# Chapter 9

1. We use Eq. 9-5 to solve for $(x_3, y_3)$.

(a) The $x$ coordinate of the system's center of mass is:

$$x_{\text{com}} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{(2.00 \text{ kg})(-1.20 \text{ m}) + (4.00 \text{ kg})(0.600 \text{ m}) + (3.00 \text{ kg})\, x_3}{2.00 \text{ kg} + 4.00 \text{ kg} + 3.00 \text{ kg}}$$
$$= -0.500 \text{ m}.$$

Solving the equation yields $x_3 = -1.50$ m.

(b) The $y$ coordinate of the system's center of mass is:

$$y_{\text{com}} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{(2.00 \text{ kg})(0.500 \text{ m}) + (4.00 \text{ kg})(-0.750 \text{ m}) + (3.00 \text{ kg})\, y_3}{2.00 \text{ kg} + 4.00 \text{ kg} + 3.00 \text{ kg}}$$
$$= -0.700 \text{ m}.$$

Solving the equation yields $y_3 = -1.43$ m.

2. Our notation is as follows: $x_1 = 0$ and $y_1 = 0$ are the coordinates of the $m_1 = 3.0$ kg particle; $x_2 = 2.0$ m and $y_2 = 1.0$ m are the coordinates of the $m_2 = 4.0$ kg particle; and $x_3 = 1.0$ m and $y_3 = 2.0$ m are the coordinates of the $m_3 = 8.0$ kg particle.

(a) The $x$ coordinate of the center of mass is

$$x_{\text{com}} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{0 + (4.0 \text{ kg})(2.0 \text{ m}) + (8.0 \text{ kg})(1.0 \text{ m})}{3.0 \text{ kg} + 4.0 \text{ kg} + 8.0 \text{ kg}} = 1.1 \text{ m}.$$

(b) The $y$ coordinate of the center of mass is

$$y_{\text{com}} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{0 + (4.0 \text{ kg})(1.0 \text{ m}) + (8.0 \text{ kg})(2.0 \text{ m})}{3.0 \text{ kg} + 4.0 \text{ kg} + 8.0 \text{ kg}} = 1.3 \text{ m}.$$

(c) As the mass of $m_3$, the topmost particle, is increased, the center of mass shifts toward that particle. As we approach the limit where $m_3$ is infinitely more massive than the others, the center of mass becomes infinitesimally close to the position of $m_3$.

3. We use Eq. 9-5 to locate the coordinates.

(a) By symmetry $x_{com} = -d_1/2 = -(13\text{ cm})/2 = -6.5$ cm. The negative value is due to our choice of the origin.

(b) We find $y_{com}$ as

$$y_{com} = \frac{m_i y_{com,i} + m_a y_{com,a}}{m_i + m_a} = \frac{\rho_i V_i y_{com,i} + \rho_a V_a y_{cm,a}}{\rho_i V_i + \rho_a V_a}$$
$$= \frac{(11\text{ cm}/2)(7.85\text{ g/cm}^3) + 3(11\text{ cm}/2)(2.7\text{ g/cm}^3)}{7.85\text{ g/cm}^3 + 2.7\text{ g/cm}^3} = 8.3\text{ cm.}$$

(c) Again by symmetry, we have $z_{com} = (2.8\text{ cm})/2 = 1.4$ cm.

4. We will refer to the arrangement as a "table." We locate the coordinate origin at the left end of the tabletop (as shown in Fig. 9-37). With $+x$ rightward and $+y$ upward, then the center of mass of the right leg is at $(x,y) = (+L, -L/2)$, the center of mass of the left leg is at $(x,y) = (0, -L/2)$, and the center of mass of the tabletop is at $(x,y) = (L/2, 0)$.

(a) The $x$ coordinate of the (whole table) center of mass is

$$x_{com} = \frac{M(+L) + M(0) + 3M(+L/2)}{M + M + 3M} = \frac{L}{2}.$$

With $L = 22$ cm, we have $x_{com} = (22\text{ cm})/2 = 11$ cm.

(b) The $y$ coordinate of the (whole table) center of mass is

$$y_{com} = \frac{M(-L/2) + M(-L/2) + 3M(0)}{M + M + 3M} = -\frac{L}{5},$$

or $y_{com} = -(22\text{ cm})/5 = -4.4$ cm.

From the coordinates, we see that the whole table center of mass is a small distance 4.4 cm directly below the middle of the tabletop.

5. Since the plate is uniform, we can split it up into three rectangular pieces, with the mass of each piece being proportional to its area and its center of mass being at its geometric center. We'll refer to the large 35 cm × 10 cm piece (shown to the left of the $y$ axis in Fig. 9-38) as section 1; it has 63.6% of the total area and its center of mass is at $(x_1, y_1) = (-5.0\text{ cm}, -2.5\text{ cm})$. The top 20 cm × 5 cm piece (section 2, in the first quadrant) has 18.2% of the total area; its center of mass is at $(x_2, y_2) = (10\text{ cm}, 12.5\text{ cm})$. The bottom 10 cm x 10 cm piece (section 3) also has 18.2% of the total area; its center of mass is at $(x_3, y_3) = (5\text{ cm}, -15\text{ cm})$.

(a) The $x$ coordinate of the center of mass for the plate is

$$x_{\text{com}} = (0.636)x_1 + (0.182)x_2 + (0.182)x_3 = -0.45 \text{ cm} .$$

(b) The $y$ coordinate of the center of mass for the plate is

$$y_{\text{com}} = (0.636)y_1 + (0.182)y_2 + (0.182)y_3 = -2.0 \text{ cm} .$$

6. The centers of mass (with centimeters understood) for each of the five sides are as follows:

| | |
|---|---|
| $(x_1, y_1, z_1) = (0, 20, 20)$ | for the side in the $yz$ plane |
| $(x_2, y_2, z_2) = (20, 0, 20)$ | for the side in the $xz$ plane |
| $(x_3, y_3, z_3) = (20, 20, 0)$ | for the side in the $xy$ plane |
| $(x_4, y_4, z_4) = (40, 20, 20)$ | for the remaining side parallel to side 1 |
| $(x_5, y_5, z_5) = (20, 40, 20)$ | for the remaining side parallel to side 2 |

Recognizing that all sides have the same mass $m$, we plug these into Eq. 9-5 to obtain the results (the first two being expected based on the symmetry of the problem).

(a) The $x$ coordinate of the center of mass is

$$x_{\text{com}} = \frac{mx_1 + mx_2 + mx_3 + mx_4 + mx_5}{5m} = \frac{0 + 20 + 20 + 40 + 20}{5} = 20 \text{ cm}$$

(b) The $y$ coordinate of the center of mass is

$$y_{\text{com}} = \frac{my_1 + my_2 + my_3 + my_4 + my_5}{5m} = \frac{20 + 0 + 20 + 20 + 40}{5} = 20 \text{ cm}$$

(c) The $z$ coordinate of the center of mass is

$$z_{\text{com}} = \frac{mz_1 + mz_2 + mz_3 + mz_4 + mz_5}{5m} = \frac{20 + 20 + 0 + 20 + 20}{5} = 16 \text{ cm}$$

7. (a) By symmetry the center of mass is located on the axis of symmetry of the molecule – the $y$ axis. Therefore $x_{\text{com}} = 0$.

(b) To find $y_{\text{com}}$, we note that $3m_H y_{\text{com}} = m_N(y_N - y_{\text{com}})$, where $y_N$ is the distance from the nitrogen atom to the plane containing the three hydrogen atoms:

$$y_N = \sqrt{\left(10.14\times10^{-11}\text{ m}\right)^2 - \left(9.4\times10^{-11}\text{ m}\right)^2} = 3.803\times10^{-11}\text{ m}.$$

Thus,

$$y_{\text{com}} = \frac{m_{\text{N}} y_{\text{N}}}{m_{\text{N}} + 3m_{\text{H}}} = \frac{(14.0067)\left(3.803\times10^{-11}\,\text{m}\right)}{14.0067 + 3(1.00797)} = 3.13\times10^{-11}\,\text{m}$$

where Appendix F has been used to find the masses.

8. (a) Since the can is uniform, its center of mass is at its geometrical center, a distance $H/2$ above its base. The center of mass of the soda alone is at its geometrical center, a distance $x/2$ above the base of the can. When the can is full this is $H/2$. Thus the center of mass of the can and the soda it contains is a distance

$$h = \frac{M(H/2) + m(H/2)}{M+m} = \frac{H}{2}$$

above the base, on the cylinder axis. With $H$ = 12 cm, we obtain $h$ = 6.0 cm.

(b) We now consider the can alone. The center of mass is $H/2$ = 6.0 cm above the base, on the cylinder axis.

(c) As $x$ decreases the center of mass of the soda in the can at first drops, then rises to $H/2$ = 6.0 cm again.

(d) When the top surface of the soda is a distance $x$ above the base of the can, the mass of the soda in the can is $m_p = m(x/H)$, where $m$ is the mass when the can is full ($x = H$). The center of mass of the soda alone is a distance $x/2$ above the base of the can. Hence

$$h = \frac{M(H/2) + m_p(x/2)}{M + m_p} = \frac{M(H/2) + m(x/H)(x/2)}{M + (mx/H)} = \frac{MH^2 + mx^2}{2(MH + mx)}.$$

We find the lowest position of the center of mass of the can and soda by setting the derivative of $h$ with respect to $x$ equal to 0 and solving for $x$. The derivative is

$$\frac{dh}{dx} = \frac{2mx}{2(MH+mx)} - \frac{\left(MH^2 + mx^2\right)m}{2(MH+mx)^2} = \frac{m^2x^2 + 2MmHx - MmH^2}{2(MH+mx)^2}.$$

The solution to $m^2x^2 + 2MmHx - MmH^2 = 0$ is

$$x = \frac{MH}{m}\left(-1 + \sqrt{1 + \frac{m}{M}}\right).$$

The positive root is used since $x$ must be positive. Next, we substitute the expression found for $x$ into $h = (MH^2 + mx^2)/2(MH + mx)$. After some algebraic manipulation we obtain

$$h=\frac{HM}{m}\left(\sqrt{1+\frac{m}{M}}-1\right)=\frac{(12\text{ cm})(0.14\text{ kg})}{0.354\text{ kg}}\left(\sqrt{1+\frac{0.354\text{ kg}}{0.14\text{ kg}}}-1\right)=4.2\text{ cm}.$$

9. We use the constant-acceleration equations of Table 2-1 (with $+y$ downward and the origin at the release point), Eq. 9-5 for $y_{com}$ and Eq. 9-17 for $\vec{v}_{com}$.

(a) The location of the first stone (of mass $m_1$) at $t = 300 \times 10^{-3}$ s is

$$y_1 = (1/2)gt^2 = (1/2)(9.8\text{ m/s}^2)\,(300 \times 10^{-3}\text{ s})^2 = 0.44\text{ m},$$

and the location of the second stone (of mass $m_2 = 2m_1$) at $t = 300 \times 10^{-3}$ s is

$$y_2 = (1/2)gt^2 = (1/2)(9.8\text{ m/s}^2)(300 \times 10^{-3}\text{ s} - 100 \times 10^{-3}\text{ s})^2 = 0.20\text{ m}.$$

Thus, the center of mass is at

$$y_{com}=\frac{m_1y_1+m_2y_2}{m_1+m_2}=\frac{m_1(0.44\text{ m})+2m_1(0.20\text{ m})}{m_1+2m_2}=0.28\text{ m}.$$

(b) The speed of the first stone at time $t$ is $v_1 = gt$, while that of the second stone is

$$v_2 = g(t - 100 \times 10^{-3}\text{ s}).$$

Thus, the center-of-mass speed at $t = 300 \times 10^{-3}$ s is

$$\begin{aligned}v_{com}&=\frac{m_1v_1+m_2v_2}{m_1+m_2}=\frac{m_1\left(9.8\text{ m/s}^2\right)\left(300\times10^{-3}\text{ s}\right)+2m_1\left(9.8\text{ m/s}^2\right)\left(300\times10^{-3}\text{ s}-100\times10^{-3}\text{ s}\right)}{m_1+2m_1}\\&=2.3\text{ m/s}.\end{aligned}$$

10. We use the constant-acceleration equations of Table 2-1 (with the origin at the traffic light), Eq. 9-5 for $x_{com}$ and Eq. 9-17 for $\vec{v}_{com}$. At $t = 3.0$ s, the location of the automobile (of mass $m_1$) is

$$x_1=\tfrac{1}{2}at^2=\tfrac{1}{2}\left(4.0\text{ m/s}^2\right)\left(3.0\text{ s}\right)^2=18\text{ m},$$

while that of the truck (of mass $m_2$) is $x_2 = vt = (8.0\text{ m/s})(3.0\text{s}) = 24$ m. The speed of the automobile then is $v_1=at=\left(4.0\text{ m/s}^2\right)\left(3.0\text{ s}\right)=12\text{ m/s}$, while the speed of the truck remains $v_2 = 8.0$ m/s.

(a) The location of their center of mass is

$$x_{com} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{(1000\text{ kg})(18\text{ m}) + (2000\text{ kg})(24\text{ m})}{1000\text{ kg} + 2000\text{ kg}} = 22\text{ m}.$$

(b) The speed of the center of mass is

$$v_{com} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{(1000\text{ kg})(12\text{ m/s}) + (2000\text{ kg})(8.0\text{ m/s})}{1000\text{ kg} + 2000\text{ kg}} = 9.3\text{ m/s}.$$

11. The implication in the problem regarding $\vec{v}_0$ is that the olive and the nut start at rest. Although we could proceed by analyzing the forces on each object, we prefer to approach this using Eq. 9-14. The total force on the nut-olive system is $\vec{F}_o + \vec{F}_n = (-\hat{\text{i}} + \hat{\text{j}})\text{ N}$. Thus, Eq. 9-14 becomes

$$(-\hat{\text{i}} + \hat{\text{j}})\text{ N} = M\vec{a}_{com}$$

where $M$ = 2.0 kg. Thus, $\vec{a}_{com} = (-\tfrac{1}{2}\hat{\text{i}} + \tfrac{1}{2}\hat{\text{j}})\text{ m/s}^2$. Each component is constant, so we apply the equations discussed in Chapters 2 and 4 and obtain

$$\Delta\vec{r}_{com} = \frac{1}{2}\vec{a}_{com}t^2 = (-4.0\text{ m})\hat{\text{i}} + (4.0\text{ m})\hat{\text{j}}$$

when $t$ = 4.0 s. It is perhaps instructive to work through this problem the *long way* (separate analysis for the olive and the nut and then application of Eq. 9-5) since it helps to point out the computational advantage of Eq. 9-14.

12. Since the center of mass of the two-skater system does not move, both skaters will end up at the center of mass of the system. Let the center of mass be a distance $x$ from the 40-kg skater, then

$$(65\text{ kg})(10\text{ m} - x) = (40\text{ kg})x \Rightarrow x = 6.2\text{ m}.$$

Thus the 40-kg skater will move by 6.2 m.

13. **THINK** A shell explodes into two segments at the top of its trajectory. Knowing the motion of one segment allows us to analyze the motion of the other using the momentum conservation principle.

**EXPRESS** We need to find the coordinates of the point where the shell explodes and the velocity of the fragment that does not fall straight down. The coordinate origin is at the firing point, the $+x$ axis is rightward, and the $+y$ direction is upward. The $y$ component of the velocity is given by $v = v_{0y} - gt$ and this is zero at time $t = v_{0y}/g = (v_0/g)\sin\theta_0$, where $v_0$ is the initial speed and $\theta_0$ is the firing angle. The coordinates of the highest point on the trajectory are

$$x = v_{0x}t = v_0 t\cos\theta_0 = \frac{v_0^2}{g}\sin\theta_0\cos\theta_0 = \frac{(20\text{ m/s})^2}{9.8\text{ m/s}^2}\sin 60°\cos 60° = 17.7\text{ m}$$

and

$$y = v_{0y}t - \frac{1}{2}gt^2 = \frac{1}{2}\frac{v_0^2}{g}\sin^2\theta_0 = \frac{1}{2}\frac{(20\text{ m/s})^2}{9.8\text{ m/s}^2}\sin^2 60° = 15.3\text{ m}.$$

Since no horizontal forces act, the horizontal component of the momentum is conserved. In addition, since one fragment has a velocity of zero after the explosion, the momentum of the other equals the momentum of the shell before the explosion. At the highest point the velocity of the shell is $v_0 \cos\theta_0$, in the positive $x$ direction. Let $M$ be the mass of the shell and let $V_0$ be the velocity of the fragment. Then

$$Mv_0\cos\theta_0 = MV_0/2,$$

since the mass of the fragment is $M/2$. This means

$$V_0 = 2v_0\cos\theta_0 = 2(20\text{ m/s})\cos 60° = 20\text{ m/s}.$$

This information is used in the form of initial conditions for a projectile motion problem to determine where the fragment lands.

**ANALYZE** Resetting our clock, we now analyze a projectile launched horizontally at time $t = 0$ with a speed of 20 m/s from a location having coordinates $x_0 = 17.7$ m, $y_0 = 15.3$ m. Its $y$ coordinate is given by $y = y_0 - \frac{1}{2}gt^2$, and when it lands this is zero. The time of landing is $t = \sqrt{2y_0/g}$ and the $x$ coordinate of the landing point is

$$x = x_0 + V_0 t = x_0 + V_0\sqrt{\frac{2y_0}{g}} = 17.7\text{ m} + (20\text{ m/s})\sqrt{\frac{2(15.3\text{ m})}{9.8\text{ m/s}^2}} = 53\text{ m}.$$

**LEARN** In the absence of explosion, the shell with a mass $M$ would have landed at

$$R = 2x_0 = \frac{v_0^2}{g}\sin 2\theta_0 = \frac{(20\text{ m/s})^2}{9.8\text{ m/s}^2}\sin[2(60°)] = 35.3\text{ m}$$

which is shorter than $x = 53$ m found above. This makes sense because the broken fragment, having a smaller mass but greater horizontal speed, can travel much farther than the original shell.

14. (a) The phrase (in the problem statement) "such that it [particle 2] always stays directly above particle 1 during the flight" means that the shadow (as if a light were directly above the particles shining down on them) of particle 2 coincides with the position of particle 1, at each moment. We say, in this case, that they are vertically

aligned. Because of that alignment, $v_{2x} = v_1 = 10.0$ m/s. Because the initial value of $v_2$ is given as 20.0 m/s, then (using the Pythagorean theorem) we must have

$$v_{2y} = \sqrt{v_2^2 - v_{2x}^2} = \sqrt{300} \text{ m/s}$$

for the initial value of the $y$ component of particle 2's velocity. Equation 2-16 (or conservation of energy) readily yields $y_{max} = 300/19.6 = 15.3$ m. Thus, we obtain

$$H_{max} = m_2 y_{max}/m_{total} = (3.00 \text{ g})(15.3 \text{ m})/(8.00 \text{ g}) = 5.74 \text{ m}.$$

(b) Since both particles have the same horizontal velocity, and particle 2's vertical component of velocity vanishes at that highest point, then the center of mass velocity then is simply $(10.0 \text{ m/s})\hat{\text{i}}$ (as one can verify using Eq. 9-17).

(c) Only particle 2 experiences any acceleration (the free fall acceleration downward), so Eq. 9-18 (or Eq. 9-19) leads to

$$a_{com} = m_2 g/m_{total} = (3.00 \text{ g})(9.8 \text{ m/s}^2)/(8.00 \text{ g}) = 3.68 \text{ m/s}^2$$

for the magnitude of the downward acceleration of the center of mass of this system. Thus, $\vec{a}_{com} = (-3.68 \text{ m/s}^2)\hat{\text{j}}$.

15. (a) The net force on the *system* (of total mass $m_1 + m_2$) is $m_2 g$. Thus, Newton's second law leads to $a = g(m_2/(m_1 + m_2)) = 0.4g$. For block 1, this acceleration is to the right (the $\hat{\text{i}}$ direction), and for block 2 this is an acceleration downward (the $-\hat{\text{j}}$ direction). Therefore, Eq. 9-18 gives

$$\vec{a}_{com} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2} = \frac{(0.6)(0.4g\hat{\text{i}}) + (0.4)(-0.4g\hat{\text{j}})}{0.6 + 0.4} = (2.35\,\hat{\text{i}} - 1.57\,\hat{\text{j}}) \text{ m/s}^2 .$$

(b) Integrating Eq. 4-16, we obtain

$$\vec{v}_{com} = (2.35\,\hat{\text{i}} - 1.57\hat{\text{j}})\,t$$

(with SI units understood), since it started at rest. We note that the *ratio* of the $y$-component to the $x$-component (for the velocity vector) does not change with time, and it is that ratio which determines the angle of the velocity vector (by Eq. 3-6), and thus the direction of motion for the center of mass of the system.

(c) The last sentence of our answer for part (b) implies that the path of the center-of-mass is a straight line.

(d) Equation 3-6 leads to $\theta = -34°$. The path of the center of mass is therefore straight, at downward angle 34°.

16. We denote the mass of Ricardo as $M_R$ and that of Carmelita as $M_C$. Let the center of mass of the two-person system (assumed to be closer to Ricardo) be a distance $x$ from the middle of the canoe of length $L$ and mass $m$. Then

$$M_R(L/2 - x) = mx + M_C(L/2 + x).$$

Now, after they switch positions, the center of the canoe has moved a distance $2x$ from its initial position. Therefore, $x$ = 40 cm/2 = 0.20 m, which we substitute into the above equation to solve for $M_C$:

$$M_C = \frac{M_R(L/2 - x) - mx}{L/2 + x} = \frac{(80)\left(\frac{3.0}{2} - 0.20\right) - (30)(0.20)}{(3.0/2) + 0.20} = 58 \text{ kg.}$$

17. There is no net horizontal force on the dog-boat system, so their center of mass does not move. Therefore by Eq. 9-16,

$$M\Delta x_{\text{com}} = 0 = m_b \Delta x_b + m_d \Delta x_d \,,$$

which implies

$$|\Delta x_b| = \frac{m_d}{m_b} |\Delta x_d|.$$

Now we express the geometrical condition that *relative to the boat* the dog has moved a distance $d$ = 2.4 m:

$$|\Delta x_b| + |\Delta x_d| = d$$

which accounts for the fact that the dog moves one way and the boat moves the other. We substitute for $|\Delta x_b|$ from above:

$$\frac{m_d}{m_b}|(\Delta x_d)| + |\Delta x_d| = d$$

which leads to $|\Delta x_d| = \dfrac{d}{1 + m_d / m_b} = \dfrac{2.4 \text{ m}}{1 + (4.5/18)} = 1.92 \text{ m.}$

The dog is therefore 1.9 m closer to the shore than initially (where it was $D$ = 6.1 m from it). Thus, it is now $D - |\Delta x_d|$ = 4.2 m from the shore.

18. The magnitude of the ball's momentum change is

$$\Delta p = m|v_i - v_f| = (0.70 \text{ kg})|(5.0 \text{ m/s}) - (-2.0 \text{ m/s})| = 4.9 \text{ kg} \cdot \text{m/s}.$$

19. (a) The change in kinetic energy is

$$\begin{aligned}\Delta K &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}(2100\text{ kg})\left((51\text{ km/h})^2 - (41\text{ km/h})^2\right)\\ &= 9.66\times10^4\text{ kg}\cdot(\text{km/h})^2\left((10^3\text{ m/km})(1\text{ h/3600 s})\right)^2\\ &= 7.5\times10^4\text{ J.}\end{aligned}$$

(b) The magnitude of the change in velocity is

$$|\Delta\vec{v}| = \sqrt{(-v_i)^2 + (v_f)^2} = \sqrt{(-41\text{ km/h})^2 + (51\text{ km/h})^2} = 65.4\text{ km/h}$$

so the magnitude of the change in momentum is

$$|\Delta\vec{p}| = m|\Delta\vec{v}| = (2100\text{ kg})(65.4\text{ km/h})\left(\frac{1000\text{ m/km}}{3600\text{ s/h}}\right) = 3.8\times10^4\text{ kg}\cdot\text{m/s.}$$

(c) The vector $\Delta\vec{p}$ points at an angle $\theta$ south of east, where

$$\theta = \tan^{-1}\left(\frac{v_i}{v_f}\right) = \tan^{-1}\left(\frac{41\text{ km/h}}{51\text{ km/h}}\right) = 39°.$$

20. We infer from the graph that the horizontal component of momentum $p_x$ is 4.0 $\text{kg}\cdot\text{m/s}$. Also, its initial magnitude of momentum $p_o$ is 6.0 $\text{kg}\cdot\text{m/s}$. Thus,

$$\cos\theta_o = \frac{p_x}{p_o} \quad\Rightarrow\quad \theta_o = 48°\ .$$

21. We use coordinates with $+x$ horizontally toward the pitcher and $+y$ upward. Angles are measured counterclockwise from the $+x$ axis. Mass, velocity, and momentum units are SI. Thus, the initial momentum can be written $\vec{p}_0 = (4.5\angle 215°)$ in magnitude-angle notation.

(a) In magnitude-angle notation, the momentum change is

$$(6.0\ \angle -90°) - (4.5\ \angle\ 215°) = (5.0\ \angle -43°)$$

(efficiently done with a vector-capable calculator in polar mode). The magnitude of the momentum change is therefore 5.0 $\text{kg}\cdot\text{m/s}$.

(b) The momentum change is $(6.0\ \angle\ 0°) - (4.5\ \angle\ 215°) = (10\ \angle\ 15°)$. Thus, the magnitude of the momentum change is 10 $\text{kg}\cdot\text{m/s}$.

22. (a) Since the force of impact on the ball is in the $y$ direction, $p_x$ is conserved:

$$p_{xi} = p_{xf} \quad\Rightarrow\quad mv_i\sin\theta_1 = mv_i\sin\theta_2\ .$$

With $\theta_1$ = 30.0°, we find $\theta_2$ = 30.0°.

(b) The momentum change is

$$\Delta\vec{p} = mv_i \cos\theta_2 \left(-\hat{\mathrm{j}}\right) - mv_i \cos\theta_2 \left(+\hat{\mathrm{j}}\right) = -2\,(0.165\text{ kg})\,(2.00\text{ m/s})\,(\cos 30°)\,\hat{\mathrm{j}}$$
$$= (-0.572\text{ kg}\cdot\text{m/s})\hat{\mathrm{j}}.$$

23. We estimate his mass in the neighborhood of 70 kg and compute the upward force $F$ of the water from Newton's second law: $F - mg = ma$, where we have chosen $+y$ upward, so that $a > 0$ (the acceleration is upward since it represents a deceleration of his downward motion through the water). His speed when he arrives at the surface of the water is found either from Eq. 2-16 or from energy conservation: $v = \sqrt{2gh}$, where $h = 12$ m, and since the deceleration $a$ reduces the speed to zero over a distance $d$ = 0.30 m we also obtain $v = \sqrt{2ad}$. We use these observations in the following.

Equating our two expressions for $v$ leads to $a = gh/d$. Our force equation, then, leads to

$$F = mg + m\left(g\frac{h}{d}\right) = mg\left(1 + \frac{h}{d}\right)$$

which yields $F \approx 2.8 \times 10^4$ kg. Since we are not at all certain of his mass, we express this as a guessed-at range (in kN) $25 < F < 30$.

Since $F \gg mg$, the impulse $\vec{J}$ due to the net force (while he is in contact with the water) is overwhelmingly caused by the upward force of the water: $\int F\,dt = \vec{J}$ to a good approximation. Thus, by Eq. 9-29,

$$\int F dt = \vec{p}_f - \vec{p}_i = 0 - m\left(-\sqrt{2gh}\right)$$

(the minus sign with the initial velocity is due to the fact that downward is the negative direction), which yields $(70\text{ kg})\sqrt{2\left(9.8\text{ m/s}^2\right)(12\text{ m})} = 1.1\times10^3\text{kg}\cdot\text{m/s}$. Expressing this as a range we estimate

$$1.0\times10^3\text{kg}\cdot\text{m/s} < \int F\,dt < 1.2\times10^3\text{kg}\cdot\text{m/s}.$$

24. We choose $+y$ upward, which implies $a > 0$ (the acceleration is upward since it represents a deceleration of his downward motion through the snow).

(a) The maximum deceleration $a_{max}$ of the paratrooper (of mass $m$ and initial speed $v$ = 56 m/s) is found from Newton's second law

$$F_{\text{snow}} - mg = ma_{\text{max}}$$

where we require $F_{\text{snow}}$ = 1.2 × $10^5$ N. Using Eq. 2-15 $v^2 = 2a_{\text{max}}d$, we find the minimum depth of snow for the man to survive:

$$d = \frac{v^2}{2a_{\text{max}}} = \frac{mv^2}{2\left(F_{\text{snow}} - mg\right)} \approx \frac{\left(85\text{kg}\right)\left(56\,\text{m/s}\right)^2}{2\left(1.2\times10^5\,\text{N}\right)} = 1.1\ \text{m}.$$

(b) His short trip through the snow involves a change in momentum

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = 0 - \left(85\,\text{kg}\right)\left(-56\,\text{m/s}\right) = -4.8\times10^3\,\text{kg}\cdot\text{m/s},$$

or $|\Delta\vec{p}| = 4.8\times10^3\,\text{kg}\cdot\text{m/s}$. The negative value of the initial velocity is due to the fact that downward is the negative direction. By the impulse-momentum theorem, this equals the impulse due to the net force $F_{\text{snow}} - mg$, but since $F_{\text{snow}} \gg mg$ we can approximate this as the impulse on him just from the snow.

25. We choose $+y$ upward, which means $\vec{v}_i = -25\,\text{m/s}$ and $\vec{v}_f = +10\,\text{m/s}$. During the collision, we make the reasonable approximation that the net force on the ball is equal to $F_{\text{avg}}$, the average force exerted by the floor up on the ball.

(a) Using the impulse momentum theorem (Eq. 9-31) we find

$$\vec{J} = m\vec{v}_f - m\vec{v}_i = (1.2)(10) - (1.2)(-25) = 42\,\text{kg}\cdot\text{m/s}.$$

(b) From Eq. 9-35, we obtain

$$\vec{F}_{\text{avg}} = \frac{\vec{J}}{\Delta t} = \frac{42}{0.020} = 2.1\times10^3\,\text{N}.$$

26. (a) By energy conservation, the speed of the victim when he falls to the floor is

$$\frac{1}{2}mv^2 = mgh \quad\Rightarrow\quad v = \sqrt{2gh} = \sqrt{2(9.8\ \text{m/s}^2)(0.50\ \text{m})} = 3.1\,\text{m/s}.$$

Thus, the magnitude of the impulse is

$$J = |\Delta p| = m|\Delta v| = mv = (70\ \text{kg})(3.1\,\text{m/s}) \approx 2.2\times10^2\ \text{N}\cdot\text{s}.$$

(b) With duration of $\Delta t = 0.082\,\text{s}$ for the collision, the average force is

$$F_{\text{avg}} = \frac{J}{\Delta t} = \frac{2.2\times10^2\ \text{N}\cdot\text{s}}{0.082\,\text{s}} \approx 2.7\times10^3\ \text{N}.$$

27. **THINK** The velocity of the ball is changing because of the external force applied. Impulse-linear momentum theorem is involved.

**EXPRESS** The initial direction of motion is in the +x direction. The magnitude of the average force $F_{\text{avg}}$ is given by

$$F_{\text{avg}} = \frac{J}{\Delta t} = \frac{32.4\ \text{N}\cdot\text{s}}{2.70\times 10^{-2}\ \text{s}} = 1.20\times 10^{3}\ \text{N}.$$

The force is in the negative direction. Using the linear momentum-impulse theorem stated in Eq. 9-31, we have

$$-F_{\text{avg}}\Delta t = J = \Delta p = m(v_f - v_i).$$

where $m$ is the mass, $v_i$ the initial velocity, and $v_f$ the final velocity of the ball. The equation can be used to solve for $v_f$.

**ANALYZE** (a) Using the above expression, we find

$$v_f = \frac{mv_i - F_{\text{avg}}\Delta t}{m} = \frac{(0.40\,\text{kg})(14\,\text{m/s}) - (1200\,\text{N})(27\times 10^{-3}\text{s})}{0.40\,\text{kg}} = -67\,\text{m/s}.$$

The final speed of the ball is $|v_f| = 67$ m/s.

(b) The negative sign in $v_f$ indicates that the velocity is in the $-x$ direction, which is opposite to the initial direction of travel.

(c) From the above, the average magnitude of the force is $F_{\text{avg}} = 1.20\times 10^{3}$ N.

(d) The direction of the impulse on the ball is $-x$, same as the applied force.

**LEARN** In vector notation, $\vec{F}_{\text{avg}}\Delta t = \vec{J} = \Delta\vec{p} = m(\vec{v}_f - \vec{v}_i)$, which gives

$$\vec{v}_f = \vec{v}_i + \frac{\vec{J}}{m} = \vec{v}_i + \frac{\vec{F}_{\text{avg}}\Delta t}{m}$$

Since $\vec{J}$ or $\vec{F}_{\text{avg}}$ is in the opposite direction of $\vec{v}_i$, the velocity of the ball will decrease under the applied force. The ball first moves in the $+x$-direction, but then slows down and comes to a stop, and then reverses its direction of travel.

28. (a) The magnitude of the impulse is

$$J = |\Delta p| = m|\Delta v| = mv = (0.70\ \text{kg})(13\ \text{m/s}) \approx 9.1\,\text{kg}\cdot\text{m/s} = 9.1\,\text{N}\cdot\text{s}.$$

(b) With duration of $\Delta t = 5.0\times10^{-3}$ s for the collision, the average force is

$$F_{\text{avg}} = \frac{J}{\Delta t} = \frac{9.1\ \text{N}\cdot\text{s}}{5.0\times10^{-3}\ \text{s}} \approx 1.8\times10^{3}\ \text{N}.$$

29. We choose the positive direction in the direction of rebound so that $\vec{v}_f > 0$ and $\vec{v}_i < 0$. Since they have the same speed $v$, we write this as $\vec{v}_f = v$ and $\vec{v}_i = -v$. Therefore, the change in momentum for each bullet of mass $m$ is $\Delta\vec{p} = m\Delta v = 2mv$. Consequently, the total change in momentum for the 100 bullets (each minute) $\Delta\vec{P} = 100\Delta\vec{p} = 200mv$. The average force is then

$$\vec{F}_{\text{avg}} = \frac{\Delta\vec{P}}{\Delta t} = \frac{(200)(3\times10^{-3}\,\text{kg})(500\,\text{m/s})}{(1\,\text{min})(60\,\text{s/min})} \approx 5\,\text{N}.$$

30. (a) By Eq. 9-30, impulse can be determined from the "area" under the $F(t)$ curve. Keeping in mind that the area of a triangle is $\frac{1}{2}$(base)(height), we find the impulse in this case is 1.00 $\text{N}\cdot\text{s}$.

(b) By definition (of the average of function, in the calculus sense) the average force must be the result of part (a) divided by the time (0.010 s). Thus, the average force is found to be 100 N.

(c) Consider ten hits. Thinking of ten hits as 10 $F(t)$ triangles, our total time interval is 10(0.050 s) = 0.50 s, and the total area is 10(1.0 $\text{N}\cdot\text{s}$). We thus obtain an average force of 10/0.50 = 20.0 N. One could consider 15 hits, 17 hits, and so on, and still arrive at this same answer.

31. (a) By energy conservation, the speed of the passenger when the elevator hits the floor is

$$\frac{1}{2}mv^2 = mgh \;\;\Rightarrow\;\; v = \sqrt{2gh} = \sqrt{2(9.8\ \text{m/s}^2)(36\ \text{m})} = 26.6\ \text{m/s}.$$

Thus, the magnitude of the impulse is

$$J = |\Delta p| = m|\Delta v| = mv = (90\ \text{kg})(26.6\ \text{m/s}) \approx 2.39\times10^{3}\ \text{N}\cdot\text{s}.$$

(b) With duration of $\Delta t = 5.0\times10^{-3}$ s for the collision, the average force is

$$F_{\text{avg}} = \frac{J}{\Delta t} = \frac{2.39\times10^{3}\ \text{N}\cdot\text{s}}{5.0\times10^{-3}\ \text{s}} \approx 4.78\times10^{5}\ \text{N}.$$

(c) If the passenger were to jump upward with a speed of $v' = 7.0\ \text{m/s}$, then the resulting downward velocity would be

$$v'' = v - v' = 26.6\ \text{m/s} - 7.0\ \text{m/s} = 19.6\ \text{m/s},$$

and the magnitude of the impulse becomes

$$J'' = |\Delta p''| = m|\Delta v''| = mv'' = (90\ \text{kg})(19.6\ \text{m/s}) \approx 1.76\times10^3\ \text{N}\cdot\text{s}.$$

(d) The corresponding average force would be

$$F''_{\text{avg}} = \frac{J''}{\Delta t} = \frac{1.76\times10^3\ \text{N}\cdot\text{s}}{5.0\times10^{-3}\ \text{s}} \approx 3.52\times10^5\ \text{N}.$$

32. (a) By the impulse-momentum theorem (Eq. 9-31) the change in momentum must equal the "area" under the $F(t)$ curve. Using the facts that the area of a triangle is $\frac{1}{2}$ (base)(height), and that of a rectangle is (height)(width), we find the momentum at $t$ = 4 s to be (30 kg·m/s)$\hat{\text{i}}$.

(b) Similarly (but keeping in mind that areas beneath the axis are counted negatively) we find the momentum at $t$ = 7 s is (38 kg·m/s)$\hat{\text{i}}$.

(c) At $t$ = 9 s, we obtain $\vec{v}$ = (6.0 m/s)$\hat{\text{i}}$.

33. We use coordinates with +$x$ rightward and +$y$ upward, with the usual conventions for measuring the angles (so that the initial angle becomes 180 + 35 = 215°). Using SI units and magnitude-angle notation (efficient to work with when using a vector-capable calculator), the change in momentum is

$$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = (3.00\angle 90°) - (3.60\angle 215°) = (5.86\angle 59.8°).$$

(a) The magnitude of the impulse is $J = \Delta p = 5.86\ \text{kg}\cdot\text{m/s} = 5.86\ \text{N}\cdot\text{s}$.

(b) The direction of $\vec{J}$ is 59.8° measured counterclockwise from the +$x$ axis.

(c) Equation 9-35 leads to

$$J = F_{\text{avg}}\Delta t = 5.86\ \text{N}\cdot\text{s} \quad\Rightarrow\quad F_{\text{avg}} = \frac{5.86\ \text{N}\cdot\text{s}}{2.00\times10^{-3}\ \text{s}} \approx 2.93\times10^3\,\text{N}.$$

We note that this force is very much larger than the weight of the ball, which justifies our (implicit) assumption that gravity played no significant role in the collision.

(d) The direction of $\vec{F}_{\text{avg}}$ is the same as $\vec{J}$, 59.8° measured counterclockwise from the +*x* axis.

34. (a) Choosing upward as the positive direction, the momentum change of the foot is

$$\Delta\vec{p} = 0 - m_{\text{foot}}\vec{v}_i = -(0.003\text{ kg})\,(-1.50\text{ m/s}) = 4.50\times10^{-3}\text{ N}\cdot\text{s}.$$

(b) Using Eq. 9-35 and now treating *downward* as the positive direction, we have

$$\vec{J} = \vec{F}_{\text{avg}}\Delta t = m_{\text{lizard}}g\,\Delta t = (0.090\text{ kg})(9.80\text{ m/s}^2)(0.60\text{ s}) = 0.529\text{ N}\cdot\text{s}.$$

(c) Push is what provides the primary support.

35. We choose our positive direction in the direction of the rebound (so the ball's initial velocity is negative-valued). We evaluate the integral $J = \int F\,dt$ by adding the appropriate areas (of a triangle, a rectangle, and another triangle) shown in the graph (but with the *t* converted to seconds). With *m* = 0.058 kg and *v* = 34 m/s, we apply the impulse-momentum theorem:

$$\int F_{\text{wall}}\,dt = m\vec{v}_f - m\vec{v}_i \quad\Rightarrow\quad \int_0^{0.002} F\,dt + \int_{0.002}^{0.004} F\,dt + \int_{0.004}^{0.006} F\,dt = m(+v) - m(-v)$$
$$\Rightarrow\quad \frac{1}{2}F_{\max}(0.002\text{s}) + F_{\max}(0.002\text{s}) + \frac{1}{2}F_{\max}(0.002\text{s}) = 2mv$$

which yields $F_{\max}(0.004\text{s}) = 2(0.058\text{kg})(34\text{m/s}) = 9.9\times10^2$ N.

36. (a) Performing the integral (from time *a* to time *b*) indicated in Eq. 9-30, we obtain

$$\int_a^b (12 - 3t^2)\,dt = 12(b-a) - (b^3 - a^3)$$

in SI units. If *b* = 1.25 s and *a* = 0.50 s, this gives $7.17\text{ N}\cdot\text{s}$.

(b) This integral (the impulse) relates to the change of momentum in Eq. 9-31. We note that the force is zero at *t* = 2.00 s. Evaluating the above expression for *a* = 0 and *b* = 2.00 gives an answer of $16.0\text{ kg}\cdot\text{m/s}$.

37. **THINK** We're given the force as a function of time, and asked to calculate the corresponding impulse, the average force and the maximum force.

**EXPRESS** Since the motion is one-dimensional, we work with the magnitudes of the vector quantities. The impulse $J$ due to a force $F(t)$ exerted on a body is

$$J = \int_{t_i}^{t_f} F(t)dt = F_{\text{avg}}\Delta t\,,$$

where $F_{\text{avg}}$ is the average force and $\Delta t = t_f - t_i$. To find the time at which the maximum force occurs, we set the derivative of $F$ with respect to time equal to zero, and solve for $t$.

**ANALYZE** (a) We take the force to be in the positive direction, at least for earlier times. Then the impulse is

$$J = \int_0^{3.0\times10^{-3}} F dt = \int_0^{3.0\times10^{-3}} \left[(6.0\times10^6)t - (2.0\times10^9)t^2\right]dt$$
$$= \left[\frac{1}{2}(6.0\times10^6)t^2 - \frac{1}{3}(2.0\times10^9)t^3\right]\Bigg|_0^{3.0\times10^{-3}} = 9.0\,\text{N}\cdot\text{s}.$$

(b) Using $J = F_{\text{avg}}\,\Delta t$, we find the average force to be

$$F_{\text{avg}}\frac{J}{\Delta t} = \frac{9.0\ \text{N}\cdot\text{s}}{3.0\times10^{-3}\ \text{s}} = 3.0\times10^3\ \text{N}.$$

(c) Differentiating $F(t)$ with respect to $t$ and setting it to zero, we have

$$\frac{dF}{dt} = \frac{d}{dt}\left[(6.0\times10^6)t - (2.0\times10^9)t^2\right] = (6.0\times10^6) - (4.0\times10^9)t = 0\,,$$

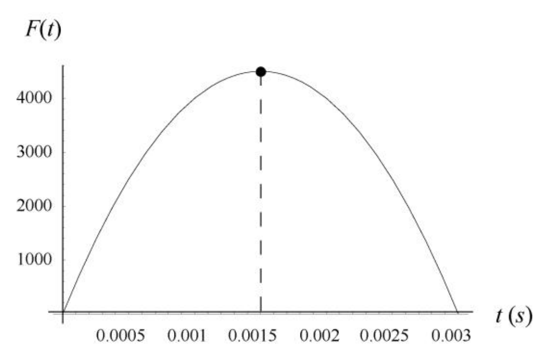
which can be solved to give $t = 1.5\times10^{-3}$ s. At that time the force is

$$F_{\max} = (6.0\times10^6)(1.5\times10^{-3}) - (2.0\times10^9)(1.5\times10^{-3})^2 = 4.5\times10^3\ \text{N}.$$

(d) Since it starts from rest, the ball acquires momentum equal to the impulse from the kick. Let $m$ be the mass of the ball and $v$ its speed as it leaves the foot. The speed of the ball immediately after it loses contact with the player's foot is

$$v = \frac{p}{m} = \frac{J}{m} = \frac{9.0\ \text{N}\cdot\text{s}}{0.45\ \text{kg}} = 20\ \text{m/s}.$$

**LEARN** The force as function of time is shown below. The area under the curve is the impulse $J$. From the plot, we readily see that $F(t)$ is a maximum at $t = 0.0015\,\text{s}$, with $F_{\max} = 4500$ N.



38. From Fig. 9-54, +*y* corresponds to the direction of the rebound (directly away from the wall) and +*x* toward the right. Using unit-vector notation, the ball's initial and final velocities are

$$\vec{v}_i = v\cos\theta\,\hat{\mathrm{i}} - v\sin\theta\,\hat{\mathrm{j}} = 5.2\,\hat{\mathrm{i}} - 3.0\,\hat{\mathrm{j}}$$
$$\vec{v}_f = v\cos\theta\,\hat{\mathrm{i}} + v\sin\theta\,\hat{\mathrm{j}} = 5.2\,\hat{\mathrm{i}} + 3.0\,\hat{\mathrm{j}}$$

respectively (with SI units understood).

(a) With $m$ = 0.30 kg, the impulse-momentum theorem (Eq. 9-31) yields

$$\vec{J} = m\vec{v}_f - m\vec{v}_i = 2\left(0.30\text{ kg}\right)(3.0\text{ m/s }\hat{\mathrm{j}}) = (1.8\text{ N}\cdot\text{s})\hat{\mathrm{j}}\,.$$

(b) Using Eq. 9-35, the force on the ball by the wall is $\vec{J}/\Delta t = (1.8/0.010)\hat{\mathrm{j}} = (180\,\text{N})\,\hat{\mathrm{j}}$. By Newton's third law, the force on the wall by the ball is $(-180\text{ N})\hat{\mathrm{j}}$ (that is, its magnitude is 180 N and its direction is directly into the wall, or "down" in the view provided by Fig. 9-54).

39. **THINK** This problem deals with momentum conservation. Since no external forces with horizontal components act on the man-stone system and the vertical forces sum to zero, the total momentum of the system is conserved.

**EXPRESS** Since the man and the stone are initially at rest, the total momentum is zero both before and after the stone is kicked. Let $m_s$ be the mass of the stone and $v_s$ be its velocity after it is kicked. Also, let $m_m$ be the mass of the man and $v_m$ be his velocity after he kicks the stone. Then, by momentum conservation,

$$m_s v_s + m_m v_m = 0 \;\;\Rightarrow\;\; v_m = -\frac{m_s}{m_m}v_s\,.$$

**ANALYZE** We take the axis to be positive in the direction of motion of the stone. Then

$$v_m = -\frac{m_s}{m_m}v_s = -\frac{0.068\text{ kg}}{91\text{ kg}}\left(4.0\text{ m/s}\right) = -3.0\times10^{-3}\text{ m/s}$$

or $|v_m| = 3.0\times10^{-3}\text{ m/s}$.

**LEARN** The negative sign in $v_m$ indicates that the man moves in the direction opposite to the motion of the stone. Note that his speed is much smaller (by a factor of $m_s/m_m$) compared to the speed of the stone.

40. Our notation is as follows: the mass of the motor is *M*; the mass of the module is *m*; the initial speed of the system is $v_0$; the relative speed between the motor and the module

is $v_r$; and, the speed of the module relative to the Earth is $v$ after the separation. Conservation of linear momentum requires

$$(M + m)v_0 = mv + M(v - v_r).$$

Therefore,

$$v = v_0 + \frac{Mv_r}{M+m} = 4300 \text{ km/h} + \frac{(4m)(82 \text{ km/h})}{4m+m} = 4.4 \times 10^3 \text{ km/h}.$$

41. (a) With SI units understood, the velocity of block $L$ (in the frame of reference indicated in the figure that goes with the problem) is $(v_1 - 3)\hat{\text{i}}$. Thus, momentum conservation (for the explosion at $t = 0$) gives

$$m_L(v_1 - 3) + (m_C + m_R)v_1 = 0$$

which leads to

$$v_1 = \frac{3\, m_L}{m_L + m_C + m_R} = \frac{3(2\text{ kg})}{10\text{ kg}} = 0.60\text{ m/s}.$$

Next, at $t = 0.80$ s, momentum conservation (for the second explosion) gives

$$m_C\, v_2 + m_R(v_2 + 3) = (m_C + m_R)v_1 = (8\text{ kg})(0.60\text{ m/s}) = 4.8\ \text{kg}\cdot\text{m/s}.$$

This yields $v_2 = -0.15$. Thus, the velocity of block $C$ after the second explosion is

$$v_2 = -(0.15\text{ m/s})\hat{\text{i}}.$$

(b) Between $t = 0$ and $t = 0.80$ s, the block moves $v_1\Delta t = (0.60\text{ m/s})(0.80\text{ s}) = 0.48$ m. Between $t = 0.80$ s and $t = 2.80$ s, it moves an additional

$$v_2\Delta t = (-0.15\text{ m/s})(2.00\text{ s}) = -0.30\text{ m}.$$

Its net displacement since $t = 0$ is therefore 0.48 m – 0.30 m = 0.18 m.

42. Our notation (and, implicitly, our choice of coordinate system) is as follows: the mass of the original body is $m$; its initial velocity is $\vec{v}_0 = v\,\hat{\text{i}}$; the mass of the less massive piece is $m_1$; its velocity is $\vec{v}_1 = 0$; and, the mass of the more massive piece is $m_2$. We note that the conditions $m_2 = 3m_1$ (specified in the problem) and $m_1 + m_2 = m$ generally assumed in classical physics (before Einstein) lead us to conclude

$$m_1 = \frac{1}{4}m \text{ and } m_2 = \frac{3}{4}m.$$

Conservation of linear momentum requires

$$m\vec{v}_0 = m_1\vec{v}_1 + m_2\vec{v}_2 \quad\Rightarrow\quad mv\,\hat{\text{i}} = 0 + \frac{3}{4}m\vec{v}_2$$

which leads to $\vec{v}_2 = \dfrac{4}{3} v\,\hat{\mathrm{i}}.$ The increase in the system's kinetic energy is therefore

$$\Delta K = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{1}{2}mv_0^2 = 0 + \frac{1}{2}\left(\frac{3}{4}m\right)\left(\frac{4}{3}v\right)^2 - \frac{1}{2}mv^2 = \frac{1}{6}mv^2.$$

43. With $\vec{v}_0 = (9.5\,\hat{\mathrm{i}} + 4.0\,\hat{\mathrm{j}})$ m/s, the initial speed is

$$v_0 = \sqrt{v_{x0}^2 + v_{y0}^2} = \sqrt{(9.5\text{ m/s})^2 + (4.0\text{ m/s})^2} = 10.31\text{ m/s}$$

and the takeoff angle of the athlete is

$$\theta_0 = \tan^{-1}\left(\frac{v_{y0}}{v_{x0}}\right) = \tan^{-1}\left(\frac{4.0}{9.5}\right) = 22.8°.$$

Using Equation 4-26, the range of the athlete without using halteres is

$$R_0 = \frac{v_0^2 \sin 2\theta_0}{g} = \frac{(10.31\text{ m/s})^2 \sin 2(22.8°)}{9.8\text{ m/s}^2} = 7.75\text{ m}.$$

On the other hand, if two halteres of mass $m$ = 5.50 kg were thrown at the maximum height, then, by momentum conservation, the subsequent speed of the athlete would be

$$(M + 2m)v_{x0} = Mv'_x \quad \Rightarrow v'_x = \frac{M + 2m}{M}v_{x0}$$

Thus, the change in the $x$-component of the velocity is

$$\Delta v_x = v'_x - v_{x0} = \frac{M + 2m}{M}v_{x0} - v_{x0} = \frac{2m}{M}v_{x0} = \frac{2(5.5\text{ kg})}{78\text{ kg}}(9.5\text{ m/s}) = 1.34\text{ m/s}.$$

The maximum height is attained when $v_y = v_{y0} - gt = 0$, or

$$t = \frac{v_{y0}}{g} = \frac{4.0\text{ m/s}}{9.8\text{ m/s}^2} = 0.41\text{ s}.$$

Therefore, the increase in range with use of halteres is

$$\Delta R = (\Delta v'_x)t = (1.34\text{ m/s})(0.41\text{ s}) = 0.55\text{ m}.$$

44. We can think of the sliding-until-stopping as an example of kinetic energy converting into thermal energy (see Eq. 8-29 and Eq. 6-2, with $F_N = mg$). This leads to $v^2 = 2\mu gd$ being true separately for each piece. Thus we can set up a ratio:

$$\left(\frac{v_L}{v_R}\right)^2 = \frac{2\mu_L g d_L}{2\mu_R g d_R} = \frac{12}{25}\ .$$

But (by the conservation of momentum) the ratio of speeds must be inversely proportional to the ratio of masses (since the initial momentum before the explosion was zero). Consequently,

$$\left(\frac{m_R}{m_L}\right)^2 = \frac{12}{25} \quad\Rightarrow\quad m_R = \tfrac{2}{5}\sqrt{3}\, m_L = 1.39 \text{ kg.}$$

Therefore, the total mass is $m_R + m_L \approx 3.4$ kg.

45. **THINK** The moving body is an isolated system with no external force acting on it. When it breaks up into three pieces, momentum remains conserved, both in the $x$- and the $y$-directions.

**EXPRESS** Our notation is as follows: the mass of the original body is $M$ = 20.0 kg; its initial velocity is $\vec{v}_0 = (200\text{ m/s})\hat{\text{i}}$; the mass of one fragment is $m_1$ = 10.0 kg; its velocity is $\vec{v}_1 = (100\text{ m/s})\hat{\text{j}}$; the mass of the second fragment is $m_2$ = 4.0 kg; its velocity is $\vec{v}_2 = (-500\text{ m/s})\hat{\text{i}}$; and, the mass of the third fragment is $m_3$ = 6.00 kg. Conservation of linear momentum requires

$$M\vec{v}_0 = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3\,.$$

The energy released in the explosion is equal to $\Delta K$, the change in kinetic energy.

**ANALYZE** (a) The above momentum-conservation equation leads to

$$\begin{aligned}\vec{v}_3 &= \frac{M\vec{v}_0 - m_1\vec{v}_1 - m_2\vec{v}_2}{m_3} \\ &= \frac{(20.0\text{ kg})(200\text{ m/s})\hat{\text{i}} - (10.0\text{ kg})(100\text{ m/s})\hat{\text{j}} - (4.0\text{ kg})(-500\text{ m/s})\hat{\text{i}}}{6.00\text{ kg}}\,. \\ &= (1.00\times10^3\text{ m/s})\,\hat{\text{i}} - (0.167\times10^3\text{ m/s})\,\hat{\text{j}}\end{aligned}$$

The magnitude of $\vec{v}_3$ is $v_3 = \sqrt{(1000\text{ m/s})^2 + (-167\text{ m/s})^2} = 1.01\times10^3\text{ m/s}$. It points at $\theta = \tan^{-1}$ (–167/1000) = –9.48° (that is, at 9.5° measured clockwise from the +$x$ axis).

(b) The energy released is $\Delta K$:

$$\Delta K = K_f - K_i = \left(\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{1}{2}m_3 v_3^2\right) - \frac{1}{2}M v_0^2 = 3.23\times 10^6 \text{ J}.$$

**LEARN** The energy released in the explosion, of chemical nature, is converted into the kinetic energy of the fragments.

46. Our $+x$ direction is east and $+y$ direction is north. The linear momenta for the two $m$ = 2.0 kg parts are then

$$\vec{p}_1 = m\vec{v}_1 = mv_1\,\hat{\text{j}}$$

where $v_1$ = 3.0 m/s, and

$$\vec{p}_2 = m\vec{v}_2 = m\left(v_{2x}\,\hat{\text{i}} + v_{2y}\,\hat{\text{j}}\right) = mv_2\left(\cos\theta\,\hat{\text{i}} + \sin\theta\,\hat{\text{j}}\right)$$

where $v_2$ = 5.0 m/s and $\theta$ = 30°. The combined linear momentum of both parts is then

$$\begin{aligned}\vec{P} &= \vec{p}_1 + \vec{p}_2 = mv_1\,\hat{\text{j}} + mv_2\left(\cos\theta\,\hat{\text{i}} + \sin\theta\,\hat{\text{j}}\right) = \left(mv_2\cos\theta\right)\hat{\text{i}} + \left(mv_1 + mv_2\sin\theta\right)\hat{\text{j}}\\ &= (2.0\text{ kg})(5.0\text{ m/s})(\cos 30°)\,\hat{\text{i}} + (2.0\text{ kg})\left(3.0\text{ m/s} + (5.0\text{ m/s})(\sin 30°)\right)\hat{\text{j}}\\ &= \left(8.66\,\hat{\text{i}} + 11\,\hat{\text{j}}\right)\text{ kg}\cdot\text{m/s}.\end{aligned}$$

From conservation of linear momentum we know that this is also the linear momentum of the whole kit before it splits. Thus the speed of the 4.0-kg kit is

$$v = \frac{P}{M} = \frac{\sqrt{P_x^2 + P_y^2}}{M} = \frac{\sqrt{(8.66\text{ kg}\cdot\text{m/s})^2 + (11\text{ kg}\cdot\text{m/s})^2}}{4.0\text{ kg}} = 3.5\text{ m/s}.$$

47. Our notation (and, implicitly, our choice of coordinate system) is as follows: the mass of one piece is $m_1 = m$; its velocity is $\vec{v}_1 = (-30\text{ m/s})\hat{\text{i}}$; the mass of the second piece is $m_2 = m$; its velocity is $\vec{v}_2 = (-30\text{ m/s})\hat{\text{j}}$; and, the mass of the third piece is $m_3 = 3m$.

(a) Conservation of linear momentum requires

$$m\vec{v}_0 = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 \quad\Rightarrow\quad 0 = m\left(-30\hat{\text{i}}\right) + m\left(-30\hat{\text{j}}\right) + 3m\vec{v}_3$$

which leads to $\vec{v}_3 = (10\hat{\text{i}} + 10\hat{\text{j}})\text{ m/s}$. Its magnitude is $v_3 = 10\sqrt{2} \approx 14\text{ m/s}$.

(b) The direction is 45° *counterclockwise* from $+x$ (in this system where we have $m_1$ flying off in the $-x$ direction and $m_2$ flying off in the $-y$ direction).

48. This problem involves both mechanical energy conservation $U_i = K_1 + K_2$, where $U_i$ = 60 J, and momentum conservation

$$0 = m_1\vec{v}_1 + m_2\vec{v}_2$$

where $m_2 = 2m_1$. From the second equation, we find $|\vec{v}_1| = 2|\vec{v}_2|$, which in turn implies (since $v_1 = |\vec{v}_1|$ and likewise for $v_2$)

$$K_1 = \frac{1}{2}m_1v_1^2 = \frac{1}{2}\left(\frac{1}{2}m_2\right)(2v_2)^2 = 2\left(\frac{1}{2}m_2v_2^2\right) = 2K_2.$$

(a) We substitute $K_1 = 2K_2$ into the energy conservation relation and find

$$U_i = 2K_2 + K_2 \Rightarrow K_2 = \frac{1}{3}U_i = 20 \text{ J}.$$

(b) And we obtain $K_1 = 2(20) = 40$ J.

49. We refer to the discussion in the textbook (see Sample Problem – "Conservation of momentum, ballistic pendulum," which uses the same notation that we use here) for many of the important details in the reasoning. Here we only present the primary computational step (using SI units):

$$v = \frac{m+M}{m}\sqrt{2gh} = \frac{2.010}{0.010}\sqrt{2(9.8)(0.12)} = 3.1\times 10^2 \text{ m/s}.$$

50. (a) We choose $+x$ along the initial direction of motion and apply momentum conservation:

$$m_{\text{bullet}}\vec{v}_i = m_{\text{bullet}}\vec{v}_1 + m_{\text{block}}\vec{v}_2$$
$$(5.2 \text{ g})(672 \text{ m/s}) = (5.2 \text{ g})(428 \text{ m/s}) + (700 \text{ g})\vec{v}_2$$

which yields $v_2 = 1.81$ m/s.

(b) It is a consequence of momentum conservation that the velocity of the center of mass is unchanged by the collision. We choose to evaluate it before the collision:

$$\vec{v}_{\text{com}} = \frac{m_{\text{bullet}}\vec{v}_i}{m_{\text{bullet}} + m_{\text{block}}} = \frac{(5.2 \text{ g})(672 \text{ m/s})}{5.2 \text{ g} + 700 \text{ g}} = 4.96 \text{ m/s}.$$

51. In solving this problem, our $+x$ direction is to the right (so all velocities are positive-valued).

(a) We apply momentum conservation to relate the situation just before the bullet strikes the second block to the situation where the bullet is embedded within the block.

$$(0.0035 \text{ kg})v = (1.8035 \text{ kg})(1.4 \text{ m/s}) \;\Rightarrow\; v = 721 \text{ m/s}.$$

(b) We apply momentum conservation to relate the situation just before the bullet strikes the first block to the instant it has passed through it (having speed $v$ found in part (a)).

$$(0.0035 \text{ kg})v_0 = (1.20 \text{ kg})(0.630 \text{ m/s}) + (0.00350 \text{ kg})(721 \text{ m/s})$$

which yields $v_0$ = 937 m/s.

52. We think of this as having two parts: the first is the collision itself – where the bullet passes through the block so quickly that the block has not had time to move through any distance yet – and then the subsequent "leap" of the block into the air (up to height $h$ measured from its initial position). The first part involves momentum conservation (with $+y$ upward):

$$(0.01\,\text{kg})(1000\,\text{m/s}) = (5.0\,\text{kg})\vec{v} + (0.01\,\text{kg})(400\,\text{m/s})$$

which yields $\vec{v} = 1.2\,\text{m/s}$. The second part involves either the free-fall equations from Ch. 2 (since we are ignoring air friction) or simple energy conservation from Ch. 8. Choosing the latter approach, we have

$$\frac{1}{2}(5.0\,\text{kg})(1.2\,\text{m/s})^2 = (5.0\,\text{kg})(9.8\,\text{m/s}^2)h$$

which gives the result $h$ = 0.073 m.

53. With an initial speed of $v_i$, the initial kinetic energy of the car is $K_i = m_c v_i^2/2$. After a totally inelastic collision with a moose of mass $m_m$, by momentum conservation, the speed of the combined system is

$$m_c v_i = (m_c + m_m)v_f \quad\Rightarrow\quad v_f = \frac{m_c v_i}{m_c + m_m},$$

with final kinetic energy

$$K_f = \frac{1}{2}(m_c + m_m)v_f^2 = \frac{1}{2}(m_c + m_m)\left(\frac{m_c v_i}{m_c + m_m}\right)^2 = \frac{1}{2}\frac{m_c^2}{m_c + m_m}v_i^2.$$

(a) The percentage loss of kinetic energy due to collision is

$$\frac{\Delta K}{K_i} = \frac{K_i - K_f}{K_i} = 1 - \frac{K_f}{K_i} = 1 - \frac{m_c}{m_c + m_m} = \frac{m_m}{m_c + m_m} = \frac{500 \text{ kg}}{1000 \text{ kg} + 500 \text{ kg}} = \frac{1}{3} = 33.3\%.$$

(b) If the collision were with a camel of mass $m_{\text{camel}} = 300 \text{ kg}$, then the percentage loss of kinetic energy would be

$$\frac{\Delta K}{K_i} = \frac{m_{\text{camel}}}{m_c + m_{\text{camel}}} = \frac{300\text{ kg}}{1000\text{ kg} + 300\text{ kg}} = \frac{3}{13} = 23\%.$$

(c) As the animal mass decreases, the percentage loss of kinetic energy also decreases.

54. The total momentum immediately before the collision (with $+x$ upward) is

$$p_i = (3.0\text{ kg})(20\text{ m/s}) + (2.0\text{ kg})(-12\text{ m/s}) = 36\ \text{kg}\cdot\text{m/s}.$$

Their momentum immediately after, when they constitute a combined mass of $M$ = 5.0 kg, is $p_f = (5.0\text{ kg})\vec{v}$. By conservation of momentum, then, we obtain $\vec{v}$ = 7.2 m/s, which becomes their "initial" velocity for their subsequent free-fall motion. We can use Ch. 2 methods or energy methods to analyze this subsequent motion; we choose the latter. The level of their collision provides the reference ($y$ = 0) position for the gravitational potential energy, and we obtain

$$K_0 + U_0 = K + U \quad \Rightarrow \quad \frac{1}{2}Mv_0^2 + 0 = 0 + Mgy_{\max}.$$

Thus, with $v_0$ = 7.2 m/s, we find $y_{\max}$ = 2.6 m.

55. We choose $+x$ in the direction of (initial) motion of the blocks, which have masses $m_1$ = 5 kg and $m_2$ = 10 kg. Where units are not shown in the following, SI units are to be understood.

(a) Momentum conservation leads to

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$
$$(5\text{ kg})(3.0\text{ m/s}) + (10\text{ kg})(2.0\text{ m/s}) = (5\text{ kg})\vec{v}_{1f} + (10\text{ kg})(2.5\text{ m/s})$$

which yields $\vec{v}_{1f} = 2.0\text{ m/s}$. Thus, the speed of the 5.0 kg block immediately after the collision is $2.0\text{ m/s}$.

(b) We find the reduction in total kinetic energy:

$$\begin{aligned} K_i - K_f &= \frac{1}{2}(5\text{ kg})(3\text{ m/s})^2 + \frac{1}{2}(10\text{ kg})(2\text{ m/s})^2 - \frac{1}{2}(5\text{ kg})(2\text{ m/s})^2 - \frac{1}{2}(10\text{ kg})(2.5\text{ m/s})^2 \\ &= -1.25\text{ J} \approx -1.3\text{ J}. \end{aligned}$$

(c) In this new scenario where $\vec{v}_{2f} = 4.0\text{ m/s}$, momentum conservation leads to $\vec{v}_{1f} = -1.0\text{ m/s}$ and we obtain $\Delta K = +40\text{ J}$.

(d) The creation of additional kinetic energy is possible if, say, some gunpowder were on the surface where the impact occurred (initially stored chemical energy would then be contributing to the result).

56. (a) The magnitude of the deceleration of each of the cars is $a = f/m = \mu_k mg/m = \mu_k g$. If a car stops in distance $d$, then its speed $v$ just after impact is obtained from Eq. 2-16:

$$v^2 = v_0^2 + 2ad \Rightarrow v = \sqrt{2ad} = \sqrt{2\mu_k gd}$$

since $v_0 = 0$ (this could alternatively have been derived using Eq. 8-31). Thus,

$$v_A = \sqrt{2\mu_k gd_A} = \sqrt{2(0.13)(9.8\ \text{m/s}^2)(8.2\ \text{m})} = 4.6\ \text{m/s}.$$

(b) Similarly, $v_B = \sqrt{2\mu_k gd_B} = \sqrt{2(0.13)(9.8\ \text{m/s}^2)(6.1\ \text{m})} = 3.9\ \text{m/s}.$

(c) Let the speed of car $B$ be $v$ just before the impact. Conservation of linear momentum gives $m_B v = m_A v_A + m_B v_B$, or

$$v = \frac{(m_A v_A + m_B v_B)}{m_B} = \frac{(1100)(4.6) + (1400)(3.9)}{1400} = 7.5\ \text{m/s}.$$

(d) The conservation of linear momentum during the impact depends on the fact that the only significant force (during impact of duration $\Delta t$) is the force of contact between the bodies. In this case, that implies that the force of friction exerted by the road on the cars is neglected during the brief $\Delta t$. This neglect would introduce some error in the analysis. Related to this is the assumption we are making that the transfer of momentum occurs at one location, that the cars do not slide appreciably during $\Delta t$, which is certainly an approximation (though probably a good one). Another source of error is the application of the friction relation Eq. 6-2 for the sliding portion of the problem (after the impact); friction is a complex force that Eq. 6-2 only partially describes.

57. (a) Let $v$ be the final velocity of the ball-gun system. Since the total momentum of the system is conserved $mv_i = (m + M)v$. Therefore,

$$v = \frac{mv_i}{m+M} = \frac{(60\ \text{g})(22\ \text{m/s})}{60\ \text{g} + 240\ \text{g}} = 4.4\ \text{m/s}.$$

(b) The initial kinetic energy is $K_i = \frac{1}{2}mv_i^2$ and the final kinetic energy is

$$K_f = \tfrac{1}{2}(m+M)v^2 = \tfrac{1}{2}m^2v_i^2/(m+M).$$

The problem indicates $\Delta E_{\text{th}} = 0$, so the difference $K_i - K_f$ must equal the energy $U_s$ stored in the spring:

$$U_s = \frac{1}{2}mv_i^2 - \frac{1}{2}\frac{m^2v_i^2}{(m+M)} = \frac{1}{2}mv_i^2\left(1 - \frac{m}{m+M}\right) = \frac{1}{2}mv_i^2\,\frac{M}{m+M}.$$

Consequently, the fraction of the initial kinetic energy that becomes stored in the spring is

$$\frac{U_s}{K_i} = \frac{M}{m+M} = \frac{240}{60+240} = 0.80\,.$$

58. We think of this as having two parts: the first is the collision itself, where the blocks "join" so quickly that the 1.0-kg block has not had time to move through any distance yet, and then the subsequent motion of the 3.0 kg system as it compresses the spring to the maximum amount $x_m$. The first part involves momentum conservation (with $+x$ rightward):

$$m_1v_1 = (m_1+m_2)v \quad\Rightarrow\quad (2.0\text{ kg})(4.0\text{ m/s}) = (3.0\text{ kg})\vec{v}$$

which yields $\vec{v} = 2.7\text{ m/s}$. The second part involves mechanical energy conservation:

$$\frac{1}{2}(3.0\text{ kg})\,(2.7\text{ m/s})^2 = \frac{1}{2}\,(200\text{ N/m})x_m^2$$

which gives the result $x_m$ = 0.33 m.

59. As hinted in the problem statement, the velocity $v$ of the system as a whole, when the spring reaches the maximum compression $x_m$, satisfies

$$m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)v.$$

The change in kinetic energy of the system is therefore

$$\Delta K = \frac{1}{2}(m_1+m_2)v^2 - \frac{1}{2}m_1v_{1i}^2 - \frac{1}{2}m_2v_{2i}^2 = \frac{(m_1v_{1i}+m_2v_{2i})^2}{2(m_1+m_2)} - \frac{1}{2}m_1v_{1i}^2 - \frac{1}{2}m_2v_{2i}^2$$

which yields $\Delta K$ = –35 J. (Although it is not necessary to do so, still it is worth noting that algebraic manipulation of the above expression leads to $|\Delta K| = \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)\, v_{\text{rel}}^2$ where $v_{\text{rel}} = v_1 - v_2$). Conservation of energy then requires

$$\frac{1}{2}kx_m^2 = -\Delta K \Rightarrow x_m = \sqrt{\frac{-2\Delta K}{k}} = \sqrt{\frac{-2(-35\text{ J})}{1120\text{ N/m}}} = 0.25\text{ m}.$$

60. (a) Let $m_A$ be the mass of the block on the left, $v_{Ai}$ be its initial velocity, and $v_{Af}$ be its final velocity. Let $m_B$ be the mass of the block on the right, $v_{Bi}$ be its initial velocity, and $v_{Bf}$ be its final velocity. The momentum of the two-block system is conserved, so

$$m_A v_{Ai} + m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf}$$

and

$$v_{Af} = \frac{m_A v_{Ai} + m_B v_{Bi} - m_B v_{Bf}}{m_A} = \frac{(1.6\ \text{kg})(5.5\ \text{m/s}) + (2.4\ \text{kg})(2.5\ \text{m/s}) - (2.4\ \text{kg})(4.9\ \text{m/s})}{1.6\ \text{kg}}$$
$$= 1.9\ \text{m/s}.$$

(b) The block continues going to the right after the collision.

(c) To see whether the collision is elastic, we compare the total kinetic energy before the collision with the total kinetic energy after the collision. The total kinetic energy before is

$$K_i = \frac{1}{2} m_A v_{Ai}^2 + \frac{1}{2} m_B v_{Bi}^2 = \frac{1}{2}(1.6\ \text{kg})(5.5\ \text{m/s})^2 + \frac{1}{2}(2.4\ \text{kg})(2.5\ \text{m/s})^2 = 31.7\ \text{J}.$$

The total kinetic energy after is

$$K_f = \frac{1}{2} m_A v_{Af}^2 + \frac{1}{2} m_B v_{Bf}^2 = \frac{1}{2}(1.6\ \text{kg})(1.9\ \text{m/s})^2 + \frac{1}{2}(2.4\ \text{kg})(4.9\ \text{m/s})^2 = 31.7\ \text{J}.$$

Since $K_i = K_f$ the collision is found to be elastic.

61. **THINK** We have a moving cart colliding with a stationary cart. Since the collision is elastic, the total kinetic energy of the system remains unchanged.

**EXPRESS** Let $m_1$ be the mass of the cart that is originally moving, $v_{1i}$ be its velocity before the collision, and $v_{1f}$ be its velocity after the collision. Let $m_2$ be the mass of the cart that is originally at rest and $v_{2f}$ be its velocity after the collision. Conservation of linear momentum gives $m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$. Similarly, the total kinetic energy is conserved and we have

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 .$$

Solving for $v_{1f}$ and $v_{2f}$, we obtain:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}, \quad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

The speed of the center of mass is $v_{\text{com}} = \dfrac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$.

**ANALYZE** (a) With $m_1 = 0.34$ kg, $v_{1i} = 1.2\ \text{m/s}$ and $v_{1f} = 0.66\ \text{m/s}$, we obtain

$$m_2 = \frac{v_{1i} - v_{1f}}{v_{1i} + v_{1f}} m_1 = \left( \frac{1.2\text{ m/s} - 0.66\text{ m/s}}{1.2\text{ m/s} + 0.66\text{ m/s}} \right) (0.34\text{ kg}) = 0.0987\text{ kg} \approx 0.099\text{ kg}.$$

(b) The velocity of the second cart is:

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \left( \frac{2(0.34\text{ kg})}{0.34\text{ kg} + 0.099\text{ kg}} \right) (1.2\text{ m/s}) = 1.9\text{ m/s}.$$

(c) From the above, we find the speed of the center of mass to be

$$v_{\text{com}} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{(0.34\text{ kg})(1.2\text{ m/s}) + 0}{0.34\text{ kg} + 0.099\text{ kg}} = 0.93\text{ m/s}.$$

**LEARN** In solving for $v_{\text{com}}$, values for the initial velocities were used. Since the system is isolated with no external force acting on it, $v_{\text{com}}$ remains the same after the collision, so the same result is obtained if values for the final velocities are used. That is,

$$v_{\text{com}} = \frac{m_1 v_{1f} + m_2 v_{2f}}{m_1 + m_2} = \frac{(0.34\text{ kg})(0.66\text{ m/s}) + (0.099\text{ kg})(1.9\text{ m/s})}{0.34\text{ kg} + 0.099\text{ kg}} = 0.93\text{ m/s}.$$

62. (a) Let $m_1$ be the mass of one sphere, $v_{1i}$ be its velocity before the collision, and $v_{1f}$ be its velocity after the collision. Let $m_2$ be the mass of the other sphere, $v_{2i}$ be its velocity before the collision, and $v_{2f}$ be its velocity after the collision. Then, according to Eq. 9-75,

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}.$$

Suppose sphere 1 is originally traveling in the positive direction and is at rest after the collision. Sphere 2 is originally traveling in the negative direction. Replace $v_{1i}$ with $v$, $v_{2i}$ with $-v$, and $v_{1f}$ with zero to obtain $0 = m_1 - 3m_2$. Thus,

$$m_2 = m_1 / 3 = (300\text{ g}) / 3 = 100\text{ g}.$$

(b) We use the velocities before the collision to compute the velocity of the center of mass:

$$v_{\text{com}} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{(300\text{ g})(2.00\text{ m/s}) + (100\text{ g})(-2.00\text{ m/s})}{300\text{ g} + 100\text{ g}} = 1.00\text{ m/s}.$$

63. (a) The center of mass velocity does not change in the absence of external forces. In this collision, only forces of one block on the other (both being part of the same system) are exerted, so the center of mass velocity is 3.00 m/s before and after the collision.

(b) We can find the velocity $v_{1i}$ of block 1 before the collision (when the velocity of block 2 is known to be zero) using Eq. 9-17:

$$(m_1 + m_2)v_{\text{com}} = m_1 v_{1i} + 0 \quad \Rightarrow \quad v_{1i} = 12.0 \text{ m/s} .$$

Now we use Eq. 9-68 to find $v_{2f}$:

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = 6.00 \text{ m/s} .$$

64. First, we find the speed $v$ of the ball of mass $m_1$ right before the collision (just as it reaches its lowest point of swing). Mechanical energy conservation (with $h$ = 0.700 m) leads to

$$m_1 gh = \frac{1}{2} m_1 v^2 \;\Rightarrow\; v = \sqrt{2gh} = 3.7 \text{ m/s}.$$

(a) We now treat the elastic collision using Eq. 9-67:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v = \frac{0.5 \text{ kg} - 2.5 \text{ kg}}{0.5 \text{ kg} + 2.5 \text{ kg}}(3.7 \text{ m/s}) = -2.47 \text{ m/s}$$

which means the final speed of the ball is $2.47 \text{ m/s}$.

(b) Finally, we use Eq. 9-68 to find the final speed of the block:

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v = \frac{2(0.5 \text{ kg})}{0.5 \text{ kg} + 2.5 \text{ kg}}(3.7 \text{ m/s}) = 1.23 \text{ m/s}.$$

65. **THINK** We have a mass colliding with another stationary mass. Since the collision is elastic, the total kinetic energy of the system remains unchanged.

**EXPRESS** Let $m_1$ be the mass of the body that is originally moving, $v_{1i}$ be its velocity before the collision, and $v_{1f}$ be its velocity after the collision. Let $m_2$ be the mass of the body that is originally at rest and $v_{2f}$ be its velocity after the collision. Conservation of linear momentum gives

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} .$$

Similarly, the total kinetic energy is conserved and we have

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 .$$

The solution to $v_{1f}$ is given by Eq. 9-67: $v_{1f} = \dfrac{m_1 - m_2}{m_1 + m_2} v_{1i}$. We solve for $m_2$ to obtain

$$m_2 = \frac{v_{1i} - v_{1f}}{v_{1i} + v_{1f}} m_1 .$$

The speed of the center of mass is

$$v_{\text{com}} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} .$$

**ANALYZE** (a) given that $v_{1f} = v_{1i}/4$, we find the second mass to be

$$m_2 = \frac{v_{1i} - v_{1f}}{v_{1i} + v_{1f}} m_1 = \left(\frac{v_{1i} - v_{1i}/4}{v_{1i} + v_{1i}/4}\right) m_1 = \frac{3}{5} m_1 = \frac{3}{5}(2.0\text{ kg}) = 1.2\text{ kg} .$$

(b) The speed of the center of mass is $v_{\text{com}} = \dfrac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \dfrac{(2.0\text{ kg})(4.0\text{ m/s})}{2.0\text{ kg} + 1.2\text{ kg}} = 2.5\text{ m/s}$.

**LEARN** The final speed of the second mass is

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \left(\frac{2(2.0\text{ kg})}{2.0\text{ kg} + 1.2\text{ kg}}\right)(4.0\text{ m/s}) = 5.0\text{ m/s}.$$

Since the system is isolated with no external force acting on it, $v_{\text{com}}$ remains the same after the collision, so the same result is obtained if values for the final velocities are used:

$$v_{\text{com}} = \frac{m_1 v_{1f} + m_2 v_{2f}}{m_1 + m_2} = \frac{(2.0\text{ kg})(1.0\text{ m/s}) + (1.2\text{ kg})(5.0\text{ kg})}{2.0\text{ kg} + 1.2\text{ kg}} = 2.5\text{ m/s} .$$

66. Using Eq. 9-67 and Eq. 9-68, we have after the collision

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{m_1 - 0.40 m_1}{m_1 + 0.40 m_1}(4.0\text{ m/s}) = 1.71\text{ m/s}$$
$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2m_1}{m_1 + 0.40 m_1}(4.0\text{ m/s}) = 5.71\text{ m/s}.$$

(a) During the (subsequent) sliding, the kinetic energy of block 1 $K_{1f} = \frac{1}{2} m_1 v_{1f}^2$ is converted into thermal form ($\Delta E_{\text{th}} = \mu_k\, m_1\, g\, d_1$). Solving for the sliding distance $d_1$ we obtain $d_1 = 0.2999\text{ m} \approx 30\text{ cm}$.

(b) A very similar computation (but with subscript 2 replacing subscript 1) leads to block 2's sliding distance $d_2 = 3.332\text{ m} \approx 3.3\text{ m}$.

67. We use Eq 9-67 and 9-68 to find the velocities of the particles after their first collision (at $x$ = 0 and $t$ = 0):

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{0.30\text{ kg} - 0.40\text{ kg}}{0.30\text{ kg} + 0.40\text{ kg}} (2.0\text{ m/s}) = -0.29\text{ m/s}$$
$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2(0.30\text{ kg})}{0.30\text{ kg} + 0.40\text{ kg}} (2.0\text{ m/s}) = 1.7\text{ m/s}.$$

At a rate of motion of 1.7 m/s, $2x_w$ = 140 cm (the distance to the wall and back to $x$ = 0) will be traversed by particle 2 in 0.82 s. At $t$ = 0.82 s, particle 1 is located at

$$x = (-2/7)(0.82) = -23\text{ cm},$$

and particle 2 is "gaining" at a rate of (10/7) m/s leftward; this is their relative velocity at that time. Thus, this "gap" of 23 cm between them will be closed after an additional time of (0.23 m)/(10/7 m/s) = 0.16 s has passed. At this time ($t$ = 0.82 + 0.16 = 0.98 s) the two particles are at $x$ = (–2/7)(0.98) = –28 cm.

68. (a) If the collision is perfectly elastic, then Eq. 9-68 applies

$$v_2 = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2m_1}{m_1 + (2.00)m_1} \sqrt{2gh} = \frac{2}{3}\sqrt{2gh}$$

where we have used the fact (found most easily from energy conservation) that the speed of block 1 at the bottom of the frictionless ramp is $\sqrt{2gh}$ (where $h$ = 2.50 m). Next, for block 2's "rough slide" we use Eq. 8-37:

$$\tfrac{1}{2} m_2 v_2^2 = \Delta E_{\text{th}} = f_k d = \mu_k m_2 g d$$

where $\mu_k$ = 0.500. Solving for the sliding distance $d$, we find that $m_2$ cancels out and we obtain $d$ = 2.22 m.

(b) In a completely inelastic collision, we apply Eq. 9-53: $v_2 = \frac{m_1}{m_1 + m_2} v_{1i}$ (where, as above, $v_{1i} = \sqrt{2gh}$ ). Thus, in this case we have $v_2 = \sqrt{2gh}/3$. Now, Eq. 8-37 (using the total mass since the blocks are now joined together) leads to a sliding distance of $d = 0.556$ m (one-fourth of the part (a) answer).

69. (a) We use conservation of mechanical energy to find the speed of either ball after it has fallen a distance $h$. The initial kinetic energy is zero, the initial gravitational potential energy is $M gh$, the final kinetic energy is $\frac{1}{2} Mv^2$, and the final potential energy is zero. Thus $Mgh = \frac{1}{2} Mv^2$ and $v = \sqrt{2gh}$. The collision of the ball of $M$ with the floor is an elastic collision of a light object with a stationary massive object. The velocity of the light object reverses direction without change in magnitude. After the collision, the ball is

traveling upward with a speed of $\sqrt{2gh}$. The ball of mass $m$ is traveling downward with the same speed. We use Eq. 9-75 to find an expression for the velocity of the ball of mass $M$ after the collision:

$$v_{Mf} = \frac{M-m}{M+m}v_{Mi} + \frac{2m}{M+m}v_{mi} = \frac{M-m}{M+m}\sqrt{2gh} - \frac{2m}{M+m}\sqrt{2gh} = \frac{M-3m}{M+m}\sqrt{2gh}\ .$$

For this to be zero, $m = M/3$. With $M = 0.63$ kg, we have $m = 0.21$ kg.

(b) We use the same equation to find the velocity of the ball of mass $m$ after the collision:

$$v_{mf} = -\frac{m-M}{M+m}\sqrt{2gh} + \frac{2M}{M+m}\sqrt{2gh} = \frac{3M-m}{M+m}\sqrt{2gh}$$

which becomes (upon substituting $M = 3m$) $v_{mf} = 2\sqrt{2gh}$. We next use conservation of mechanical energy to find the height $h'$ to which the ball rises. The initial kinetic energy is $\frac{1}{2}mv_{mf}^2$, the initial potential energy is zero, the final kinetic energy is zero, and the final potential energy is $mgh'$. Thus,

$$\frac{1}{2}mv_{mf}^2 = mgh' \Rightarrow h' = \frac{v_{mf}^2}{2g} = 4h\ .$$

With $h = 1.8$ m, we have $h' = 7.2\ \text{m}$.

70. We use Eqs. 9-67, 9-68, and 4-21 for the elastic collision and the subsequent projectile motion. We note that both pucks have the same time-of-fall $t$ (during their projectile motions). Thus, we have

$$\Delta x_2 = v_2 t \quad \text{where } \Delta x_2 = d \text{ and } v_2 = \frac{2m_1}{m_1 + m_2}v_{1i}$$

$$\Delta x_1 = v_1 t \quad \text{where } \Delta x_1 = -2d \text{ and } v_1 = \frac{m_1 - m_2}{m_1 + m_2}v_{1i}\ .$$

Dividing the first equation by the second, we arrive at

$$\frac{d}{-2d} = \frac{\frac{2m_1}{m_1+m_2}v_{1i}\,t}{\frac{m_1-m_2}{m_1+m_2}v_{1i}\,t}\ .$$

After canceling $v_{1i}$, $t$, and $d$, and solving, we obtain $m_2 = 1.0$ kg.

71. We apply the conservation of linear momentum to the $x$ and $y$ axes respectively.

$$m_1 v_{1i} = m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2$$
$$0 = m_1 v_{1f} \sin\theta_1 - m_2 v_{2f} \sin\theta_2.$$

We are given $v_{2f} = 1.20\times10^5$ m/s, $\theta_1 = 64.0°$ and $\theta_2 = 51.0°$. Thus, we are left with two unknowns and two equations, which can be readily solved.

(a) We solve for the final alpha particle speed using the *y*-momentum equation:

$$v_{1f} = \frac{m_2 v_{2f} \sin\theta_2}{m_1 \sin\theta_1} = \frac{(16.0)\left(1.20\times10^5\right)\sin(51.0°)}{(4.00)\sin(64.0°)} = 4.15\times10^5 \text{ m/s}.$$

(b) Plugging our result from part (a) into the *x*-momentum equation produces the initial alpha particle speed:

$$\begin{aligned} v_{1i} &= \frac{m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2}{m_{1i}} \\ &= \frac{(4.00)\left(4.15\times10^5\right)\cos(64.0°) + (16.0)\left(1.2\times10^5\right)\cos(51.0°)}{4.00} \\ &= 4.84\times10^5 \text{ m/s}. \end{aligned}$$

72. We orient our +*x* axis along the initial direction of motion, and specify angles in the "standard" way — so θ = –90° for the particle *B*, which is assumed to scatter "downward" and ϕ > 0 for particle *A*, which presumably goes into the first quadrant. We apply the conservation of linear momentum to the *x* and *y* axes, respectively.

$$m_B v_B = m_B v'_B \cos\theta + m_A v'_A \cos\phi$$
$$0 = m_B v'_B \sin\theta + m_A v'_A \sin\phi$$

(a) Setting $v_B = v$ and $v'_B = v/2$, the *y*-momentum equation yields

$$m_A v'_A \sin\phi = m_B \frac{v}{2}$$

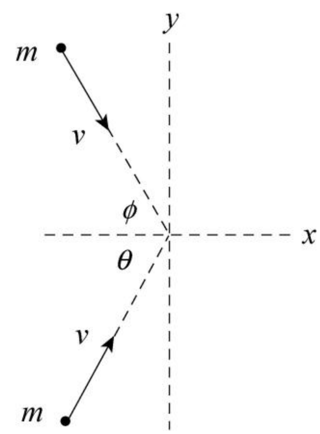
and the *x*-momentum equation yields $m_A v'_A \cos\phi = m_B v$. Dividing these two equations, we find $\tan\phi = \frac{1}{2}$, which yields $\phi = 27°$.

(b) We can *formally* solve for $v'_A$ (using the *y*-momentum equation and the fact that $\phi = 1/\sqrt{5}$)

$$v'_A = \frac{\sqrt{5}}{2}\frac{m_B}{m_A} v$$

but lacking numerical values for $v$ and the mass ratio, we cannot fully determine the final speed of $A$. Note: substituting $\cos\phi = 2/\sqrt{5}$, into the $x$-momentum equation leads to exactly this same relation (that is, no new information is obtained that might help us determine an answer).

73. Suppose the objects enter the collision along lines that make the angles $\theta > 0$ and $\phi > 0$ with the $x$ axis, as shown in the diagram that follows. Both have the same mass $m$ and the same initial speed $v$. We suppose that after the collision the combined object moves in the positive $x$ direction with speed $V$.




Since the $y$ component of the total momentum of the two-object system is conserved,

$$mv \sin\theta - mv \sin\phi = 0.$$

This means $\phi = \theta$. Since the $x$ component is conserved,

$$2mv \cos\theta = 2mV.$$

We now use $V = v/2$ to find that $\cos\theta = 1/2$. This means $\theta = 60°$. The angle between the initial velocities is 120°.

74. (a) Conservation of linear momentum implies

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}\,'_A + m_B\vec{v}\,'_B .$$

Since $m_A = m_B = m = 2.0$ kg, the masses divide out and we obtain

$$\begin{aligned}\vec{v}\,'_B &= \vec{v}_A + \vec{v}_B - \vec{v}\,'_A = (15\hat{\text{i}} + 30\hat{\text{j}})\text{ m/s} + (-10\hat{\text{i}} + 5\hat{\text{j}})\text{ m/s} - (-5\hat{\text{i}} + 20\hat{\text{j}})\text{ m/s} \\ &= (10\hat{\text{i}} + 15\hat{\text{j}})\text{ m/s} .\end{aligned}$$

(b) The final and initial kinetic energies are

$$\begin{aligned}K_f &= \frac{1}{2}mv'^2_A + \frac{1}{2}mv'^2_B = \frac{1}{2}(2.0)\left((-5)^2 + 20^2 + 10^2 + 15^2\right) = 8.0\times10^2\text{ J} \\ K_i &= \frac{1}{2}mv^2_A + \frac{1}{2}mv^2_B = \frac{1}{2}(2.0)\left(15^2 + 30^2 + (-10)^2 + 5^2\right) = 1.3\times10^3\text{ J} .\end{aligned}$$

The change kinetic energy is then $\Delta K = -5.0 \times 10^2$ J (that is, 500 J of the initial kinetic energy is lost).

75. We orient our +$x$ axis along the initial direction of motion, and specify angles in the "standard" way — so $\theta$ = +60° for the proton (1), which is assumed to scatter into the first quadrant and $\phi$ = –30° for the target proton (2), which scatters into the fourth quadrant (recall that the problem has told us that this is perpendicular to $\theta$). We apply the conservation of linear momentum to the $x$ and $y$ axes, respectively.

$$\begin{aligned} m_1 v_1 &= m_1 v_1' \cos\theta + m_2 v_2' \cos\phi \\ 0 &= m_1 v_1' \sin\theta + m_2 v_2' \sin\phi. \end{aligned}$$

We are given $v_1$ = 500 m/s, which provides us with two unknowns and two equations, which is sufficient for solving. Since $m_1 = m_2$ we can cancel the mass out of the equations entirely.

(a) Combining the above equations and solving for $v_2'$ we obtain

$$v_2' = \frac{v_1 \sin\theta}{\sin(\theta - \phi)} = \frac{(500 \text{ m/s})\sin(60°)}{\sin(90°)} = 433 \text{ m/s}.$$

We used the identity sin $\theta\cos\phi$ – $\cos\theta$ $\sin\phi$ = sin ($\theta$– $\phi$) in simplifying our final expression.

(b) In a similar manner, we find

$$v_1' = \frac{v_1 \sin\theta}{\sin(\phi - \theta)} = \frac{(500 \text{ m/s})\sin(-30°)}{\sin(-90°)} = 250 \text{ m/s} .$$

76. We use Eq. 9-88. Then

$$v_f = v_i + v_{\text{rel}} \ln\left(\frac{M_i}{M_f}\right) = 105 \text{ m/s} + (253 \text{ m/s}) \ln\left(\frac{6090 \text{ kg}}{6010 \text{ kg}}\right) = 108 \text{ m/s}.$$

77. **THINK** The mass of the faster barge is increasing at a constant rate. Additional force must be provided in order to maintain a constant speed.

**EXPRESS** We consider what must happen to the coal that lands on the faster barge during a time interval $\Delta t$. In that time, a total of $\Delta m$ of coal must experience a change of velocity (from slow to fast) $\Delta v = v_{\text{fast}} - v_{\text{slow}}$, where rightwards is considered the positive direction. The rate of change in momentum for the coal is therefore

$$\frac{\Delta p}{\Delta t} = \frac{(\Delta m)}{\Delta t}\Delta v = \left(\frac{\Delta m}{\Delta t}\right)(v_{\text{fast}} - v_{\text{slow}})$$

which, by Eq. 9-23, must equal the force exerted by the (faster) barge on the coal. The processes (the shoveling, the barge motions) are constant, so there is no ambiguity in equating $\frac{\Delta p}{\Delta t}$ with $\frac{dp}{dt}$. Note that we ignore the transverse speed of the coal as it is shoveled from the slower barge to the faster one.

**ANALYZE** (a) With $v_{\text{fast}} = 20\,\text{km/h} = 5.56\,\text{m/s}$, $v_{\text{slow}} = 10\,\text{km/h} = 2.78\,\text{m/s}$ and the rate of mass change $(\Delta m/\Delta t) = 1000\,\text{kg/min} = (16.67\,\text{kg/s})$, the force that must be applied to the faster barge is

$$F_{\text{fast}} = \left(\frac{\Delta m}{\Delta t}\right)(v_{\text{fast}} - v_{\text{slow}}) = (16.67\text{ kg/s})(5.56\text{ m/s} - 2.78\text{ m/s}) = 46.3\,\text{N}$$

(b) The problem states that the frictional forces acting on the barges does not depend on mass, so the loss of mass from the slower barge does not affect its motion (so no extra force is required as a result of the shoveling).

**LEARN** The force that must be applied to the faster barge in order to maintain a constant speed is equal to the rate of change of momentum of the coal.

78. We use Eq. 9-88 and simplify with $v_i = 0$, $v_f = v$, and $v_{\text{rel}} = u$.

$$v_f - v_i = v_{\text{rel}} \ln \frac{M_i}{M_f} \Rightarrow \frac{M_i}{M_f} = e^{v/u}$$

(a) If $v = u$ we obtain $\frac{M_i}{M_f} = e^1 \approx 2.7$.

(b) If $v = 2u$ we obtain $\frac{M_i}{M_f} = e^2 \approx 7.4$.

79. **THINK** As fuel is consumed, both the mass and the speed of the rocket will change.

**EXPRESS** The thrust of the rocket is given by $T = Rv_{\text{rel}}$ where $R$ is the rate of fuel consumption and $v_{\text{rel}}$ is the speed of the exhaust gas relative to the rocket. On the other hand, the mass of fuel ejected is given by $M_{\text{fuel}} = R\Delta t$, where $\Delta t$ is the time interval of the burn. Thus, the mass of the rocket after the burn is

$$M_f = M_i - M_{\text{fuel}}\,.$$

**ANALYZE** (a) Given that $R = 480$ kg/s and $v_{\text{rel}} = 3.27 \times 10^3$ m/s, we find the thrust to be

$$T = Rv_{\text{rel}} = (480\,\text{kg/s})(3.27\times10^3\,\text{m/s}) = 1.57\times10^6\,\text{N}.$$

(b) With the mass of fuel ejected given by $M_{\text{fuel}} = R\Delta t =$ (480 kg/s)(250 s) = $1.20\times10^5$ kg, the final mass of the rocket is

$$M_f = M_i - M_{\text{fuel}} = (2.55 \times 10^5 \text{ kg}) - (1.20 \times 10^5 \text{ kg}) = 1.35 \times 10^5 \text{ kg}.$$

(c) Since the initial speed is zero, the final speed of the rocket is

$$v_f = v_{\text{rel}} \ln \frac{M_i}{M_f} = \left(3.27\times10^3 \text{ m/s}\right) \ln\left(\frac{2.55\times10^5 \text{ kg}}{1.35\times10^5 \text{ kg}}\right) = 2.08\times10^3 \text{ m/s}.$$

**LEARN** The speed of the rocket continues to rise as the fuel is consumed. From the first rocket equation given in Eq. 9-87, the thrust of the rocket is related to the acceleration by $T = Ma$ . Using this equation, we find the initial acceleration to be

$$a_i = \frac{T}{M_i} = \frac{1.57\times10^6 \text{ N}}{2.55\times10^5 \text{ kg}} = 6.16 \text{ m/s}^2.$$

80. The velocity of the object is

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\Big((3500 - 160t)\,\hat{\text{i}} + 2700\,\hat{\text{j}} + 300\,\hat{\text{k}}\Big) = -(160 \text{ m/s})\hat{\text{i}}.$$

(a) The linear momentum is $\vec{p} = m\vec{v} = \left(250 \text{ kg}\right)\left(-160 \text{ m/s}\,\hat{\text{i}}\right) = (-4.0\times10^4 \text{ kg}\cdot\text{m/s})\,\hat{\text{i}}.$

(b) The object is moving west (our $-\hat{\text{i}}$ direction).

(c) Since the value of $\vec{p}$ does not change with time, the net force exerted on the object is zero, by Eq. 9-23.

81. We assume no external forces act on the system composed of the two parts of the last stage. Hence, the total momentum of the system is conserved. Let $m_c$ be the mass of the rocket case and $m_p$ the mass of the payload. At first they are traveling together with velocity $v$. After the clamp is released $m_c$ has velocity $v_c$ and $m_p$ has velocity $v_p$. Conservation of momentum yields

$$(m_c + m_p)v = m_c v_c + m_p v_p.$$

(a) After the clamp is released the payload, having the lesser mass, will be traveling at the greater speed. We write $v_p = v_c + v_{\text{rel}}$, where $v_{\text{rel}}$ is the relative velocity. When this expression is substituted into the conservation of momentum condition, the result is

$$\left(m_c + m_p\right)v = m_c v_c + m_p v_c + m_p v_{\text{rel}}.$$

Therefore,

$$v_c = \frac{(m_c + m_p)v - m_p v_{\text{rel}}}{m_c + m_p} = \frac{(290.0\text{ kg} + 150.0\text{ kg})(7600\text{ m/s}) - (150.0\text{ kg})(910.0\text{ m/s})}{290.0\text{ kg} + 150.0\text{ kg}}$$
$$= 7290\text{ m/s}.$$

(b) The final speed of the payload is $v_p = v_c + v_{\text{rel}}$ = 7290 m/s + 910.0 m/s = 8200 m/s.

(c) The total kinetic energy before the clamp is released is

$$K_i = \frac{1}{2}(m_c + m_p)v^2 = \frac{1}{2}(290.0\text{ kg} + 150.0\text{ kg})(7600\text{ m/s})^2 = 1.271 \times 10^{10}\text{ J}.$$

(d) The total kinetic energy after the clamp is released is

$$K_f = \frac{1}{2}m_c v_c^2 + \frac{1}{2}m_p v_p^2 = \frac{1}{2}(290.0\text{ kg})(7290\text{ m/s})^2 + \frac{1}{2}(150.0\text{ kg})(8200\text{ m/s})^2$$
$$= 1.275 \times 10^{10}\text{ J}.$$

The total kinetic energy increased slightly. Energy originally stored in the spring is converted to kinetic energy of the rocket parts.

82. Let $m$ be the mass of the higher floors. By energy conservation, the speed of the higher floors just before impact is

$$mgd = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{2gd}.$$

The magnitude of the impulse during the impact is

$$J = |\Delta p| = m|\Delta v| = mv = m\sqrt{2gd} = mg\sqrt{\frac{2d}{g}} = W\sqrt{\frac{2d}{g}}$$

where $W = mg$ represents the weight of the higher floors. Thus, the average force exerted on the lower floor is

$$F_{\text{avg}} = \frac{J}{\Delta t} = \frac{W}{\Delta t}\sqrt{\frac{2d}{g}}$$

With $F_{\text{avg}} = sW$, where $s$ is the safety factor, we have

$$s = \frac{1}{\Delta t}\sqrt{\frac{2d}{g}} = \frac{1}{1.5 \times 10^{-3}\text{ s}}\sqrt{\frac{2(4.0\text{ m})}{9.8\text{ m/s}^2}} = 6.0 \times 10^2.$$

83. (a) Momentum conservation gives

$$m_R v_R + m_L v_L = 0 \;\Rightarrow\; (0.500 \text{ kg})\, v_R + (1.00 \text{ kg})(-1.20 \text{ m/s}) = 0$$

which yields $v_R$ = 2.40 m/s. Thus, $\Delta x = v_R\, t$ = (2.40 m/s)(0.800 s) = 1.92 m.

(b) Now we have $m_R v_R + m_L (v_R - 1.20 \text{ m/s}) = 0$, which yields

$$v_R = \frac{(1.2 \text{ m/s})m_L}{m_L + m_R} = \frac{(1.20 \text{ m/s})(1.00 \text{ kg})}{1.00 \text{ kg} + 0.500 \text{ kg}} = 0.800 \text{ m/s}.$$

Consequently, $\Delta x = v_R\, t$ = 0.640 m.

84. (a) This is a highly symmetric collision, and when we analyze the $y$-components of momentum we find their net value is zero. Thus, the stuck-together particles travel along the $x$ axis.

(b) Since it is an elastic collision with identical particles, the final speeds are the same as the initial values. Conservation of momentum along each axis then assures that the angles of approach are the same as the angles of scattering. Therefore, one particle travels along line 2, the other along line 3.

(c) Here the final speeds are less than they were initially. The total $x$-component cannot be less, however, by momentum conservation, so the loss of speed shows up as a decrease in their $y$-velocity-components. This leads to smaller angles of scattering. Consequently, one particle travels through region $B$, the other through region $C$; the paths are symmetric about the $x$-axis. We note that this is intermediate between the final states described in parts (b) and (a).

(d) Conservation of momentum along the $x$-axis leads (because these are identical particles) to the simple observation that the $x$-component of each particle remains constant:

$$v_{fx} = v\cos\theta = 3.06 \text{ m/s}.$$

(e) As noted above, in this case the speeds are unchanged; both particles are moving at 4.00 m/s in the final state.

85. Using Eq. 9-67 and Eq. 9-68, we have after the first collision

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{m_1 - 2m_1}{m_1 + 2m_1} v_{1i} = -\frac{1}{3} v_{1i}$$
$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2m_1}{m_1 + 2m_1} v_{1i} = \frac{2}{3} v_{1i}.$$

After the second collision, the velocities are

$$v_{2ff} = \frac{m_2 - m_3}{m_2 + m_3} v_{2f} = \frac{-m_2}{3m_2} \frac{2}{3} v_{1i} = -\frac{2}{9} v_{1i}$$

$$v_{3ff} = \frac{2m_2}{m_2 + m_3} v_{2f} = \frac{2m_2}{3m_2} \frac{2}{3} v_{1i} = \frac{4}{9} v_{1i} \ .$$

(a) Setting $v_{1i}$ = 4 m/s, we find $v_{3ff} \approx 1.78$ m/s.

(b) We see that $v_{3ff}$ is less than $v_{1i}$ .

(c) The final kinetic energy of block 3 (expressed in terms of the initial kinetic energy of block 1) is

$$K_{3ff} = \frac{1}{2} m_3 v_3^2 = \frac{1}{2}(4m_1)\left(\frac{4}{9}\right)^2 v_{1i}^2 = \frac{64}{81} K_{1i} \, .$$

We see that this is less than $K_{1i}$ .

(d) The final momentum of block 3 is $p_{3ff} = m_3 v_{3ff} = (4m_1)\left(\frac{16}{9}\right) v_1 > m_1 v_1$.

86. (a) We use Eq. 9-68 twice:

$$v_2 = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2m_1}{1.5m_1} (4.00 \text{ m/s}) = \frac{16}{3} \text{ m/s}$$

$$v_3 = \frac{2m_2}{m_2 + m_3} v_2 = \frac{2m_2}{1.5m_2} (16/3 \text{ m/s}) = \frac{64}{9} \text{ m/s} = 7.11 \text{ m/s} \, .$$

(b) Clearly, the speed of block 3 is greater than the (initial) speed of block 1.

(c) The kinetic energy of block 3 is

$$K_{3f} = \frac{1}{2} m_3 v_3^2 = \left(\frac{1}{2}\right)^3 m_1 \left(\frac{16}{9}\right)^2 v_{1i}^2 = \frac{64}{81} K_{1i} \, .$$

We see the kinetic energy of block 3 is less than the (initial) $K$ of block 1. In the final situation, the initial $K$ is being shared among the three blocks (which are all in motion), so this is not a surprising conclusion.

(d) The momentum of block 3 is

$$p_{3f} = m_3 v_3 = \left(\frac{1}{2}\right)^2 m_1 \left(\frac{16}{9}\right) v_{1i} = \frac{4}{9} p_{1i}$$

and is therefore less than the initial momentum (both of these being considered in magnitude, so questions about ± sign do not enter the discussion).

87. We choose our positive direction in the direction of the rebound (so the ball's initial velocity is negative-valued $\vec{v}_i = -5.2 \text{ m/s}$).

(a) The speed of the ball right after the collision is

$$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(K_i/2)}{m}} = \sqrt{\frac{mv_i^2/2}{m}} = \frac{v_i}{\sqrt{2}} \approx 3.7 \text{ m/s}.$$

(b) With $m$ = 0.15 kg, the impulse-momentum theorem (Eq. 9-31) yields

$$\vec{J} = m\vec{v}_f - m\vec{v}_i = (0.15 \text{ kg})(3.7 \text{ m/s}) - (0.15 \text{ kg})(-5.2 \text{ m/s}) = 1.3 \text{ N}\cdot\text{s}.$$

(c) Equation 9-35 leads to $F_{avg} = J/\Delta t = 1.3/0.0076 = 1.8 \times 10^2$ N.

88. We first consider the 1200 kg part. The impulse has magnitude $J$ and is (by our choice of coordinates) in the positive direction. Let $m_1$ be the mass of the part and $v_1$ be its velocity after the bolts are exploded. We assume both parts are at rest before the explosion. Then $J = m_1v_1$, so

$$v_1 = \frac{J}{m_1} = \frac{300 \text{ N}\cdot\text{s}}{1200 \text{ kg}} = 0.25 \text{ m/s}.$$

The impulse on the 1800 kg part has the same magnitude but is in the opposite direction, so $-J = m_2v_2$, where $m_2$ is the mass and $v_2$ is the velocity of the part. Therefore,

$$v_2 = -\frac{J}{m_2} = -\frac{300\text{N}\cdot\text{s}}{1800\,\text{kg}} = -0.167 \text{ m/s}.$$

Consequently, the relative speed of the parts after the explosion is

$$u = 0.25 \text{ m/s} - (-0.167 \text{ m/s}) = 0.417 \text{ m/s}.$$

89. **THINK** The momentum of the car changes as it turns and collides with a tree.

**EXPRESS** Let the initial and final momenta of the car be $\vec{p}_i = m\vec{v}_i$ and $\vec{p}_f = m\vec{v}_f$, respectively. The impulse on it equals the change in its momentum:

$$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = m(\vec{v}_f - \vec{v}_i).$$

The average force over the duration $\Delta t$ is given by $\vec{F}_{avg} = \vec{J}/\Delta t$.

**ANALYZE** (a) The initial momentum of the car is

$$\vec{p}_i = m\vec{v}_i = (1400\,\text{kg})(5.3\,\text{m/s})\hat{\text{j}} = (7400\,\text{kg}\cdot\text{m/s})\hat{\text{j}}$$

and the final momentum after making the turn is $\vec{p}_f = (7400\,\text{kg}\cdot\text{m/s})\hat{\text{i}}$ (note that the magnitude remains the same, only the direction is changed). Thus, the impulse is

$$\vec{J} = \vec{p}_f - \vec{p}_i = (7.4\times10^3\,\text{N}\cdot\text{s})(\hat{\text{i}} - \hat{\text{j}}).$$

(b) The initial momentum of the car after the turn is $\vec{p}\,'_i = (7400\,\text{kg}\cdot\text{m/s})\hat{\text{i}}$ and the final momentum after colliding with a tree is $\vec{p}\,'_f = 0$. The impulse acting on it is

$$\vec{J}\,' = \vec{p}\,'_f - \vec{p}\,'_i = (-7.4\times10^3\,\text{N}\cdot\text{s})\hat{\text{i}}.$$

(c) The average force on the car during the turn is

$$\vec{F}_{\text{avg}} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\vec{J}}{\Delta t} = \frac{(7400\,\text{kg}\cdot\text{m/s})(\hat{\text{i}} - \hat{\text{j}})}{4.6\ \text{s}} = (1600\,\text{N})(\hat{\text{i}} - \hat{\text{j}})$$

and its magnitude is

$$F_{\text{avg}} = (1600\,\text{N})\sqrt{2} = 2.3\times10^3\,\text{N}.$$

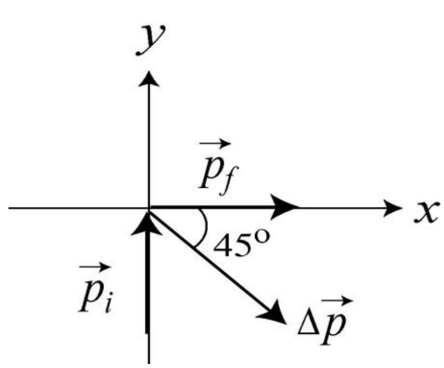
(d) The average force during the collision with the tree is

$$\vec{F}\,'_{\text{avg}} = \frac{\vec{J}\,'}{\Delta t} = \frac{(-7400\,\text{kg}\cdot\text{m/s})\hat{\text{i}}}{350\times10^{-3}\text{s}} = (-2.1\times10^4\,\text{N})\hat{\text{i}}$$

and its magnitude is $F'_{\text{avg}} = 2.1\times10^4\,\text{N}$.

(e) As shown in (c), the average force during the turn, in unit vector notation, is $\vec{F}_{\text{avg}} = (1600\,\text{N})(\hat{\text{i}} - \hat{\text{j}})$. The force is 45° below the positive *x* axis.

**LEARN** During the turn, the average force $\vec{F}_{\text{avg}}$ is in the same direction as $\vec{J}$, or $\Delta\vec{p}$. Its *x* and *y* components have equal magnitudes. The *x* component is positive and the *y* component is negative, so the force is 45° below the positive *x* axis.



90. (a) We find the momentum $\vec{p}_{n\,r}$ of the residual nucleus from momentum conservation.

$$\vec{p}_{n\,i} = \vec{p}_e + \vec{p}_v + \vec{p}_{n\,r} \;\;\Rightarrow\;\; 0 = (-1.2\times10^{-22}\ \text{kg}\cdot\text{m/s})\,\hat{\text{i}} + (-6.4\times10^{-23}\ \text{kg}\cdot\text{m/s})\,\hat{\text{j}} + \vec{p}_{n\,r}$$

Thus, $\vec{p}_{n\,r} = (1.2\times10^{-22}\,\text{kg}\cdot\text{m/s})\,\hat{\text{i}} + (6.4\times10^{-23}\,\text{kg}\cdot\text{m/s})\,\hat{\text{j}}$. Its magnitude is

$$|\,\vec{p}_{n\,r}\,| = \sqrt{\left(1.2\times10^{-22}\ \text{kg}\cdot\text{m/s}\right)^2 + \left(6.4\times10^{-23}\ \text{kg}\cdot\text{m/s}\right)^2} = 1.4\times10^{-22}\ \text{kg}\cdot\text{m/s}.$$

(b) The angle measured from the +$x$ axis to $\vec{p}_{n\,r}$ is

$$\theta = \tan^{-1}\left(\frac{6.4\times10^{-23}\ \text{kg}\cdot\text{m/s}}{1.2\times10^{-22}\ \text{kg}\cdot\text{m/s}}\right) = 28°.$$

(c) Combining the two equations $p = mv$ and $K = \frac{1}{2}mv^2$, we obtain (with $p = p_{n\,r}$ and $m = m_{n\,r}$)

$$K = \frac{p^2}{2m} = \frac{\left(1.4\times10^{-22}\ \text{kg}\cdot\text{m/s}\right)^2}{2\left(5.8\times10^{-26}\ \text{kg}\right)} = 1.6\times10^{-19}\ \text{J}.$$

91. No external forces with horizontal components act on the cart-man system and the vertical forces sum to zero, so the total momentum of the system is conserved. Let $m_c$ be the mass of the cart, $v$ be its initial velocity, and $v_c$ be its final velocity (after the man jumps off). Let $m_m$ be the mass of the man. His initial velocity is the same as that of the cart and his final velocity is zero. Conservation of momentum yields $(m_m + m_c)v = m_c v_c$. Consequently, the final speed of the cart is

$$v_c = \frac{v(m_m + m_c)}{m_c} = \frac{(2.3\ \text{m/s})(75\ \text{kg} + 39\ \text{kg})}{39\ \text{kg}} = 6.7\ \text{m/s}.$$

The cart speeds up by 6.7 m/s – 2.3 m/s = + 4.4 m/s. In order to slow himself, the man gets the cart to push backward on him by pushing forward on it, so the cart speeds up.

92. The fact that they are connected by a spring is not used in the solution. We use Eq. 9-17 for $\vec{v}_{\text{com}}$:

$$M\vec{v}_{\text{com}} = m_1\vec{v}_1 + m_2\vec{v}_2 = (1.0\ \text{kg})(1.7\ \text{m/s}) + (3.0\ \text{kg})\vec{v}_2$$

which yields $|\vec{v}_2| = 0.57\ \text{m/s}$. The direction of $\vec{v}_2$ is opposite that of $\vec{v}_1$ (that is, they are both headed toward the center of mass, but from opposite directions).

93. **THINK** A completely inelastic collision means that the railroad freight car and the caboose car move together after the collision. The motion is one-dimensional.

**EXPRESS** Let $m_F$ be the mass of the freight car and $v_F$ be its initial velocity. Let $m_C$ be the mass of the caboose and $v$ be the common final velocity of the two when they are coupled. Conservation of the total momentum of the two-car system leads to

$$m_F v_F = (m_F + m_C)v \;\Rightarrow\; v = \frac{m_F v_F}{m_F + m_C}.$$

The initial kinetic energy of the system is $K_i = \frac{1}{2} m_F v_F^2$ and the final kinetic energy is

$$K_f = \frac{1}{2}(m_F + m_C)v^2 = \frac{1}{2}(m_F + m_C)\frac{m_F^2 v_F^2}{(m_F + m_C)^2} = \frac{1}{2}\frac{m_F^2 v_F^2}{(m_F + m_C)}.$$

Since 27% of the original kinetic energy is lost, we have $K_f = 0.73K_i$. Combining with the two equations above allows us to solve for $m_C$, the mass of the caboose.

**ANALYZE** With $K_f = 0.73K_i$, or

$$\frac{1}{2}\frac{m_F^2 v_F^2}{(m_F + m_C)} = (0.73)\left(\frac{1}{2} m_F v_F^2\right)$$

we obtain $m_F/(m_F + m_C) = 0.73,$ which we use in solving for the mass of the caboose:

$$m_C = \frac{0.27}{0.73} m_F = 0.37 m_F = (0.37)(3.18 \times 10^4 \text{ kg}) = 1.18 \times 10^4 \text{ kg}.$$

**LEARN** Energy is lost during an inelastic collision, but momentum is still conserved because there's no external force acting on the two-car system.

94. Let $m_c$ be the mass of the Chrysler and $v_c$ be its velocity. Let $m_f$ be the mass of the Ford and $v_f$ be its velocity. Then the velocity of the center of mass is

$$v_{\text{com}} = \frac{m_c v_c + m_f v_f}{m_c + m_f} = \frac{(2400 \text{ kg})(80 \text{ km/h}) + (1600 \text{ kg})(60 \text{ km/h})}{2400 \text{ kg} + 1600 \text{ kg}} = 72 \text{ km/h}.$$

We note that the two velocities are in the same direction, so the two terms in the numerator have the same sign.

95. **THINK** A billiard ball undergoes glancing collision with another identical billiard ball. The collision is two-dimensional.

**EXPRESS** The mass of each ball is *m*, and the initial speed of one of the balls is $v_{1i} = 2.2\,\text{m/s}$. We apply the conservation of linear momentum to the *x* and *y* axes respectively:

$$mv_{1i} = mv_{1f}\cos\theta_1 + mv_{2f}\cos\theta_2$$
$$0 = mv_{1f}\sin\theta_1 - mv_{2f}\sin\theta_2$$

The mass *m* cancels out of these equations, and we are left with two unknowns and two equations, which is sufficient to solve.

**ANALYZE** (a) Solving the simultaneous equations leads to

$$v_{1f} = \frac{\sin\theta_2}{\sin(\theta_1+\theta_2)}v_{1i}, \quad v_{2f} = \frac{\sin\theta_1}{\sin(\theta_1+\theta_2)}v_{1i}$$

Since $v_{2f} = v_{1i}/2 = 1.1\,\text{m/s}$ and $\theta_2 = 60°$, we have

$$\frac{\sin\theta_1}{\sin(\theta_1+60°)} = \frac{1}{2} \quad \Rightarrow \quad \tan\theta_1 = \frac{1}{\sqrt{3}}$$

or $\theta_1 = 30°$. Thus, the speed of ball 1 after collision is

$$v_{1f} = \frac{\sin\theta_2}{\sin(\theta_1+\theta_2)}v_{1i} = \frac{\sin 60°}{\sin(30°+60°)}v_{1i} = \frac{\sqrt{3}}{2}v_{1i} = \frac{\sqrt{3}}{2}(2.2\,\text{m/s}) = 1.9\,\text{m/s}\,.$$

(b) From the above, we have $\theta_1$ = 30°, measured *clockwise* from the +*x*-axis, or equivalently, –30°, measured *counterclockwise* from the +*x*-axis.

(c) The kinetic energy before collision is $K_i = \frac{1}{2}mv_{1i}^2$. After the collision, we have

$$K_f = \frac{1}{2}m\left(v_{1f}^2 + v_{2f}^2\right)$$

Substituting the expressions for $v_{1f}$ and $v_{2f}$ found above gives

$$K_f = \frac{1}{2}m\left[\frac{\sin^2\theta_2}{\sin^2(\theta_1+\theta_2)} + \frac{\sin^2\theta_1}{\sin^2(\theta_1+\theta_2)}\right]v_{1i}^2$$

Since $\theta_1 = 30°$ and $\theta_2 = 60°$, $\sin(\theta_1+\theta_2) = 1$ and $\sin^2\theta_1 + \sin^2\theta_2 = \sin^2\theta_1 + \cos^2\theta_1 = 1$, and indeed, we have $K_f = \frac{1}{2}mv_{1i}^2 = K_i$, which means that energy is conserved.

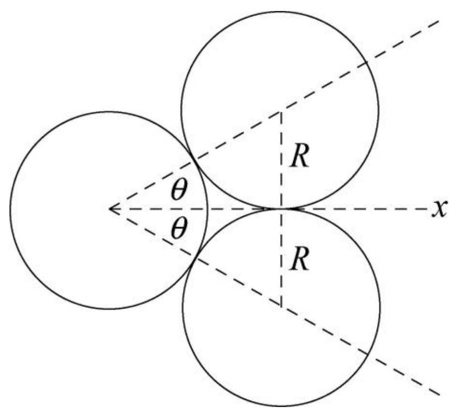
**LEARN** One may verify that when two identical masses collide elastically, they will move off perpendicularly to each other with $\theta_1 + \theta_2 = 90°$.

96. (a) We use Eq. 9-87. The thrust is

$$Rv_{\text{rel}} = Ma = \left(4.0\times10^4\ \text{kg}\right)\left(2.0\ \text{m/s}^2\right) = 8.0\times10^4\ \text{N}.$$

(b) Since $v_{\text{rel}}$ = 3000 m/s, we see from part (a) that $R \approx 27$ kg/s.

97. The diagram below shows the situation as the incident ball (the left-most ball) makes contact with the other two.




It exerts an impulse of the same magnitude on each ball, along the line that joins the centers of the incident ball and the target ball. The target balls leave the collision along those lines, while the incident ball leaves the collision along the $x$ axis. The three dashed lines that join the centers of the balls in contact form an equilateral triangle, so both of the angles marked $\theta$ are 30°. Let $v_0$ be the velocity of the incident ball before the collision and $V$ be its velocity afterward. The two target balls leave the collision with the same speed. Let $v$ represent that speed. Each ball has mass $m$. Since the $x$ component of the total momentum of the three-ball system is conserved,

$$mv_0 = mV + 2mv\cos\theta$$

and since the total kinetic energy is conserved,

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mV^2 + 2\left(\frac{1}{2}mv^2\right).$$

We know the directions in which the target balls leave the collision so we first eliminate $V$ and solve for $v$. The momentum equation gives $V = v_0 - 2v\cos\theta$, so

$$V^2 = v_0^2 - 4v_0v\cos\theta + 4v^2\cos^2\theta$$

and the energy equation becomes $v_0^2 = v_0^2 - 4v_0 v\cos\theta + 4v^2\cos^2\theta + 2v^2$. Therefore,

$$v = \frac{2v_0\cos\theta}{1+2\cos^2\theta} = \frac{2(10\ \text{m/s})\cos 30°}{1+2\cos^2 30°} = 6.93\ \text{m/s}.$$

(a) The discussion and computation above determines the final speed of ball 2 (as labeled in Fig. 9-76) to be 6.9 m/s.

(b) The direction of ball 2 is at 30° counterclockwise from the +$x$ axis.

(c) Similarly, the final speed of ball 3 is 6.9 m/s.

(d) The direction of ball 3 is at –30° counterclockwise from the +$x$ axis.

(e) Now we use the momentum equation to find the final velocity of ball 1:

$$V = v_0 - 2v\cos\theta = 10\ \text{m/s} - 2(6.93\ \text{m/s})\ \cos 30° = -2.0\,\text{m/s}.$$

So the speed of ball 1 is $|V| = 2.0$ m/s.

(f) The minus sign indicates that it bounces back in the – $x$ direction. The angle is –180°.

98. (a) The momentum change for the 0.15 kg object is

$$\Delta\vec{p} = (0.15)[2\,\hat{\text{i}} + 3.5\,\hat{\text{j}} - 3.2\,\hat{\text{k}} - (5\,\hat{\text{i}} + 6.5\,\hat{\text{j}} + 4\,\hat{\text{k}})] = (-0.450\hat{\text{i}} - 0.450\hat{\text{j}} - 1.08\hat{\text{k}})\ \text{kg}\cdot\text{m/s}\,.$$

(b) By the impulse-momentum theorem (Eq. 9-31), $\vec{J} = \Delta\vec{p}$ , we have

$$\vec{J} = (-0.450\hat{\text{i}} - 0.450\hat{\text{j}} - 1.08\hat{\text{k}})\ \text{N}\cdot\text{s}\,.$$

(c) Newton's third law implies $\vec{J}_{\text{wall}} = -\vec{J}_{\text{ball}}$ (where $\vec{J}_{\text{ball}}$ is the result of part (b)), so

$$\vec{J}_{\text{wall}} = (0.450\hat{\text{i}} + 0.450\hat{\text{j}} + 1.08\hat{\text{k}})\ \text{N}\cdot\text{s}\,.$$

99. (a) We place the origin of a coordinate system at the center of the pulley, with the $x$ axis horizontal and to the right and with the $y$ axis downward. The center of mass is halfway between the containers, at $x = 0$ and $y = \ell$, where $\ell$ is the vertical distance from the pulley center to either of the containers. Since the diameter of the pulley is 50 mm, the center of mass is at a horizontal distance of 25 mm from each container.

(b) Suppose 20 g is transferred from the container on the left to the container on the right. The container on the left has mass $m_1$ = 480 g and is at $x_1$ = –25 mm. The container on the right has mass $m_2$ = 520 g and is at $x_2$ = +25 mm. The $x$ coordinate of the center of mass is then

$$x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{(480\text{ g})(-25\text{ mm}) + (520\text{ g})(25\text{ mm})}{480\text{ g} + 520\text{ g}} = 1.0\text{ mm}.$$

The $y$ coordinate is still $\ell$. The center of mass is 26 mm from the lighter container, along the line that joins the bodies.

(c) When they are released the heavier container moves downward and the lighter container moves upward, so the center of mass, which must remain closer to the heavier container, moves downward.

(d) Because the containers are connected by the string, which runs over the pulley, their accelerations have the same magnitude but are in opposite directions. If $a$ is the acceleration of $m_2$, then $-a$ is the acceleration of $m_1$. The acceleration of the center of mass is

$$a_{\text{com}} = \frac{m_1(-a) + m_2 a}{m_1 + m_2} = a\,\frac{m_2 - m_1}{m_1 + m_2}.$$

We must resort to Newton's second law to find the acceleration of each container. The force of gravity $m_1 g$, down, and the tension force of the string $T$, up, act on the lighter container. The second law for it is $m_1 g - T = -m_1 a$. The negative sign appears because $a$ is the acceleration of the heavier container. The same forces act on the heavier container and for it the second law is $m_2 g - T = m_2 a$. The first equation gives $T = m_1 g + m_1 a$. This is substituted into the second equation to obtain $m_2 g - m_1 g - m_1 a = m_2 a$, so

$$a = (m_2 - m_1)g/(m_1 + m_2).$$

Thus,

$$a_{\text{com}} = \frac{g(m_2 - m_1)^2}{(m_1 + m_2)^2} = \frac{(9.8\text{ m/s}^2)(520\text{ g} - 480\text{ g})^2}{(480\text{ g} + 520\text{ g})^2} = 1.6 \times 10^{-2}\text{ m/s}^2.$$

The acceleration is downward.

100. (a) We use Fig. 9-21 of the text (which treats both angles as positive-valued, even though one of them is in the fourth quadrant; this is why there is an explicit minus sign in Eq. 9-80 as opposed to it being implicitly in the angle). We take the cue ball to be body 1 and the other ball to be body 2. Conservation of the $x$ and the components of the total momentum of the two-ball system leads to:

$$mv_{1i} = mv_{1f}\cos\theta_1 + mv_{2f}\cos\theta_2$$

$$0 = -mv_{1f}\sin\theta_1 + mv_{2f}\sin\theta_2.$$

The masses are the same and cancel from the equations. We solve the second equation for $\sin\theta_2$:

$$\sin\theta_2 = \frac{v_{1f}}{v_{2f}}\sin\theta_1 = \left(\frac{3.50 \text{ m/s}}{2.00 \text{ m/s}}\right)\sin 22.0° = 0.656 \ .$$

Consequently, the angle between the second ball and the initial direction of the first is $\theta_2$ = 41.0°.

(b) We solve the first momentum conservation equation for the initial speed of the cue ball.

$$v_{1i} = v_{1f}\cos\theta_1 + v_{2f}\cos\theta_2 = (3.50 \text{ m/s})\cos 22.0° + (2.00 \text{ m/s})\cos 41.0° = 4.75 \text{ m/s} \ .$$

(c) With SI units understood, the initial kinetic energy is

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}m(4.75)^2 = 11.3m$$

and the final kinetic energy is

$$K_f = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2 = \frac{1}{2}m\left((3.50)^2 + (2.00)^2\right) = 8.1m.$$

Kinetic energy is not conserved.

101. This is a completely inelastic collision, followed by projectile motion. In the collision, we use momentum conservation.

$$\vec{p}_{\text{shoes}} = \vec{p}_{\text{together}} \quad \Rightarrow \quad (3.2 \text{ kg})(3.0 \text{ m/s}) = (5.2 \text{ kg})\vec{v}$$

Therefore, $\vec{v} = 1.8 \text{ m/s}$ toward the right as the combined system is projected from the edge of the table. Next, we can use the projectile motion material from Ch. 4 or the energy techniques of Ch. 8; we choose the latter.

$$K_{\text{edge}} + U_{\text{edge}} = K_{\text{floor}} + U_{\text{floor}}$$

$$\frac{1}{2}(5.2 \text{ kg})(1.8 \text{ m/s})^2 + (5.2 \text{ kg})(9.8 \text{ m/s}^2)(0.40 \text{ m}) = K_{\text{floor}} + 0$$

Therefore, the kinetic energy of the system right before hitting the floor is $K_{\text{floor}}$ = 29 J.

102. (a) Since the center of mass of the man-balloon system does not move, the balloon will move downward with a certain speed $u$ relative to the ground as the man climbs up the ladder.

(b) The speed of the man relative to the ground is $v_g = v - u$. Thus, the speed of the center of mass of the system is

$$v_{\text{com}} = \frac{mv_g - Mu}{M+m} = \frac{m(v-u) - Mu}{M+m} = 0.$$

This yields

$$u = \frac{mv}{M+m} = \frac{(80 \text{ kg})(2.5 \text{ m/s})}{320 \text{ kg} + 80 \text{ kg}} = 0.50 \text{ m/s}.$$

(c) Now that there is no relative motion within the system, the speed of both the balloon and the man is equal to $v_{\text{com}}$, which is zero. So the balloon will again be stationary.

103. The velocities of $m_1$ and $m_2$ just after the collision with each other are given by Eq. 9-75 and Eq. 9-76 (setting $v_{1i} = 0$):

$$v_{1f} = \frac{2m_2}{m_1+m_2} v_{2i}, \quad v_{2f} = \frac{m_2 - m_1}{m_1+m_2} v_{2i}$$

After bouncing off the wall, the velocity of $m_2$ becomes $-v_{2f}$. In these terms, the problem requires $v_{1f} = -v_{2f}$, or

$$\frac{2m_2}{m_1+m_2} v_{2i} = -\frac{m_2 - m_1}{m_1+m_2} v_{2i}$$

which simplifies to

$$2m_2 = -(m_2 - m_1) \Rightarrow m_2 = \frac{m_1}{3} \ .$$

With $m_1$ = 6.6 kg, we have $m_2$ = 2.2 kg.

104. We treat the car (of mass $m_1$) as a "point-mass" (which is initially 1.5 m from the right end of the boat). The left end of the boat (of mass $m_2$) is initially at $x$ = 0 (where the dock is), and its left end is at $x$ = 14 m. The boat's center of mass (in the absence of the car) is initially at $x$ = 7.0 m. We use Eq. 9-5 to calculate the center of mass of the system:

$$x_{\text{com}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{(1500 \text{ kg})(14 \text{ m} - 1.5 \text{ m}) + (4000 \text{ kg})(7 \text{ m})}{1500 \text{ kg} + 4000 \text{ kg}} = 8.5 \text{ m}.$$

In the absence of *external* forces, the center of mass of the system does not change. Later, when the car (about to make the jump) is near the left end of the boat (which has moved from the shore an amount $\delta x$), the value of the system center of mass is still 8.5 m. The car (at this moment) is thought of as a "point-mass" 1.5 m from the left end, so we must have

$$x_{\text{com}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{(1500 \text{ kg})(\delta x + 1.5 \text{ m}) + (4000 \text{ kg})(7 \text{ m} + \delta x)}{1500 \text{ kg} + 4000 \text{ kg}} = 8.5 \text{ m}.$$

Solving this for $\delta x$, we find $\delta x$ = 3.0 m.

105. **THINK** Both momentum and energy are conserved during an elastic collision.

**EXPRESS** Let $m_1$ be the mass of the object that is originally moving, $v_{1i}$ be its velocity before the collision, and $v_{1f}$ be its velocity after the collision. Let $m_2 = M$ be the mass of the object that is originally at rest and $v_{2f}$ be its velocity after the collision. Conservation of linear momentum gives $m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$. Similarly, the total kinetic energy is conserved and we have

$$\frac{1}{2}m_1 v_{1i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2 .$$

Solving for $v_{1f}$ and $v_{2f}$, we obtain:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}, \quad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

The second equation can be inverted to give $m_2 = m_1\left(\dfrac{2v_{1i}}{v_{2f}} - 1\right)$.

**ANALYZE** With $m_1$ = 3.0 kg, $v_{1i}$ = 8.0 m/s and $v_{2f}$ = 6.0 m/s, the above expression leads to

$$m_2 = M = m_1\left(\frac{2v_{1i}}{v_{2f}} - 1\right) = (3.0\text{ kg})\left(\frac{2(8.0\text{ m/s})}{6.0\text{ m/s}} - 1\right) = 5.0\text{ kg}$$

**LEARN** Our analytic expression for $m_2$ shows that if the two masses are equal, then $v_{2f} = v_{1i}$, and the pool player's result is recovered.

106. We denote the mass of the car as $M$ and that of the sumo wrestler as $m$. Let the initial velocity of the sumo wrestler be $v_0 > 0$ and the final velocity of the car be $v$. We apply the momentum conservation law.

(a) From $mv_0 = (M + m)v$ we get

$$v = \frac{mv_0}{M+m} = \frac{(242\text{ kg})(5.3\text{ m/s})}{2140\text{ kg} + 242\text{ kg}} = 0.54\text{ m/s}.$$

(b) Since $v_{\text{rel}} = v_0$, we have

$$mv_0 = Mv + m(v + v_{\text{rel}}) = mv_0 + (M+m)v,$$

and obtain $v = 0$ for the final speed of the flatcar.

(c) Now $mv_0 = Mv + m(v - v_{\text{rel}})$, which leads to

$$v = \frac{m(v_0 + v_{\text{rel}})}{m+M} = \frac{(242\text{ kg})(5.3\text{ m/s} + 5.3\text{ m/s})}{242\text{ kg} + 2140\text{ kg}} = 1.1\text{ m/s}.$$

107. **THINK** To successfully launch a rocket from the ground, fuel is consumed at a rate that results in a thrust big enough to overcome the gravitational force.

**EXPRESS** The thrust of the rocket is given by $T = Rv_{\text{rel}}$ where $R$ is the rate of fuel consumption and $v_{\text{rel}}$ is the speed of the exhaust gas relative to the rocket.

**ANALYZE** (a) The exhaust speed is $v_{\text{rel}}$ = 1200 m/s. For the thrust to equal the weight $Mg$ where $M$ = 6100 kg, we must have

$$T = Rv_{\text{rel}} = Mg \quad \Rightarrow \quad R = \frac{Mg}{v_{\text{rel}}} = \frac{(6100\,\text{kg})(9.8\,\text{m/s}^2)}{1200\,\text{m/s}} = 49.8\,\text{kg/s} \approx 50\,\text{kg/s}\,.$$

(b) Using Eq. 9-42 with the additional effect due to gravity, we have

$$Rv_{\text{rel}} - Mg = Ma$$

so that requiring $a$ = 21 m/s$^2$ leads to

$$R = \frac{M(g+a)}{v_{\text{rel}}} = \frac{(6100\,\text{kg})(9.8\,\text{m/s}^2 + 21\,\text{m/s}^2)}{1200\,\text{m/s}} = 156.6\,\text{kg/s} \approx 1.6\times 10^2\ \text{kg/s}\,.$$

**LEARN** A greater upward acceleration requires a greater fuel consumption rate. To be launched from Earth's surface, the initial acceleration of the rocket must exceed $g = 9.8\,\text{m/s}^2$. This means that the rate $R$ must be greater than 50 kg/s.

108. Conservation of momentum leads to

$$(900\ \text{kg})(1000\ \text{m/s}) = (500\ \text{kg})(v_{\text{shuttle}} - 100\ \text{m/s}) + (400\ \text{kg})(v_{\text{shuttle}})$$

which yields $v_{\text{shuttle}}$ = 1055.6 m/s for the shuttle speed and $v_{\text{shuttle}}$ – 100 m/s = 955.6 m/s for the module speed (all measured in the frame of reference of the stationary main spaceship). The fractional increase in the kinetic energy is

$$\frac{\Delta K}{K_i} = \frac{K_f}{K_i} - 1 = \frac{(500\ \text{kg})(955.6\ \text{m/s})^2/2 + (400\ \text{kg})(1055.6\ \text{m/s})^2/2}{(900\ \text{kg})(1000\ \text{m/s})^2/2} = 2.5\times 10^{-3}.$$

109. **THINK** In this problem, we are asked to locate the center of mass of the Earth-Moon system.

**EXPRESS** We locate the coordinate origin at the center of Earth. Then the distance $r_{\text{com}}$ of the center of mass of the Earth-Moon system is given by

$$r_{\text{com}} = \frac{m_M r_{ME}}{m_M + m_E}$$

where $m_M$ is the mass of the Moon, $m_E$ is the mass of Earth, and $r_{ME}$ is their separation.

**ANALYZE** (a) With $m_E = 5.98\times10^{24}$ kg , $m_M = 7.36\times10^{22}$ kg and $r_{ME} = 3.82\times10^8$ m (these values are given in Appendix C), we find the center of mass to be at

$$r_{\rm com} = \frac{\left(7.36\times10^{22}\ \text{kg}\right)\left(3.82\times10^{8}\ \text{m}\right)}{7.36\times10^{22}\ \text{kg}+5.98\times10^{24}\ \text{kg}} = 4.64\times10^{6}\ \text{m} \approx 4.6\times10^{3}\ \text{km}.$$

(b) The radius of Earth is $R_E = 6.37 \times 10^6$ m, so $r_{\rm com}/R_E = 0.73 = 73\%$ .

**LEARN** The center of mass of the Earth-Moon system is located inside the Earth!

110. (a) The magnitude of the impulse is equal to the change in momentum:

$$J = mv - m(-v) = 2mv = 2(0.140\ \text{kg})(7.80\ \text{m/s}) = 2.18\ \text{kg}\cdot\text{m/s}$$

(b) Since in the calculus sense the average of a function is the integral of it divided by the corresponding interval, then the average force is the impulse divided by the time $\Delta t$. Thus, our result for the magnitude of the average force is $2mv/\Delta t$. With the given values, we obtain

$$F_{\rm avg} = \frac{2(0.140\ \text{kg})(7.80\ \text{m/s})}{0.00380\ \text{s}} = 575\ \text{N}\ .$$

111. **THINK** The water added to the sled will move at the same speed as the sled.

**EXPRESS** Let the mass of the sled be $m_s$ and its initial speed be $v_i$ . If the total mass of water being scooped up is $m_w$ , then by momentum conservation, $m_s v_i = (m_s + m_w) v_f$ , where $v_f$ is the final speed of the sled-water system.

**ANALYZE** With $m_s = 2900\ \text{kg}$ , $m_w = 920\ \text{kg}$ and $v_i = 250\ \text{m/s}$ , we obtain

$$v_f = \frac{m_s v_i}{m_s + m_w} = \frac{\left(2900\ \text{kg}\right)\left(250\ \text{m/s}\right)}{2900\ \text{kg}+920\ \text{kg}} = 189.8\ \text{m/s} \approx 190\ \text{m/s}\ .$$

**LEARN** The water added to the sled can be regarded as undergoing completely inelastic collision with the sled. Some kinetic energy is converted into other forms of energy (thermal, sound, etc.) and the final speed of the sled-water system is smaller than the initial speed of the sled alone.

112. **THINK** The pellets that were fired carry both kinetic energy and momentum. Force is exerted by the rigid wall in stopping the pellets.

**EXPRESS** Let $m$ be the mass of a pellet and $v$ be its velocity as it hits the wall, then its momentum is $p = mv$, toward the wall. The kinetic energy of a pellet is $K = mv^2/2$. The force on the wall is given by the rate at which momentum is transferred from the pellets to the wall. Since the pellets do not rebound, each pellet that hits transfers $p$. If $\Delta N$ pellets hit in time $\Delta t$, then the average rate at which momentum is transferred would be $F_{\text{avg}} = p(\Delta N/\Delta t)$.

**ANALYZE** (a) With $m = 2.0 \times 10^{-3}$ kg and $v = 500$ m/s, the momentum of a pellet is

$$p = mv = (2.0 \times 10^{-3}\ \text{kg})(500\ \text{m/s}) = 1.0\ \text{kg} \cdot \text{m/s}.$$

(b) The kinetic energy of a pellet is $K = \frac{1}{2}mv^2 = \frac{1}{2}(2.0\times10^{-3}\,\text{kg})(500\,\text{m/s})^2 = 2.5\times10^2\,\text{J}$ .

(c) With $(\Delta N/\Delta t) = 10/\text{s}$, the average force on the wall from the stream of pellets is

$$F_{\text{avg}} = p\left(\frac{\Delta N}{\Delta t}\right) = (1.0\,\text{kg}\cdot\text{m/s})(10\,\text{s}^{-1}) = 10\,\text{N}.$$

The force on the wall is in the direction of the initial velocity of the pellets.

(d) If $\Delta t'$ is the time interval for a pellet to be brought to rest by the wall, then the average force exerted on the wall by a pellet is

$$F'_{\text{avg}} = \frac{p}{\Delta t'} = \frac{1.0\,\text{kg}\cdot\text{m/s}}{0.6\times10^{-3}\,\text{s}} = 1.7\times10^3\,\text{N}.$$

The force is in the direction of the initial velocity of the pellet.

(e) In part (d) the force is averaged over the time a pellet is in contact with the wall, while in part (c) it is averaged over the time for many pellets to hit the wall. Hence, $F'_{\text{avg}} \neq F_{\text{avg}}$ .

**LEARN** During the majority of this time, no pellet is in contact with the wall, so the average force in part (c) is much less than the average force in part (d).

113. We convert mass rate to SI units: $R$ = (540 kg/min)/(60 s/min) = 9.00 kg/s. In the absence of the asked-for additional force, the car would decelerate with a magnitude given by Eq. 9-87: $Rv_{\text{rel}} = M|a|$, so that if $a = 0$ is desired then the additional force must have a magnitude equal to $R\,v_{\text{rel}}$ (so as to cancel that effect):

$$F = Rv_{\text{rel}} = (9.00\ \text{kg/s})(3.20\ \text{m/s}) = 28.8\,\text{N}.$$

114. First, we imagine that the small square piece (of mass $m$) that was cut from the large plate is returned to it so that the large plate is again a complete 6 m × 6 m ($d$ =1.0 m) square plate (which has its center of mass at the origin). Then we “add” a square piece of

"negative mass" ($-m$) at the appropriate location to obtain what is shown in the figure. If the mass of the whole plate is $M$, then the mass of the small square piece cut from it is obtained from a simple ratio of areas:

$$m = \left(\frac{2.0\,\text{m}}{6.0\,\text{m}}\right)^2 M \Rightarrow M = 9m.$$

(a) The $x$ coordinate of the small square piece is $x$ = 2.0 m (the middle of that square "gap" in the figure). Thus the $x$ coordinate of the center of mass of the remaining piece is

$$x_{\text{com}} = \frac{(-m)x}{M+(-m)} = \frac{-m(2.0\ \text{m})}{9m - m} = -0.25\ \text{m}.$$

(b) Since the $y$ coordinate of the small square piece is zero, we have $y_{\text{com}} = 0$.

115. **THINK** We have two forces acting on two masses separately. The masses will move according to Newton's second law.

**EXPRESS** Let $\vec{F}_1$ be the force acting on $m_1$, and $\vec{F}_2$ the force acting on $m_2$. According to Newton's second law, their displacements are

$$\vec{d}_1 = \frac{1}{2}\vec{a}_1 t^2 = \frac{1}{2}\left(\frac{\vec{F}_1}{m_1}\right)t^2, \quad \vec{d}_2 = \frac{1}{2}\vec{a}_2 t^2 = \frac{1}{2}\left(\frac{\vec{F}_2}{m_2}\right)t^2$$

The corresponding displacement of the center of mass is

$$\vec{d}_{\text{cm}} = \frac{m_1\vec{d}_1 + m_2\vec{d}_2}{m_1+m_2} = \frac{1}{2}\frac{m_1}{m_1+m_2}\left(\frac{\vec{F}_1}{m_1}\right)t^2 + \frac{1}{2}\frac{m_2}{m_1+m_2}\left(\frac{\vec{F}_2}{m_2}\right)t^2 = \frac{1}{2}\left(\frac{\vec{F}_1+\vec{F}_2}{m_1+m_2}\right)t^2.$$

**ANALYZE** (a) The two masses are $m_1 = 2.00\times10^{-3}$ kg and $m_2 = 4.00\times10^{-3}$ kg. With the forces given by $\vec{F}_1 = (-4.00\ \text{N})\hat{\text{i}} + (5.00\ \text{N})\hat{\text{j}}$ and $\vec{F}_2 = (2.00\ \text{N})\hat{\text{i}} - (4.00\ \text{N})\hat{\text{j}}$, and $t = 2.00\times10^{-3}$ s, we obtain

$$\begin{aligned}\vec{d}_{\text{cm}} &= \frac{1}{2}\left(\frac{\vec{F}_1+\vec{F}_2}{m_1+m_2}\right)t^2 = \frac{1}{2}\frac{(-4.00\ \text{N}+2.00\ \text{N})\hat{\text{i}} + (5.00\ \text{N}-4.00\ \text{N})\hat{\text{j}}}{2.00\times10^{-3}\ \text{kg} + 4.00\times10^{-3}\ \text{kg}}(2.00\times10^{-3}\ \text{s})^2 \\ &= (-6.67\times10^{-4}\ \text{m})\hat{\text{i}} + (3.33\times10^{-4}\ \text{m})\hat{\text{j}}.\end{aligned}$$

The magnitude of $\vec{d}_{\text{cm}}$ is

$$d_{\text{cm}} = \sqrt{(-6.67\times10^{-4}\ \text{m})^2 + (3.33\times10^{-4}\ \text{m})^2} = 7.45\times10^{-4}\ \text{m}$$

or 0.745 mm.

(b) The angle of $\vec{d}_{\text{cm}}$ is given by

$$\theta = \tan^{-1}\left(\frac{3.33\times10^{-4}\ \text{m}}{-6.67\times10^{-4}\ \text{m}}\right) = \tan^{-1}\left(-\frac{1}{2}\right) = 153°,$$

measured counterclockwise from $+x$-axis.

(c) The velocities of the two masses are

$$\vec{v}_1 = \vec{a}_1 t = \frac{\vec{F}_1 t}{m_1}, \quad \vec{v}_2 = \vec{a}_2 t = \frac{\vec{F}_2 t}{m_2},$$

and the velocity of the center of mass is

$$\vec{v}_{\text{cm}} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2} = \frac{m_1}{m_1+m_2}\left(\frac{\vec{F}_1 t}{m_1}\right) + \frac{m_2}{m_1+m_2}\left(\frac{\vec{F}_2 t}{m_2}\right) = \left(\frac{\vec{F}_1+\vec{F}_2}{m_1+m_2}\right)t .$$

The corresponding kinetic energy of the center of mass is

$$K_{\text{cm}} = \frac{1}{2}(m_1+m_2)v_{\text{cm}}^2 = \frac{1}{2}\frac{|\vec{F}_1+\vec{F}_2|^2}{m_1+m_2}t^2$$

With $|\vec{F}_1+\vec{F}_2| = |(-2.00\ \text{N})\hat{\text{i}} + (1.00\ \text{N})\hat{\text{j}}| = \sqrt{5}\ \text{N}$, we get

$$K_{\text{cm}} = \frac{1}{2}\frac{|\vec{F}_1+\vec{F}_2|^2}{m_1+m_2}t^2 = \frac{1}{2}\frac{(\sqrt{5}\ \text{N})^2}{2.00\times10^{-3}\ \text{kg}+4.00\times10^{-3}\ \text{kg}}(2.00\times10^{-3}\ \text{s})^2 = 1.67\times10^{-3}\ \text{J}.$$

**LEARN** The motion of the center of the mass could be analyzed as though a force $\vec{F} = \vec{F}_1 + \vec{F}_2$ is acting on a mass $M = m_1 + m_2$. Thus, the acceleration of the center of the mass is $\vec{a}_{\text{cm}} = \dfrac{\vec{F}_1+\vec{F}_2}{m_1+m_2}$.

116. (a) The center of mass does not move in the absence of external forces (since it was initially at rest).

(b) They collide at their center of mass. If the initial coordinate of $P$ is $x = 0$ and the initial coordinate of $Q$ is $x = 1.0$ m, then Eq. 9-5 gives

$$x_{\text{com}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{0 + (0.30 \text{ kg})(1.0 \text{ m})}{0.1 \text{ kg} + 0.3 \text{ kg}} = 0.75 \text{ m}.$$

Thus, they collide at a point 0.75 m from *P*'s original position.

117. This is a completely inelastic collision, but Eq. 9-53 ($V = \frac{m_1}{m_1 + m_2} v_{1i}$) is not easily applied since that equation is designed for use when the struck particle is initially stationary. To deal with this case (where particle 2 is already in motion), we return to the principle of momentum conservation:

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{V} \quad \Rightarrow \quad \vec{V} = \frac{2(4\hat{\text{i}} - 5\hat{\text{j}}) + 4(6\hat{\text{i}} - 2\hat{\text{j}})}{2+4}.$$

(a) In unit-vector notation, then, $\vec{V} = (2.67 \text{ m/s})\hat{\text{i}} + (-3.00 \text{ m/s})\hat{\text{j}}$ .

(b) The magnitude of $\vec{V}$ is $|\vec{V}| = 4.01$ m/s.

(c) The direction of $\vec{V}$ is 48.4° (measured *clockwise* from the $+x$ axis).

118. We refer to the discussion in the textbook (Sample Problem – "Elastic collision, two pendulums," which uses the same notation that we use here) for some important details in the reasoning. We choose rightward in Fig. 9-20 as our $+x$ direction. We use the notation $\vec{v}$ when we refer to velocities and $v$ when we refer to speeds (which are necessarily positive). Since the algebra is fairly involved, we find it convenient to introduce the notation $\Delta m = m_2 - m_1$ (which, we note for later reference, is a positive-valued quantity).

(a) Since $\vec{v}_{1i} = +\sqrt{2gh_1}$ where $h_1 = 9.0$ cm, we have

$$\vec{v}_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = -\frac{\Delta m}{m_1 + m_2}\sqrt{2gh_1}$$

which is to say that the *speed* of sphere 1 immediately after the collision is

$$v_{1f} = \left(\Delta m / (m_1 + m_2)\right)\sqrt{2gh_1}$$

and that $\vec{v}_{1f}$ points in the $-x$ direction. This leads (by energy conservation $m_1gh_{1f} = \frac{1}{2}m_1v_{1f}^2$) to

$$h_{1f} = \frac{v_{1f}^2}{2g} = \left(\frac{\Delta m}{m_1 + m_2}\right)^2 h_1 .$$

With $m_1 = 50$ g and $m_2 = 85$ g, this becomes $h_{1f} \approx 0.60$ cm.

(b) Equation 9-68 gives

$$v_{2f} = \frac{2m_1}{m_1+m_2} v_{1i} = \frac{2m_1}{m_1+m_2}\sqrt{2gh_1}$$

which leads (by energy conservation $m_2 g h_{2f} = \frac{1}{2} m_2 v_{2f}^2$) to

$$h_{2f} = \frac{v_{2f}^2}{2g} = \left(\frac{2m_1}{m_1+m_2}\right)^2 h_1 \ .$$

With $m_1$ = 50 g and $m_2$ = 85 g, this becomes $h_{2f} \approx 4.9 \text{ cm}$.

(c) Fortunately, they hit again at the lowest point (as long as their amplitude of swing was "small," this is further discussed in Chapter 16). At the risk of using cumbersome notation, we refer to the *next* set of heights as $h_{1ff}$ and $h_{2ff}$. At the lowest point (before this second collision) sphere 1 has velocity $+\sqrt{2gh_{1f}}$ (rightward in Fig. 9-20) and sphere 2 has velocity $-\sqrt{2gh_{1f}}$ (that is, it points in the $-x$ direction). Thus, the velocity of sphere 1 immediately after the second collision is, using Eq. 9-75,

$$\begin{aligned}
\vec{v}_{1ff} &= \frac{m_1-m_2}{m_1+m_2}\sqrt{2gh_{1f}} + \frac{2m_2}{m_1+m_2}\left(-\sqrt{2gh_{2f}}\right) \\
&= \frac{-\Delta m}{m_1+m_2}\left(\frac{\Delta m}{m_1+m_2}\sqrt{2gh_1}\right) - \frac{2m_2}{m_1+m_2}\left(\frac{2m_1}{m_1+m_2}\sqrt{2gh_1}\right) \\
&= -\frac{(\Delta m)^2 + 4m_1m_2}{(m_1+m_2)^2}\sqrt{2gh_1} \ .
\end{aligned}$$

This can be greatly simplified (by expanding $(\Delta m)^2$ and $(m_1 + m_2)^2$) to arrive at the conclusion that the speed of sphere 1 immediately after the second collision is simply $v_{1ff} = \sqrt{2gh_1}$ and that $\vec{v}_{1ff}$ points in the $-x$ direction. Energy conservation $\left(m_1 g h_{1ff} = \frac{1}{2} m_1 v_{1ff}^2\right)$ leads to

$$h_{1ff} = \frac{v_{1ff}^2}{2g} = h_1 = 9.0 \text{ cm} \, .$$

(d) One can reason (energy-wise) that $h_{1ff}$ = 0 simply based on what we found in part (c). Still, it might be useful to see how this shakes out of the algebra. Equation 9-76 gives the velocity of sphere 2 immediately after the second collision:

$$v_{2ff} = \frac{2m_1}{m_1+m_2}\sqrt{2gh_{1f}} + \frac{m_2-m_1}{m_1+m_2}\left(-\sqrt{2gh_{2f}}\right)$$
$$= \frac{2m_1}{m_1+m_2}\left(\frac{\Delta m}{m_1+m_2}\sqrt{2gh_1}\right) + \frac{\Delta m}{m_1+m_2}\left(\frac{-2m_1}{m_1+m_2}\sqrt{2gh_1}\right)$$

which vanishes since $(2m_1)(\Delta m)-(\Delta m)(2m_1)=0$. Thus, the second sphere (after the second collision) stays at the lowest point, which basically recreates the conditions at the start of the problem (so all subsequent swings-and-impacts, neglecting friction, can be easily predicted, as they are just replays of the first two collisions).

119. (a) Each block is assumed to have uniform density, so that the center of mass of each block is at its geometric center (the positions of which are given in the table [see problem statement] at $t = 0$). Plugging these positions (and the block masses) into Eq. 9-29 readily gives $x_{com} = -0.50$ m (at $t = 0$).

(b) Note that the left edge of block 2 (the middle of which is still at $x = 0$) is at $x = -2.5$ cm, so that at the moment they touch the right edge of block 1 is at $x = -2.5$ cm and thus the middle of block 1 is at $x = -5.5$ cm. Putting these positions (for the middles) and the block masses into Eq. 9-29 leads to $x_{com} = -1.83$ cm or $-0.018$ m (at $t = (1.445\text{ m})/(0.75\text{ m/s}) = 1.93$ s).

(c) We could figure where the blocks are at $t = 4.0$ s and use Eq. 9-29 again, but it is easier (and provides more insight) to note that in the absence of *external* forces on the system the center of mass should move at constant velocity:

$$\vec{v}_{com} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2} = 0.25\text{ m/s }\hat{\text{i}}$$

as can be easily verified by putting in the values at $t = 0$. Thus,

$$x_{com} = x_{com\ initial} + \vec{v}_{com}\,t = (-0.50\text{ m}) + (0.25\text{ m/s})(4.0\text{ s}) = +0.50\text{ m}.$$

120. One approach is to choose a *moving* coordinate system that travels the center of mass of the body, and another is to do a little extra algebra analyzing it in the original coordinate system (in which the speed of the $m = 8.0$ kg mass is $v_0 = 2$ m/s, as given). Our solution is in terms of the latter approach since we are assuming that this is the approach most students would take. Conservation of linear momentum (along the direction of motion) requires

$$mv_0 = m_1v_1 + m_2v_2 \quad\Rightarrow\quad (8.0)(2.0) = (4.0)v_1 + (4.0)v_2$$

which leads to $v_2 = 4 - v_1$ in SI units (m/s). We require

$$\Delta K = \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right) - \frac{1}{2}mv_0^2 \quad\Rightarrow\quad 16 = \left(\frac{1}{2}(4.0)v_1^2 + \frac{1}{2}(4.0)v_2^2\right) - \frac{1}{2}(8.0)(2.0)^2$$

which simplifies to $v_2^2 = 16 - v_1^2$ in SI units. If we substitute for $v_2$ from above, we find

$$(4 - v_1)^2 = 16 - v_1^2$$

which simplifies to $2v_1^2 - 8v_1 = 0$, and yields either $v_1 = 0$ or $v_1 = 4$ m/s. If $v_1 = 0$ then $v_2 = 4 - v_1 = 4$ m/s, and if $v_1 = 4$ m/s then $v_2 = 0$.

(a) Since the forward part continues to move in the original direction of motion, the speed of the rear part must be zero.

(b) The forward part has a velocity of 4.0 m/s along the original direction of motion.

121. We use $m_1$ for the mass of the electron and $m_2 = 1840m_1$ for the mass of the hydrogen atom. Using Eq. 9-68,

$$v_{2f} = \frac{2m_1}{m_1 + 1840m_1}v_{1i} = \frac{2}{1841}v_{1i}$$

we compute the final kinetic energy of the hydrogen atom:

$$K_{2f} = \frac{1}{2}(1840m_1)\left(\frac{2v_{1i}}{1841}\right)^2 = \frac{(1840)(4)}{1841^2}\left(\frac{1}{2}(1840m_1)v_{1i}^2\right)$$

so we find the fraction to be $(1840)(4)/1841^2 \approx 2.2 \times 10^{-3}$, or 0.22%.

122. Denoting the new speed of the car as $v$, then the new speed of the man relative to the ground is $v - v_{\text{rel}}$. Conservation of momentum requires

$$\left(\frac{W}{g} + \frac{w}{g}\right)v_0 = \left(\frac{W}{g}\right)v + \left(\frac{w}{g}\right)(v - v_{\text{rel}}).$$

Consequently, the change of velocity is

$$\Delta\vec{v} = v - v_0 = \frac{w\,v_{\text{rel}}}{W + w} = \frac{(915\text{ N})(4.00\text{ m/s})}{(2415\text{ N}) + (915\text{ N})} = 1.10\text{ m/s}.$$

123. Conservation of linear momentum gives $mv + MV_J = mv_f + MV_{Jf}$. Similarly, the total kinetic energy is conserved:

$$\frac{1}{2}mv^2+\frac{1}{2}MV_J^2=\frac{1}{2}mv_f^2+\frac{1}{2}MV_{J\,f}^2 .$$

Solving for $v_f$ and $V_{J\,f}$, we obtain:

$$v_{1f}=\frac{m-M}{m+M}v+\frac{2M}{m+M}V_J,\quad V_{J\,f}=\frac{2m}{m+M}v+\frac{M-m}{m+M}V_J$$

Since $m \ll M,$ the above expressions can be simplified to

$$v_{1f}\approx -v+2V_J,\quad V_{J\,f}\approx V_J$$

The velocity of the probe relative to the Sun is

$$v_{1f}\approx -v+2V_J=-(10.5\text{ km/s})+2(-13.0\text{ km/s})=-36.5\text{ km/s} .$$

The speed is $|v_{1f}|=36.5\text{ km/s}.$

124. (a) The change in momentum (taking upwards to be the positive direction) is

$$\Delta\vec{p} = (0.550\text{ kg})[(3\text{ m/s})\hat{\text{j}} - (-12\text{ m/s})\hat{\text{j}}] = (+8.25\text{ kg}\cdot\text{m/s})\,\hat{\text{j}} .$$

(b) By the impulse-momentum theorem (Eq. 9-31) $\vec{J} = \Delta\vec{p} = (+8.25\text{ N}\cdot\text{s})\,\hat{\text{j}}$ .

(c) By Newton's third law, $\vec{J}_\text{c} = -\vec{J}_\text{b} = (-8.25\text{ N}\cdot\text{s})\,\hat{\text{j}}$ .

125. (a) Since the initial momentum is zero, then the final momenta must add (in the vector sense) to 0. Therefore, with SI units understood, we have

$$\begin{aligned}\vec{p}_3&=-\vec{p}_1-\vec{p}_2=-m_1\vec{v}_1-m_2\vec{v}_2\\&=-\left(16.7\times10^{-27}\right)\left(6.00\times10^{6}\,\hat{\text{i}}\right)-\left(8.35\times10^{-27}\right)\left(-8.00\times10^{6}\,\hat{\text{j}}\right)\\&=\left(-1.00\times10^{-19}\,\hat{\text{i}}+0.67\times10^{-19}\,\hat{\text{j}}\right)\text{kg}\cdot\text{m/s}.\end{aligned}$$

(b) Dividing by $m_3$ = 11.7 × $10^{-27}$ kg and using the Pythagorean theorem we find the speed of the third particle to be $v_3$ = 1.03 × $10^7$ m/s. The total amount of kinetic energy is

$$\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2+\frac{1}{2}m_3v_3^2=1.19\times10^{-12}\text{ J}.$$

126. Using Eq. 9-67, we have after the elastic collision

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{-200\text{ g}}{600\text{ g}} v_{1i} = -\tfrac{1}{3}(3.00\text{ m/s}) = -1.00\text{ m/s}\,.$$

(a) The impulse is therefore

$$J = m_1 v_{1f} - m_1 v_{1i} = (0.200\text{ kg})(-1.00\text{ m/s}) - (0.200\text{ kg})(3.00\text{ m/s}) = -0.800\text{ N}\cdot\text{s}$$
$$= -0.800\text{ kg}\cdot\text{m/s},$$

or $|J| = -0.800$ kg·m/s.

(b) For the completely inelastic collision Eq. 9-75 applies

$$v_{1f} = V = \frac{m_1}{m_1 + m_2} v_{1i} = +1.00\text{ m/s}\,.$$

Now the impulse is

$$J = m_1 v_{1f} - m_1 v_{1i} = (0.200\text{ kg})(1.00\text{ m/s}) - (0.200\text{ kg})(3.00\text{ m/s}) = 0.400\text{ N}\cdot\text{s}$$
$$= 0.400\text{ kg}\cdot\text{m/s}.$$

127. We use Eq. 9-88 and simplify with $v_f - v_i = \Delta v$, and $v_{\text{rel}} = u$.

$$v_f - v_i = v_{\text{rel}} \ln\left(\frac{M_i}{M_f}\right) \quad \Rightarrow \quad \frac{M_f}{M_i} = e^{-\Delta v/u}$$

If $\Delta v = 2.2$ m/s and $u = 1000$ m/s, we obtain $\dfrac{M_i - M_f}{M_i} = 1 - e^{-0.0022} \approx 0.0022.$

128. Using the linear momentum-impulse theorem, we have

$$J = F_{\text{avg}}\Delta t = \Delta p = m(v_f - v_i)\,.$$

where $m$ is the mass, $v_i$ the initial velocity, and $v_f$ the final velocity of the ball. With $v_i = 0$, we obtain

$$v_f = \frac{F_{\text{avg}}\Delta t}{m} = \frac{(32\text{ N})(14\times 10^{-3}\text{s})}{0.20\,\text{kg}} = 2.24\,\text{m/s}.$$