3-60. Let $x = \frac{\varepsilon}{kT} = \frac{hf}{kT}$ in Equation 3-15:

$$\sum_{n=0}^{\infty} f_n = A\sum_{n=0}^{\infty} e^{-nx} = A\left[e^0 + e^{-x} + \left(e^{-x}\right)^2 + \left(e^{-x}\right)^3 + \cdots\right] = A\left(1 + y + y^2 + y^3 + \cdots\right) = 1$$

Where $y = e^{-x}$. This sum is the series expansion of

$(1-y)^{-1}$, i.e., $(1-y)^{-1} = 1 + y + y^2 + y^3 + \cdots$. Then $\sum f_n = A(1-y)^{-1} = 1$ gives $A = 1-y$.

Writing Equation 3-16 in terms of x and y.

$$\overline{E} = \sum_{n=0}^{\infty} E_n A e^{-E_n/kT} = A\sum_{n=0}^{\infty} nhf e^{-nhf/kT} = Ahf\sum_{n=0}^{\infty} ne^{-nx}$$

Note that $\sum ne^{-nx} = -(d/dx)\sum e^{-nx}$. But $\sum e^{-nx} = (1-y)^{-1}$, so we have

$$\sum ne^{-nx} = -\frac{d}{dx}\sum e^{-nx} = -\frac{d}{dx}(1-y)^{-1} = (1-y)^{-2}\left(-\frac{dy}{dx}\right) = y(1-y)^{-2}$$

Since $\frac{dy}{dx} = \frac{d\left(e^{-x}\right)}{dx} = -e^{-x} = -y.$

Multiplying this sum by hf and by $A = (1-y)$, the average energy is

$$\overline{E} = hfA\sum_{n=0}^{\infty} ne^{-nx} = hf\,(1-y)\,y(1-y)^{-2} = \frac{hfy}{1-y} = \frac{hfe^{-x}}{1-e^{-x}}$$

Multiplying the numerator and the denominator by e^{-x} and substituting for x, we obtain

$\overline{E} = \frac{hf}{e^{hf/kT} - 1}$, which is Equation 3-17.