac{kq_\alpha Q}{(1/2)m_\alpha v^2} = \dfrac{ke^2 \bullet 2 \bullet 79}{E_{k\alpha}}$ (Equation 4-11)

$E_{k\alpha} = \dfrac{(1.44 MeV \bullet fm)(2)(13)}{4 fm} = 9.4 MeV$

4-17. $f_{rev} = \dfrac{mk^2Z^2e^4}{2\pi\hbar^3n^3}$ (Equation 4-29)

$$= \frac{mc^2Z^2\left(ke^2\right)^2}{2\pi\hbar n^3\left(\hbar c\right)^2} = \frac{cZ^2}{\left(h/mc\right)n^3}\left(\frac{ke^2}{\hbar c}\right)^2 = \frac{cZ^2\alpha^2}{\lambda_c n^3}$$

$$= \frac{\left(3.00\times10^8\,m/s\right)\left(1\right)^2}{\left(0.00243\times10^{-9}\,m\right)\left(2\right)^3}\left(\frac{1}{137}\right)^2 = 8.22\times10^{14}\,Hz$$

$$N = f_{rev}t = \left(8.22\times10^{14}\,Hz\right)\left(10^{-8}\,s\right) = 8.22\times10^6 \text{ revolutions}$$

4-13. (a) $r_n = \frac{n^2 a_0}{Z}$ (Equation 4-18)

$$r_6 = \frac{6^2\left(0.053nm\right)}{1} = 1.91nm$$

(b) $r_6\left(He^+\right) = \frac{6^2\left(0.053nm\right)}{2} = 0.95nm$

4-15. $\frac{1}{\lambda} = Z^2 R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$ (Equation 4-22)

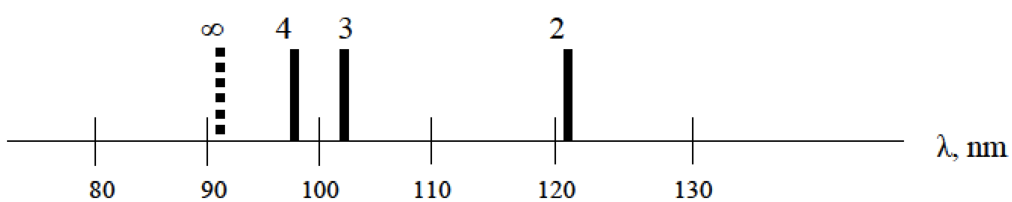
$$\frac{1}{\lambda_{ni}} = R\left(\frac{1}{1^2} - \frac{1}{n_i^2}\right) = R\left(\frac{n_i^2 - 1}{n_i^2}\right)$$

$$\lambda_{ni} = \frac{n_i^2}{R\left(n_i^2 - 1\right)} = \frac{n_i^2}{\left(1.0968\times10^7 m\right)\left(n_i^2 - 1\right)} = \left(91.17nm\right)\left(\frac{n_i^2}{n_i^2 - 1}\right)$$

$$\lambda_2 = \frac{4}{3}\left(91.17nm\right) = 121.57nm \qquad \lambda_3 = \frac{9}{8}\left(91.17nm\right) = 102.57nm$$

$$\lambda_4 = \frac{16}{15}\left(91.17nm\right) = 97.25nm \qquad \lambda_\infty = 91.17nm$$

None of these are in the visible; all are in the ultraviolet.



4-19. (a) $a_\mu = \frac{\hbar^2}{\mu_\mu k e^2} = \frac{\mu_e}{\mu_\mu}\bullet\frac{\hbar^2}{\mu_e k e^2} = \frac{\mu_e}{\mu_\mu} a_0 = \frac{9.11\times10^{-31}kg}{1.69\times10^{-28}kg}\left(0.0529nm\right) = 2.56\times10^{-4}nm$

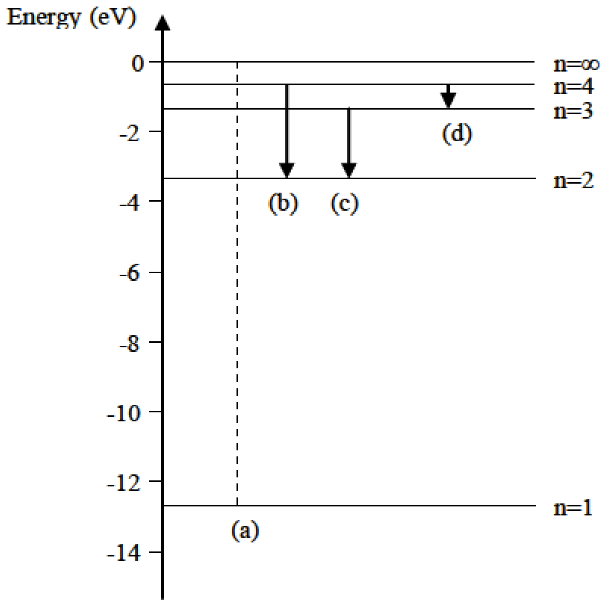
(b) $E_\mu = \frac{\mu_\mu k^2 e^4}{2\hbar^2} = \frac{\mu_\mu}{\mu_e}\bullet\frac{\mu_e k^2 e^4}{2\hbar^2} = \frac{\mu_\mu}{\mu_e}\bullet E_0 = \frac{1.69\times10^{-28}kg}{9.11\times10^{-31}kg}\left(13.6eV\right) = 2520eV$

(c) The shortest wavelength in the Lyman series is the series limit ($n_i = \infty$, $n_f = 1$). The photon energy is equal in magnitude to the ground state energy $-E_\mu$.

$$\lambda_\infty = \frac{hc}{E_\mu} = \frac{1240 eV\bullet nm}{2520 eV} = 0.492 nm$$

(The reduced masses have been used in this solution.)

4-21.



(a) Lyman limit, (b) H_β line, (c) H_α line, (d) longest wavelength line of Paschen series

4-24. (a) The reduced mass correction to the Rydberg constant is important in this case.

$$R = R_\infty\left(\frac{1}{1+m/M}\right) = R_\infty\left(\frac{1}{2}\right) = 5.4869\times10^6 m^{-1} \quad \text{(from Equation 4-26)}$$

$$E_n = -hcR/n^2 \quad \text{(from Equations 4-23 and 4-24)}$$

$$E_1 = -(1240 eV\bullet nm)(5.4869\times10^6 m^{-1})(10^{-9} m/nm)/(1)^2 = -6.804 eV$$

Similarly, $E_2 = -1.701 eV$ and $E_3 = -0.756 eV$

(b) Lyman α is the $n=2 \rightarrow n=1$ transition.

$$\frac{hc}{\lambda}=E_2-E_1 \quad \rightarrow \quad \lambda_\alpha=\frac{hc}{E_2-E_1}=\frac{1240eV\bullet nm}{-1.701eV-(-6.804eV)}=243nm$$

Lyman β is the $n=3 \rightarrow n=1$ transition.

$$\lambda_\beta=\frac{hc}{E_3-E_1}=\frac{1240eV\bullet nm}{-0.756eV-(-6.804eV)}=205nm$$

4-25. (a) The radii of the Bohr orbits are given by (see Equation 4-18)

$r=n^2a_0/Z$ where $a_0=0.0529nm$ and $Z=1$ for hydrogen.

For $n=600$, $r=(600)^2(0.0529nm)=1.90\times10^4nm=19.0\mu m$

This is about the size of a tiny grain of sand.

(b) The electron's speed in a Bohr orbit is given by

$v^2=ke^2/mr$ with $Z=1$

Substituting r for the $n=600$ orbit from (a), then taking the square root,

$$v^2=(8.99\times10^9N\bullet m^2)(1.609\times10^{-19}C)^2/(9.11\times10^{-31}kg)(19.0\times10^{-6}m)$$

$$v^2=1.33\times10^7m^2/s^2 \quad \rightarrow \quad v=3.65\times10^3m/s$$

For comparison, in the $n=1$ orbit, v is about $2\times10^6m/s$

4-26. (a) $\frac{1}{\lambda}=R(Z-1)^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$

$$\lambda_3=\left[(1.097\times10^7m^{-1})(42-1)^2\left(\frac{1}{1^2}-\frac{1}{3^2}\right)\right]^{-1}=6.10\times10^{-11}m=0.0610nm$$

$$\lambda_4=\left[(1.097\times10^7m^{-1})(42-1)^2\left(\frac{1}{1^2}-\frac{1}{4^2}\right)\right]^{-1}=5.78\times10^{-11}m=0.0578nm$$

(b) $\lambda_{limit}=\left[(1.097\times10^7m^{-1})(42-1)^2\left(\frac{1}{1^2}-0\right)\right]^{-1}=5.42\times10^{-11}m=0.0542nm$

4-32. (a) $-E_1 = E_0 Z^2 / n^2$ (Equation 4-20)

$$= 13.6eV\left(74-1\right)^2 / \left(1\right)^2 = 7.25\times 10^4 eV = 72.5keV$$

(b) $-E_1 = E_0\left(Z-\sigma\right)^2 / n^2 = 69.5\times 10^3 eV = 13.6eV\left(74-\sigma\right)^2 / \left(1\right)^2$

$$\left(74-\sigma\right)^2 = 69.5\times 10^3 eV / 13.6eV$$

$$\sigma = 74 - \left(69.5\times 10^3 eV / 13.6eV\right)^{1/2} = 2.5$$

4-27. $\frac{1}{\lambda} = R(Z-1)^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right) = R(Z-1)^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$ for K_α

$$Z-1 = \left[\frac{1}{\lambda R\left(1-\frac{1}{4}\right)}\right]^{1/2} = \left[\frac{1}{(0.0794nm)(1.097\times10^{-2}/nm)(3/4)}\right]^{1/2}$$

$Z = 1+39.1 \approx 40$ Zirconium

4-29. $r_n = \frac{n^2 a_0}{Z}$ (Equation 4-18)

The n =1 electrons "see" a nuclear charge of approximately $Z-1$, or 78 for Au.

$r_1 = 0.0529nm/78 = 6.8\times10^{-4}nm(10^{-9}m/nm)(10^{15}fm/m) = 680fm$, or about 100 times the radius of the Au nucleus.

4-36. $\Delta E = \frac{hc}{\lambda} = \frac{1240eV\cdot nm}{790nm} = 1.610eV$. The first decrease in current will occur when the voltage reaches $1.61V$.

4-42. Those scattered at $\theta = 180°$ obeyed the Rutherford formula. This is a head-on collision where the α comes instantaneously to rest before reversing direction. At that point its kinetic energy has been converted entirely to electrostatic potential energy, so

$\frac{1}{2}m_\alpha v^2 = 7.7MeV = \frac{k(2e)(79e)}{r}$ where r = upper limit of the nuclear radius.

$$r = \frac{k(2)(79)e^2}{7.7MeV} = \frac{2(79)(1.440MeV\cdot fm)}{7.7MeV} = 29.5fm$$

4-45. $\lambda = \left[R\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)\right]^{-1}$ $\quad\Delta\lambda = \frac{d\lambda}{d\mu}\Delta\mu = (-R^{-2})\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)^{-1}\frac{dR}{d\mu}\Delta\mu$

Because $R \propto \mu$, $dR/d\mu = R/\mu$. $\Delta\lambda \approx (-R^{-2})\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)^{-1}(R/\mu)\Delta\mu = -\lambda(\Delta\mu/\mu)$

$$\mu_H = \frac{m_e m_p}{m_e + m_p} \qquad \mu_D = \frac{m_e m_d}{m_e + m_d}$$

$$\frac{\Delta\mu}{\mu} = \frac{\mu_D - \mu_H}{\mu_H} = \frac{\mu_D}{\mu_H} - 1 = \frac{m_e m_d / (m_e + m_d)}{m_e m_p / (m_e + m_p)} - 1 = \frac{m_d / (m_e + m_d)}{m_p / (m_e + m_p)} - 1 = \frac{m_e (m_d - m_p)}{m_p (m_e + m_d)}$$

If we approximate $m_d = 2m_p$ and $m_e \ll m_d$, then $\frac{\Delta\mu}{\mu} \approx \frac{m_e}{2m_p}$ and

$$\Delta\lambda = -\lambda(\Delta\mu / \mu) = -(656.3nm)\frac{0.511MeV}{2(938.28MeV)} = -0.179nm$$

4-54. (a) $E_n = -\frac{ke^2}{2r_n} = -\frac{ke^2}{2n^2 r_o}$ $\qquad E_{n-1} = -\frac{ke^2}{2(n-1)^2 r_o}$

$$hf = E_n - E_{n-1} = -\frac{ke^2}{2n^2 r_o} - \left(-\frac{ke^2}{2(n-1)^2 r_o}\right)$$

$$f = \frac{ke^2}{2hr_o}\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right] = \frac{ke^2}{2hr_o}\frac{n^2 - (n^2 - 2n + 1)}{n^2(n-1)^2}$$

$$= \frac{ke^2}{2hr_o}\frac{2n-1}{n^2(n-1)^2} \approx \frac{ke^2}{r_o hn^3} \quad \text{for n} \gg 1$$

(b) $f_{rev} = \frac{v}{2\pi r} \quad \rightarrow \quad f_{rev}^2 = \frac{v^2}{4\pi^2 r^2} = \frac{1}{4\pi^2 mr}\frac{mv^2}{r} = \frac{1}{4\pi^2 mr}\frac{ke^2}{r^2} = \frac{ke^2}{4\pi^2 mr_o^3 n^6}$

(c) The correspondence principle implies that the frequencies of radiation and revolution are equal.

$$f^2 = \left(\frac{ke^2}{r_o h n^3}\right)^2 = \frac{ke^2}{4\pi^2 m r_o^3 n^6} = f_{rev}^2 \qquad r_o = \frac{ke^2}{4\pi^2 m n^6}\left(\frac{hn^3}{ke^2}\right)^2 = \frac{h^2}{4\pi^2 mke^2} = \frac{\hbar^2}{mke^2}$$

which is the same as a_0 in Equation 4-19.

4-55. $\dfrac{kZe^2}{r} = \dfrac{mv^2}{r} \rightarrow \dfrac{kZe^2}{r^2} = \dfrac{(\gamma mv)^2}{mr}$ (from Equation 4-12)

$$\gamma v = \left(\frac{kZe^2}{mr}\right)^{1/2} = \frac{v}{\sqrt{1-\beta^2}}$$

$$\frac{c^2\beta^2}{1-\beta^2} = \left(\frac{kZe^2}{mr}\right) \text{ Therefore, } \beta^2\left[c^2 + \left(\frac{kZe^2}{mr}\right)\right] = \left(\frac{kZe^2}{mr}\right)$$

$$\beta^2 \approx \frac{1}{c^2}\left(\frac{kZe^2}{ma_o}\right) \rightarrow \beta = 0.0075Z^{1/2} \rightarrow v = 0.0075cZ^{1/2} = 2.25\times10^6 m/s \times Z^{1/2}$$

$$E = KE - kZe^2/r = mc^2(\gamma - 1) - \frac{kZe^2}{r} = mc^2\left[\frac{1}{\sqrt{1-\beta^2}} - 1\right] - \frac{kZe^2}{r}$$

And substituting $\beta = 0.0075$ and $r = a_o$

$$E = 511\times10^3 eV\left[\frac{1}{\sqrt{1-(0.0075)^2}} - 1\right] - 28.8Z\ eV$$

$$= 14.4eV - 28.8Z\ eV = -14.4Z\ eV$$

4-59. Refer to Figure 4-16. All possible transitions starting at $n = 5$ occur.

$n = 5$ to $n = 4, 3, 2, 1$

$n = 4$ to $n = 3, 2, 1$

$n = 3$ to $n = 2, 1$

$n = 2$ to $n = 1$

Thus, there are 10 different photon energies emitted.

n_i	n_f	fraction	no. of photons
5	4	1/4	125
5	3	1/4	125
5	2	1/4	125
5	1	1/4	125
4	3	$1/4 \times 1/3$	42
4	2	$1/4 \times 1/3$	42
4	1	$1/4 \times 1/3$	42
3	2	$1/2\left[1/4+1/4(1/3)\right]$	83
3	1	$1/2\left[1/4+1/4(1/3)\right]$	83
2	1	$\left[\left(1/2(1/4+1/4)(1/3)\right)+1/4(1/3)+1/4\right]$	250

Total = 1,042

Note that the number of electrons arriving at the $n = 1$ level (125+42+83+250) is 500, as it should be.