Chapter 7

1. **THINK** As the proton is being accelerated, its speed increases, and so does its kinetic energy.

EXPRESS To calculate the speed of the proton at a later time, we use the equation $v^2 = v_0^2 + 2a\Delta x$ from Table 2-1. The change in kinetic energy is then equal to

$$\Delta K = \frac{1}{2} m(v_f^2 - v_i^2) .$$

ANALYZE (a) With $\Delta x = 3.5 \text{ cm} = 0.035 \text{ m}$ and $a = 3.6\times 10^{15} \text{ m/s}^2$, we find the proton speed to be

$$v = \sqrt{v_0^2 + 2a\Delta x} = \sqrt{\left(2.4\times 10^7 \text{ m/s}\right)^2 + 2\ \left(3.6\times 10^{15} \text{ m/s}^2\right)\left(0.035 \text{ m}\right)} = 2.9\times 10^7 \text{ m/s.}$$

(b) The initial kinetic energy is

$$K_i = \frac{1}{2} m v_0^2 = \frac{1}{2}\ \left(1.67\times 10^{-27} \text{ kg}\right)\left(2.4\times 10^7 \text{ m/s}\right)^2 = 4.8\times 10^{-13} \text{ J},$$

and the final kinetic energy is

$$K_f = \frac{1}{2} m v^2 = \frac{1}{2}\ \left(1.67\times 10^{-27} \text{ kg}\right)\left(2.9\times 10^7 \text{ m/s}\right)^2 = 6.9\times 10^{-13} \text{ J.}$$

Thus, the change in kinetic energy is

$$\Delta K = K_f - K_i = 6.9 \times 10^{-13} \text{ J} - 4.8 \times 10^{-13} \text{ J} = 2.1 \times 10^{-13} \text{ J.}$$

LEARN The change in kinetic energy can be rewritten as

$$\Delta K = \frac{1}{2} m(v_f^2 - v_i^2) = \frac{1}{2} m(2a\Delta x) = ma\Delta x = F\Delta x = W$$

which, according to the work-kinetic energy theorem, is simply the work done on the particle.

2. With speed v = 11200 m/s, we find

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(2.9\times10^5 \text{ kg})\ (11200 \text{ m/s})^2 = 1.8\times10^{13} \text{ J}.$$

3. (a) The change in kinetic energy for the meteorite would be

$$\Delta K = K_f - K_i = -K_i = -\frac{1}{2}m_i v_i^2 = -\frac{1}{2}\left(4\times10^6 \text{ kg}\right)\left(15\times10^3 \text{ m/s}\right)^2 = -5\times10^{14} \text{ J},$$

or $|\Delta K| = 5\times10^{14}$ J. The negative sign indicates that kinetic energy is lost.

(b) The energy loss in units of megatons of TNT would be

$$-\Delta K = \left(5\times10^{14} \text{ J}\right)\left(\frac{1 \text{ megaton TNT}}{4.2\times10^{15} \text{ J}}\right) = 0.1 \text{megaton TNT}.$$

(c) The number of bombs N that the meteorite impact would correspond to is found by noting that megaton = 1000 kilotons and setting up the ratio:

$$N = \frac{0.1\times1000 \text{kiloton TNT}}{13 \text{kiloton TNT}} = 8.$$

4. (a) We set up the ratio

$$\frac{50 \text{ km}}{1 \text{ km}} = \left(\frac{E}{1 \text{ megaton}}\right)^{1/3}$$

and find $E = 50^3 \approx 1 \times 10^5$ megatons of TNT.

(b) We note that 15 kilotons is equivalent to 0.015 megatons. Dividing the result from part (a) by 0.013 yields about ten million (10^7) bombs.

5. We denote the mass of the father as m and his initial speed v_i. The initial kinetic energy of the father is

$$K_i = \frac{1}{2}K_{\text{son}}$$

and his final kinetic energy (when his speed is $v_f = v_i$ + 1.0 m/s) is $K_f = K_{\text{son}}$. We use these relations along with Eq. 7-1 in our solution.

(a) We see from the above that $K_i = \frac{1}{2}K_f$, which (with SI units understood) leads to

$$\frac{1}{2}mv_i^2 = \frac{1}{2}\left[\frac{1}{2}m\ \left(v_i + 1.0\text{ m/s}\right)^2\right].$$

The mass cancels and we find a second-degree equation for v_i : $\frac{1}{2}v_i^2 - v_i - \frac{1}{2} = 0$. The positive root (from the quadratic formula) yields v_i = 2.4 m/s.

(b) From the first relation above $\left(K_i = \frac{1}{2}K_{\text{son}}\right)$, we have

$$\frac{1}{2}mv_i^2 = \frac{1}{2}\left(\frac{1}{2}\ (m/2)\ v_{\text{son}}^2\right)$$

and (after canceling m and one factor of 1/2) are led to $v_{\text{son}} = 2v_i = 4.8\ \text{m/s}$.

6. We apply the equation $x(t) = x_0 + v_0 t + \frac{1}{2}at^2$, found in Table 2-1. Since at t = 0 s, x_0 = 0, and $v_0 = 12\ \text{m/s}$, the equation becomes (in unit of meters)

$$x(t) = 12t + \tfrac{1}{2}at^2.$$

With $x = 10\ \text{m}$ when $t = 1.0\ \text{s}$, the acceleration is found to be $a = -4.0\ \text{m/s}^2$. The fact that $a < 0$ implies that the bead is decelerating. Thus, the position is described by $x(t) = 12t - 2.0t^2$. Differentiating x with respect to t then yields

$$v(t) = \frac{dx}{dt} = 12 - 4.0t.$$

Indeed at t =3.0 s, $v(t = 3.0) = 0$ and the bead stops momentarily. The speed at $t = 10\ \text{s}$ is $v(t = 10) = -28\ \text{m/s}$, and the corresponding kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1.8\times 10^{-2}\text{kg})(-28\ \text{m/s})^2 = 7.1\ \text{J}.$$

7. Since this involves constant-acceleration motion, we can apply the equations of Table 2-1, such as $x = v_0 t + \frac{1}{2}at^2$ (where $x_0 = 0$). We choose to analyze the third and fifth points, obtaining

$$0.2\,\text{m} = v_0(1.0\text{ s}) + \frac{1}{2}a\ (1.0\text{ s})^2$$

$$0.8\,\text{m} = v_0(2.0\text{ s}) + \frac{1}{2}a\ (2.0\text{ s})^2.$$

Simultaneous solution of the equations leads to $v_0 = 0$ and $a = 0.40\,\text{m/s}^2$. We now have two ways to finish the problem. One is to compute force from $F = ma$ and then obtain the work from Eq. 7-7. The other is to find ΔK as a way of computing W (in accordance with Eq. 7-10). In this latter approach, we find the velocity at $t = 2.0\,\text{s}$ from $v = v_0 + at\,(\text{so } v = 0.80\,\text{m/s})$. Thus,

$$W = \Delta K = \frac{1}{2}(3.0\,\text{kg})(0.80\,\text{m/s})^2 = 0.96\ \text{J}.$$

8. Using Eq. 7-8 (and Eq. 3-23), we find the work done by the water on the ice block:

$$W = \vec{F}\cdot\vec{d} = \left[(210\ \text{N})\,\hat{\text{i}} - (150\ \text{N})\,\hat{\text{j}}\right]\cdot\left[(15\ \text{m})\,\hat{\text{i}} - (12\ \text{m})\,\hat{\text{j}}\right] = (210\ \text{N})(15\ \text{m}) + (-150\ \text{N})(-12\ \text{m})$$
$$= 5.0\times10^3\ \text{J}.$$

9. By the work-kinetic energy theorem,

$$W = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}(2.0\,\text{kg})\left((6.0\,\text{m/s})^2 - (4.0\,\text{m/s})^2\right) = 20\ \text{J}.$$

We note that the *directions* of $\vec{v}_f$ and $\vec{v}_i$ play no role in the calculation.

10. Equation 7-8 readily yields

$$W = F_x\,\Delta x + F_y\,\Delta y = (2.0\ \text{N})\cos(100^\circ)(3.0\ \text{m}) + (2.0\ \text{N})\sin(100^\circ)(4.0\ \text{m}) = 6.8\ \text{J}.$$

11. Using the work-kinetic energy theorem, we have

$$\Delta K = W = \vec{F}\cdot\vec{d} = Fd\cos\phi.$$

In addition, $F = 12\ \text{N}$ and $d = \sqrt{(2.00\ \text{m})^2 + (-4.00\ \text{m})^2 + (3.00\ \text{m})^2} = 5.39\ \text{m}$.

(a) If $\Delta K = +30.0\ \text{J}$, then

$$\phi = \cos^{-1}\left(\frac{\Delta K}{Fd}\right) = \cos^{-1}\left(\frac{30.0\ \text{J}}{(12.0\ \text{N})(5.39\ \text{m})}\right) = 62.3^\circ.$$

(b) $\Delta K = -30.0\ \text{J}$, then

$$\phi = \cos^{-1}\left(\frac{\Delta K}{Fd}\right) = \cos^{-1}\left(\frac{-30.0\ \text{J}}{(12.0\ \text{N})(5.39\ \text{m})}\right) = 118^\circ.$$

12. (a) From Eq. 7-6, $F = W/x = 3.00$ N (this is the slope of the graph).

(b) Equation 7-10 yields $K = K_i + W = 3.00$ J + 6.00 J = 9.00 J.

13. We choose +x as the direction of motion (so $\vec{a}$ and $\vec{F}$ are negative-valued).

(a) Newton's second law readily yields $\vec{F} = (85\,\text{kg})(-2.0\,\text{m/s}^2)$ so that

$$F = |\vec{F}| = 1.7\times10^2\ \text{N}.$$

(b) From Eq. 2-16 (with $v = 0$) we have

$$0 = v_0^2 + 2a\Delta x \quad\Rightarrow\quad \Delta x = -\frac{(37\,\text{m/s})^2}{2\left(-2.0\,\text{m/s}^2\right)} = 3.4\times10^2\ \text{m}.$$

Alternatively, this can be worked using the work-energy theorem.

(c) Since $\vec{F}$ is opposite to the direction of motion (so the angle ϕ between $\vec{F}$ and $\vec{d} = \Delta x$ is 180°) then Eq. 7-7 gives the work done as $W = -F\Delta x = -5.8\times10^4\ \text{J}$.

(d) In this case, Newton's second law yields $\vec{F} = (85\,\text{kg})\left(-4.0\,\text{m/s}^2\right)$ so that $F = |\vec{F}| = 3.4\times10^2\ \text{N}$.

(e) From Eq. 2-16, we now have

$$\Delta x = -\frac{(37\,\text{m/s})^2}{2\left(-4.0\,\text{m/s}^2\right)} = 1.7\times10^2\ \text{m}.$$

(f) The force $\vec{F}$ is again opposite to the direction of motion (so the angle ϕ is again 180°) so that Eq. 7-7 leads to $W = -F\Delta x = -5.8\times10^4$ J. The fact that this agrees with the result of part (c) provides insight into the concept of work.

14. The forces are all constant, so the total work done by them is given by $W = F_{\text{net}}\Delta x$, where F_{net} is the magnitude of the net force and Δx is the magnitude of the displacement. We add the three vectors, finding the x and y components of the net force:

$$\begin{aligned} F_{\text{net}\,x} &= -F_1 - F_2 \sin 50.0° + F_3 \cos 35.0° = -3.00\,\text{N} - (4.00\,\text{N})\sin 35.0° + (10.0\,\text{N})\cos 35.0° \\ &= 2.13\,\text{N} \end{aligned}$$

$$\begin{aligned} F_{\text{net}\,y} &= -F_2 \cos 50.0° + F_3 \sin 35.0° = -(4.00\,\text{N})\ \cos 50.0° + (10.0\,\text{N})\sin 35.0° \\ &= 3.17\,\text{N}. \end{aligned}$$

The magnitude of the net force is

$$F_{\text{net}} = \sqrt{F_{\text{net}\,x}^2 + F_{\text{net}\,y}^2} = \sqrt{(2.13\text{ N})^2 + (3.17\text{ N})^2} = 3.82\text{ N}.$$

The work done by the net force is

$$W = F_{\text{net}} d = (3.82\text{ N})(4.00\text{ m}) = 15.3\text{ J}$$

where we have used the fact that $\vec{d} \parallel \vec{F}_{\text{net}}$ (which follows from the fact that the canister started from rest and moved horizontally under the action of horizontal forces — the resultant effect of which is expressed by $\vec{F}_{\text{net}}$).

15. (a) The forces are constant, so the work done by any one of them is given by $W = \vec{F} \cdot \vec{d}$, where $\vec{d}$ is the displacement. Force $\vec{F}_1$ is in the direction of the displacement, so

$$W_1 = F_1 d \cos\phi_1 = (5.00\text{ N})(3.00\text{ m})\cos 0° = 15.0\text{ J}.$$

Force $\vec{F}_2$ makes an angle of 120° with the displacement, so

$$W_2 = F_2 d \cos\phi_2 = (9.00\text{ N})(3.00\text{ m})\cos 120° = -13.5\text{ J}.$$

Force $\vec{F}_3$ is perpendicular to the displacement, so

$$W_3 = F_3 d \cos\phi_3 = 0 \text{ since } \cos 90° = 0.$$

The net work done by the three forces is

$$W = W_1 + W_2 + W_3 = 15.0\text{ J} - 13.5\text{ J} + 0 = +1.50\text{ J}.$$

(b) If no other forces do work on the box, its kinetic energy increases by 1.50 J during the displacement.

16. The change in kinetic energy can be written as

$$\Delta K = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}m(2a\Delta x) = ma\Delta x$$

where we have used $v_f^2 = v_i^2 + 2a\Delta x$ from Table 2-1. From the figure, we see that $\Delta K = (0-30)\text{ J} = -30\text{ J}$ when $\Delta x = +5\text{ m}$. The acceleration can then be obtained as

$$a = \frac{\Delta K}{m\Delta x} = \frac{(-30\text{ J})}{(8.0\text{ kg})(5.0\text{ m})} = -0.75\text{ m/s}^2.$$

The negative sign indicates that the mass is decelerating. From the figure, we also see that when $x = 5$ m the kinetic energy becomes zero, implying that the mass comes to rest momentarily. Thus,

$$v_0^2 = v^2 - 2a\Delta x = 0 - 2(-0.75\ \text{m/s}^2)(5.0\ \text{m}) = 7.5\ \text{m}^2/\text{s}^2,$$

or $v_0 = 2.7$ m/s. The speed of the object when $x = -3.0$ m is

$$v = \sqrt{v_0^2 + 2a\Delta x} = \sqrt{7.5\ \text{m}^2/\text{s}^2 + 2(-0.75\ \text{m/s}^2)(-3.0\ \text{m})} = \sqrt{12}\ \text{m/s} = 3.5\ \text{m/s}.$$

17. **THINK** The helicopter does work to lift the astronaut upward against gravity. The work done on the astronaut is converted to the kinetic energy of the astronaut.

EXPRESS We use $\vec{F}$ to denote the upward force exerted by the cable on the astronaut. The force of the cable is upward and the force of gravity is mg downward. Furthermore, the acceleration of the astronaut is $a = g/10$ upward. According to Newton's second law, the force is given by

$$F - mg = ma \quad \Rightarrow \quad F = m(g+a) = \frac{11}{10}mg,$$

in the same direction as the displacement. On the other hand, the force of gravity has magnitude $F_g = mg$ and is opposite in direction to the displacement.

ANALYZE (a) Since the force of the cable $\vec{F}$ and the displacement $\vec{d}$ are in the same direction, the work done by $\vec{F}$ is

$$W_F = Fd = \frac{11mgd}{10} = \frac{11\,(72\ \text{kg})(9.8\ \text{m/s}^2)(15\ \text{m})}{10} = 1.164\times10^4\ \text{J} \approx 1.2\times10^4\ \text{J}.$$

(b) Using Eq. 7-7, the work done by gravity is

$$W_g = -F_g d = -mgd = -\,(72\ \text{kg})(9.8\ \text{m/s}^2)(15\ \text{m}) = -1.058\times10^4\ \text{J} \approx -1.1\times10^4\ \text{J}.$$

(c) The total work done is the sum of the two works:

$$W_{\text{net}} = W_F + W_g = 1.164\times10^4\ \text{J} - 1.058\times10^4\ \text{J} = 1.06\times10^3\ \text{J} \approx 1.1\times10^3\ \text{J}.$$

Since the astronaut started from rest, the work-kinetic energy theorem tells us that this is her final kinetic energy.

(d) Since $K = \frac{1}{2}mv^2$, her final speed is $v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2(1.06\times10^3\ \text{J})}{72\ \text{kg}}} = 5.4\ \text{m/s}$.

LEARN For a general upward acceleration *a*, the net work done is

$$W_{\text{net}} = W_F + W_g = Fd - F_g d = m(g+a)d - mgd = mad.$$

Since $W_{\text{net}} = \Delta K = mv^2/2$ by the work-kinetic energy theorem, the speed of the astronaut would be $v = \sqrt{2ad}$, which is independent of the mass of the astronaut. In our case, $v = \sqrt{2(9.8\text{ m/s}^2/10)(15\text{ m})} = 5.4\text{ m/s}$, which agrees with that calculated in (d).

18. In both cases, there is no acceleration, so the lifting force is equal to the weight of the object.

(a) Equation 7-8 leads to $W = \vec{F}\cdot\vec{d} = (360\,\text{kN})(0.10\,\text{m}) = 36\text{ kJ}$.

(b) In this case, we find W = (4000 N)(0.050 m) $= 2.0\times10^2\text{ J}$.

19. Equation 7-15 applies, but the wording of the problem suggests that it is only necessary to examine the contribution from the rope (which would be the "W_a" term in Eq. 7-15):

$$W_a = -(50\text{ N})(0.50\text{ m}) = -25\text{ J}$$

(the minus sign arises from the fact that the pull from the rope is anti-parallel to the direction of motion of the block). Thus, the kinetic energy would have been 25 J greater if the rope had not been attached (given the same displacement).

20. From the figure, one may write the kinetic energy (in units of J) as a function of *x* as

$$K = K_s - 20x = 40 - 20x.$$

Since $W = \Delta K = \vec{F}_x\cdot\Delta x$, the component of the force along the force along $+x$ is $F_x = dK/dx = -20$ N. The normal force on the block is $F_N = F_y$, which is related to the gravitational force by

$$mg = \sqrt{F_x^2 + (-F_y)^2}\,.$$

(Note that F_N points in the opposite direction of the component of the gravitational force.) With an initial kinetic energy $K_s = 40.0$ J and $v_0 = 4.00$ m/s, the mass of the block is

$$m = \frac{2K_s}{v_0^2} = \frac{2(40.0\text{ J})}{(4.00\text{ m/s})^2} = 5.00\text{ kg}.$$

Thus, the normal force is

$$F_y = \sqrt{(mg)^2 - F_x^2} = \sqrt{(5.0\text{ kg})^2(9.8\text{ m/s}^2)^2 - (20\text{ N})^2} = 44.7\text{ N} \approx 45\text{ N}.$$

21. **THINK** In this problem the cord is doing work on the block so that it does not undergo free fall.

EXPRESS We use F to denote the magnitude of the force of the cord on the block. This force is upward, opposite to the force of gravity (which has magnitude $F_g = Mg$), to prevent the block from undergoing free fall. The acceleration is $\vec{a} = g/4$ downward. Taking the downward direction to be positive, then Newton's second law yields

$$\vec{F}_{\text{net}} = m\vec{a} \Rightarrow Mg - F = M\left(\frac{g}{4}\right),$$

so $F = 3Mg/4$, in the opposite direction of the displacement. On the other hand, the force of gravity $F_g = mg$ is in the same direction to the displacement.

ANALYZE (a) Since the displacement is downward, the work done by the cord's force is, using Eq. 7-7,

$$W_F = -Fd = -\frac{3}{4}Mgd.$$

(b) Similarly, the work done by the force of gravity is $W_g = F_g d = Mgd$.

(c) The total work done on the block is simply the sum of the two works:

$$W_{\text{net}} = W_F + W_g = -\frac{3}{4}Mgd + Mgd = \frac{1}{4}Mgd .$$

Since the block starts from rest, we use Eq. 7-15 to conclude that this $(M\,gd/4)$ is the block's kinetic energy K at the moment it has descended the distance d.

(d) With $K = \frac{1}{2}Mv^2$, the speed is

$$v = \sqrt{\frac{2K}{M}} = \sqrt{\frac{2(Mgd/4)}{M}} = \sqrt{\frac{gd}{2}}$$

at the moment the block has descended the distance d.

LEARN For a general downward acceleration a, the force exerted by the cord is $F = m(g-a)$, so that the net work done on the block is $W_{\text{net}} = F_{\text{net}}d = mad$. The speed of the block after falling a distance d is $v = \sqrt{2ad}$. In the special case where the block hangs still, $a = 0$, $F = mg$ and $v = 0$. In our case, $a = g/4$, and $v = \sqrt{2(g/4)d} = \sqrt{gd/2}$, which agrees with that calculated in (d).

22. We use d to denote the magnitude of the spelunker's displacement during each stage. The mass of the spelunker is m = 80.0 kg. The work done by the lifting force is denoted W_i where i = 1, 2, 3 for the three stages. We apply the work-energy theorem, Eq. 17-15.

(a) For stage 1, $W_1 - mgd = \Delta K_1 = \frac{1}{2}mv_1^2$, where $v_1 = 5.00$ m/s. This gives

$$W_1 = mgd + \frac{1}{2}mv_1^2 = (80.0\text{ kg})(9.80\text{ m/s}^2)(10.0\text{ m}) + \frac{1}{2}(80.0\text{ kg})(5.00\text{ m/s})^2 = 8.84\times10^3\text{ J.}$$

(b) For stage 2, $W_2 - mgd = \Delta K_2 = 0$, which leads to

$$W_2 = mgd = (80.0\text{ kg})(9.80\text{ m/s}^2)(10.0\text{ m}) = 7.84\times10^3\text{ J.}$$

(c) For stage 3, $W_3 - mgd = \Delta K_3 = -\frac{1}{2}mv_1^2$. We obtain

$$W_3 = mgd - \frac{1}{2}mv_1^2 = (80.0\text{ kg})(9.80\text{ m/s}^2)(10.0\text{ m}) - \frac{1}{2}(80.0\text{ kg})(5.00\text{ m/s})^2 = 6.84\times10^3\text{ J.}$$

23. The fact that the applied force $\vec{F}_a$ causes the box to move up a frictionless ramp at a constant speed implies that there is no net change in the kinetic energy: $\Delta K = 0$. Thus, the work done by $\vec{F}_a$ must be equal to the negative work done by gravity: $W_a = -W_g$. Since the box is displaced vertically upward by $h = 0.150$ m, we have

$$W_a = +mgh = (3.00\text{ kg})(9.80\text{ m/s}^2)(0.150\text{ m}) = 4.41\text{ J}$$

24. (a) Using notation common to many vector-capable calculators, we have (from Eq. 7-8) W = dot([20.0,0] + [0, –(3.00)(9.8)], [0.500 ∠ 30.0º]) = +1.31 J , where "dot" stands for dot product.

(b) Eq. 7-10 (along with Eq. 7-1) then leads to $v = \sqrt{2(1.31\text{ J})/(3.00\text{ kg})}$ = 0.935 m/s.

25. (a) The net upward force is given by

$$F + F_N - (m+M)g = (m+M)a$$

where m = 0.250 kg is the mass of the cheese, M = 900 kg is the mass of the elevator cab, F is the force from the cable, and $F_N = 3.00$ N is the normal force on the cheese. On the cheese alone, we have

$$F_N - mg = ma \quad \Rightarrow \quad a = \frac{3.00\text{ N} - (0.250\text{ kg})(9.80\text{ m/s}^2)}{0.250\text{ kg}} = 2.20\text{ m/s}^2.$$

Thus the force from the cable is $F = (m+M)(a+g) - F_N = 1.08\times10^4$ N, and the work done by the cable on the cab is

$$W = Fd_1 = (1.80\times10^4 \text{ N})(2.40 \text{ m}) = 2.59\times10^4 \text{ J}.$$

(b) If $W = 92.61$ kJ and $d_2 = 10.5$ m, the magnitude of the normal force is

$$F_N = (m+M)g - \frac{W}{d_2} = (0.250 \text{ kg} + 900 \text{ kg})(9.80 \text{ m/s}^2) - \frac{9.261\times10^4 \text{ J}}{10.5 \text{ m}} = 2.45 \text{ N}.$$

26. We make use of Eq. 7-25 and Eq. 7-28 since the block is stationary before and after the displacement. The work done by the applied force can be written as

$$W_a = -W_s = \frac{1}{2}k(x_f^2 - x_i^2).$$

The spring constant is $k = (80 \text{ N})/(2.0 \text{ cm}) = 4.0\times10^3 \text{N/m}$. With $W_a = 4.0$ J, and $x_i = -2.0$ cm, we have

$$x_f = \pm\sqrt{\frac{2W_a}{k} + x_i^2} = \pm\sqrt{\frac{2(4.0 \text{ J})}{(4.0\times10^3 \text{ N/m})} + (-0.020 \text{ m})^2} = \pm 0.049 \text{ m} = \pm 4.9 \text{ cm}.$$

27. From Eq. 7-25, we see that the work done by the spring force is given by

$$W_s = \frac{1}{2}k(x_i^2 - x_f^2).$$

The fact that 360 N of force must be applied to pull the block to x = + 4.0 cm implies that the spring constant is

$$k = \frac{360 \text{ N}}{4.0 \text{ cm}} = 90 \text{ N/cm} = 9.0\times10^3 \text{ N/m}.$$

(a) When the block moves from $x_i = +5.0$ cm to $x = +3.0$ cm, we have

$$W_s = \frac{1}{2}(9.0\times10^3 \text{ N/m})[(0.050 \text{ m})^2 - (0.030 \text{ m})^2] = 7.2 \text{ J}.$$

(b) Moving from $x_i = +5.0$ cm to $x = -3.0$ cm, we have

$$W_s = \frac{1}{2}(9.0\times10^3 \text{ N/m})[(0.050 \text{ m})^2 - (-0.030 \text{ m})^2] = 7.2 \text{ J}.$$

(c) Moving from $x_i = +5.0\ \text{cm}$ to $x = -5.0\ \text{cm}$, we have

$$W_s = \frac{1}{2}(9.0\times10^3\ \text{N/m})[(0.050\ \text{m})^2 - (-0.050\ \text{m})^2] = 0\ \text{J}.$$

(d) Moving from $x_i = +5.0\ \text{cm}$ to $x = -9.0\ \text{cm}$, we have

$$W_s = \frac{1}{2}(9.0\times10^3\ \text{N/m})[(0.050\ \text{m})^2 - (-0.090\ \text{m})^2] = -25\ \text{J}.$$

28. The spring constant is k = 100 N/m and the maximum elongation is x_i = 5.00 m. Using Eq. 7-25 with x_f = 0, the work is found to be

$$W = \frac{1}{2}kx_i^2 = \frac{1}{2}(100\ \text{N/m})(5.00\ \text{m})^2 = 1.25\times10^3\ \text{J}.$$

29. The work done by the spring force is given by Eq. 7-25: $W_s = \frac{1}{2}k(x_i^2 - x_f^2)$. The spring constant k can be deduced from the figure which shows the amount of work done to pull the block from 0 to x = 3.0 cm. The parabola $W_a = kx^2/2$ contains (0,0), (2.0 cm, 0.40 J) and (3.0 cm, 0.90 J). Thus, we may infer from the data that $k = 2.0\times10^3\ \text{N/m}$.

(a) When the block moves from $x_i = +5.0\ \text{cm}$ to $x = +4.0\ \text{cm}$, we have

$$W_s = \frac{1}{2}(2.0\times10^3\ \text{N/m})[(0.050\ \text{m})^2 - (0.040\ \text{m})^2] = 0.90\ \text{J}.$$

(b) Moving from $x_i = +5.0\ \text{cm}$ to $x = -2.0\ \text{cm}$, we have

$$W_s = \frac{1}{2}(2.0\times10^3\ \text{N/m})[(0.050\ \text{m})^2 - (-0.020\ \text{m})^2] = 2.1\ \text{J}.$$

(c) Moving from $x_i = +5.0\ \text{cm}$ to $x = -5.0\ \text{cm}$, we have

$$W_s = \frac{1}{2}(2.0\times10^3\ \text{N/m})[(0.050\ \text{m})^2 - (-0.050\ \text{m})^2] = 0\ \text{J}.$$

30. Hooke's law and the work done by a spring is discussed in the chapter. We apply the work-kinetic energy theorem, in the form of $\Delta K = W_a + W_s$, to the points in the figure at x = 1.0 m and x = 2.0 m, respectively. The "applied" work W_a is that due to the constant force $\vec{P}$.

$$4\text{ J} = P(1.0\text{ m}) - \frac{1}{2}k(1.0\text{ m})^2$$
$$0 = P(2.0\text{ m}) - \frac{1}{2}k(2.0\text{ m})^2.$$

(a) Simultaneous solution leads to $P = 8.0$ N.

(b) Similarly, we find $k = 8.0$ N/m.

31. **THINK** The applied force varies with x, so an integration is required to calculate the work done on the body.

EXPRESS As the body moves along the x axis from $x_i = 3.0$ m to $x_f = 4.0$ m the work done by the force is

$$W = \int_{x_i}^{x_f} F_x\, dx = \int_{x_i}^{x_f} -6x\, dx = -3(x_f^2 - x_i^2) = -3\,(4.0^2 - 3.0^2) = -21\text{ J}.$$

According to the work-kinetic energy theorem, this gives the change in the kinetic energy:

$$W = \Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right)$$

where v_i is the initial velocity (at x_i) and v_f is the final velocity (at x_f). Given v_i, we can readily calculate v_f.

ANALYZE (a) The work-kinetic theorem yields

$$v_f = \sqrt{\frac{2W}{m} + v_i^2} = \sqrt{\frac{2(-21\text{ J})}{2.0\text{ kg}} + (8.0\text{ m/s})^2} = 6.6\text{ m/s}.$$

(b) The velocity of the particle is $v_f = 5.0$ m/s when it is at $x = x_f$. The work-kinetic energy theorem is used to solve for x_f. The net work done on the particle is $W = -3\left(x_f^2 - x_i^2\right)$, so the theorem leads to

$$W = \Delta K \quad \Rightarrow \quad -3\left(x_f^2 - x_i^2\right) = \frac{1}{2}m\,\left(v_f^2 - v_i^2\right).$$

Thus,

$$x_f = \sqrt{-\frac{m}{6}\left(v_f^2 - v_i^2\right) + x_i^2} = \sqrt{-\frac{2.0\text{ kg}}{6\text{ N/m}}\left((5.0\text{ m/s})^2 - (8.0\text{ m/s})^2\right) + (3.0\text{ m})^2} = 4.7\text{ m}.$$

LEARN Since $x_f > x_i$, $W = -3(x_f^2 - x_i^2) < 0$, i.e., the work done by the force is negative. From the work-kinetic energy theorem, this implies $\Delta K < 0$. Hence, the speed of the particle will continue to decrease as it moves in the $+x$-direction.

32. The work done by the spring force is given by Eq. 7-25: $W_s = \frac{1}{2}k(x_i^2 - x_f^2)$. Since $F_x = -kx$, the slope in Fig. 7-37 corresponds to the spring constant *k*. Its value is given by $k = 80\ \text{N/cm} = 8.0\times10^3\ \text{N/m}$.

(a) When the block moves from $x_i = +8.0\ \text{cm}$ to $x = +5.0\ \text{cm}$, we have

$$W_s = \frac{1}{2}(8.0\times10^3\ \text{N/m})[(0.080\ \text{m})^2 - (0.050\ \text{m})^2] = 15.6\ \text{J} \approx 16\ \text{J}.$$

(b) Moving from $x_i = +8.0\ \text{cm}$ to $x = -5.0\ \text{cm}$, we have

$$W_s = \frac{1}{2}(8.0\times10^3\ \text{N/m})[(0.080\ \text{m})^2 - (-0.050\ \text{m})^2] = 15.6\ \text{J} \approx 16\ \text{J}.$$

(c) Moving from $x_i = +8.0\ \text{cm}$ to $x = -8.0\ \text{cm}$, we have

$$W_s = \frac{1}{2}(8.0\times10^3\ \text{N/m})[(0.080\ \text{m})^2 - (-0.080\ \text{m})^2] = 0\ \text{J}.$$

(d) Moving from $x_i = +8.0\ \text{cm}$ to $x = -10.0\ \text{cm}$, we have

$$W_s = \frac{1}{2}(8.0\times10^3\ \text{N/m})[(0.080\ \text{m})^2 - (-0.10\ \text{m})^2] = -14.4\ \text{J} \approx -14\ \text{J}.$$

33. (a) This is a situation where Eq. 7-28 applies, so we have

$$Fx = \tfrac{1}{2}kx^2 \ \Rightarrow\ (3.0\ \text{N})\,x = \tfrac{1}{2}(50\ \text{N/m})x^2$$

which (other than the trivial root) gives $x = (3.0/25)\ \text{m} = 0.12\ \text{m}$.

(b) The work done by the applied force is $W_a = Fx = (3.0\ \text{N})(0.12\ \text{m}) = 0.36\ \text{J}$.

(c) Eq. 7-28 immediately gives $W_s = -W_a = -0.36\ \text{J}$.

(d) With $K_f = K$ considered variable and $K_i = 0$, Eq. 7-27 gives $K = Fx - \tfrac{1}{2}kx^2$. We take the derivative of *K* with respect to *x* and set the resulting expression equal to zero, in order to find the position x_c taht corresponds to a maximum value of *K*:

$$x_c = \frac{F}{k} = (3.0/50)\ \text{m} = 0.060\ \text{m}.$$

We note that x_c is also the point where the applied and spring forces “balance.”

(e) At x_c we find $K = K_{max} = 0.090$ J.

34. According to the graph the acceleration a varies linearly with the coordinate x. We may write $a = \alpha x$, where α is the slope of the graph. Numerically,

$$\alpha = \frac{20 \text{ m/s}^2}{8.0 \text{ m}} = 2.5 \text{ s}^{-2}.$$

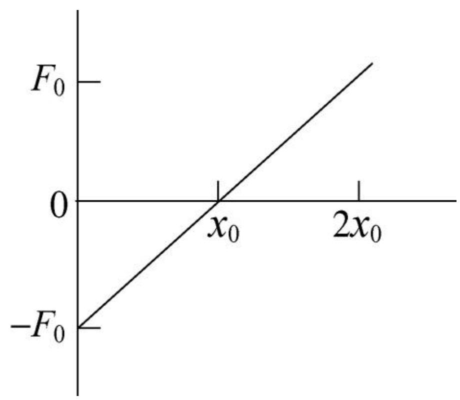
The force on the brick is in the positive x direction and, according to Newton's second law, its magnitude is given by $F = ma = m\alpha x$. If x_f is the final coordinate, the work done by the force is

$$W = \int_0^{x_f} F\ dx = m\alpha \int_0^{x_f} x\ dx = \frac{m\alpha}{2} x_f^2 = \frac{(10 \text{ kg})(2.5 \text{ s}^{-2})}{2}(8.0 \text{ m})^2 = 8.0 \times 10^2 \text{ J}.$$

35. **THINK** We have an applied force that varies with x. An integration is required to calculate the work done on the particle.

EXPRESS Given a one-dimensional force $F(x)$, the work done is simply equal to the area under the curve: $W = \int_{x_i}^{x_f} F(x)\ dx$.

ANALYZE (a) The plot of $F(x)$ is shown to the right. Here we take x_0 to be positive. The work is negative as the object moves from $x = 0$ to $x = x_0$ and positive as it moves from $x = x_0$ to $x = 2x_0$.



Since the area of a triangle is (base)(altitude)/2, the work done from $x = 0$ to $x = x_0$ is $W_1 = -(x_0)(F_0)/2$ and the work done from $x = x_0$ to $x = 2x_0$ is

$$W_2 = (2x_0 - x_0)(F_0)/2 = (x_0)(F_0)/2$$

The total work is the sum of the two:

$$W = W_1 + W_2 = -\frac{1}{2}F_0 x_0 + \frac{1}{2}F_0 x_0 = 0.$$

(b) The integral for the work is

$$W = \int_0^{2x_0} F_0\left(\frac{x}{x_0} - 1\right)dx = F_0\left(\frac{x^2}{2x_0} - x\right)\Bigg|_0^{2x_0} = 0.$$

LEARN If the particle starts out at $x = 0$ with an initial speed v_i, with a negative work $W_1 = -F_0 x_0/2 < 0$, its speed at $x = x_0$ will decrease to

$$v=\sqrt{v_i^2+\frac{2W_1}{m}}=\sqrt{v_i^2-\frac{F_0x_0}{m}}<v_i\,,$$

but return to v_i again at $x=2x_0$ with a positive work $W_2=F_0x_0/2>0$.

36. From Eq. 7-32, we see that the "area" in the graph is equivalent to the work done. Finding that area (in terms of rectangular [length × width] and triangular [$\frac{1}{2}$ base × height] areas) we obtain

$$W=W_{0<x<2}+W_{2<x<4}+W_{4<x<6}+W_{6<x<8}=(20+10+0-5)\text{ J}=25\text{ J.}$$

37. (a) We first multiply the vertical axis by the mass, so that it becomes a graph of the applied force. Now, adding the triangular and rectangular "areas" in the graph (for $0 \le x \le 4$) gives 42 J for the work done.

(b) Counting the "areas" under the axis as negative contributions, we find (for $0 \le x \le 7$) the work to be 30 J at x = 7.0 m.

(c) And at x = 9.0 m, the work is 12 J.

(d) Equation 7-10 (along with Eq. 7-1) leads to speed v = 6.5 m/s at x = 4.0 m. Returning to the original graph (where a was plotted) we note that (since it started from rest) it has received acceleration(s) (up to this point) only in the $+x$ direction and consequently must have a velocity vector pointing in the $+x$ direction at x = 4.0 m.

(e) Now, using the result of part (b) and Eq. 7-10 (along with Eq. 7-1) we find the speed is 5.5 m/s at x = 7.0 m. Although it has experienced some deceleration during the $0 \le x \le 7$ interval, its velocity vector still points in the $+x$ direction.

(f) Finally, using the result of part (c) and Eq. 7-10 (along with Eq. 7-1) we find its speed v = 3.5 m/s at x = 9.0 m. It certainly has experienced a significant amount of deceleration during the $0 \le x \le 9$ interval; nonetheless, its velocity vector *still* points in the $+x$ direction.

38. (a) Using the work-kinetic energy theorem

$$K_f=K_i+\int_0^{2.0}(2.5-x^2)\;dx=0+(2.5)(2.0)-\frac{1}{3}(2.0)^3=2.3\text{ J.}$$

(b) For a variable end-point, we have K_f as a function of x, which could be differentiated to find the extremum value, but we recognize that this is equivalent to solving F = 0 for x:

$$F=0\;\Rightarrow\;2.5-x^2=0\,.$$

Thus, K is extremized at $x = \sqrt{2.5} \approx 1.6$ m and we obtain

$$K_f = K_i + \int_0^{\sqrt{2.5}} (2.5 - x^2)dx = 0 + (2.5)(\sqrt{2.5}) - \frac{1}{3}(\sqrt{2.5})^3 = 2.6 \text{ J.}$$

Recalling our answer for part (a), it is clear that this extreme value is a maximum.

39. As the body moves along the x axis from x_i = 0 m to x_f = 3.00 m the work done by the force is

$$W = \int_{x_i}^{x_f} F_x \; dx = \int_{x_i}^{x_f} (cx - 3.00x^2)dx = \left(\frac{c}{2}x^2 - x^3\right)_0^3 = \frac{c}{2}(3.00)^2 - (3.00)^3$$
$$= 4.50c - 27.0.$$

However, $W = \Delta K = (11.0 - 20.0) = -9.00$ J from the work-kinetic energy theorem. Thus,

$$4.50c - 27.0 = -9.00$$

or $c = 4.00$ N/m.

40. Using Eq. 7-32, we find

$$W = \int_{0.25}^{1.25} e^{-4x^2} \; dx = 0.21 \text{ J}$$

where the result has been obtained numerically. Many modern calculators have that capability, as well as most math software packages that a great many students have access to.

41. We choose to work this using Eq. 7-10 (the work-kinetic energy theorem). To find the initial and final kinetic energies, we need the speeds, so

$$v = \frac{dx}{dt} = 3.0 - 8.0t + 3.0t^2$$

in SI units. Thus, the initial speed is v_i = 3.0 m/s and the speed at t = 4 s is v_f = 19 m/s. The change in kinetic energy for the object of mass m = 3.0 kg is therefore

$$\Delta K = \frac{1}{2}m \left(v_f^2 - v_i^2\right) = 528 \text{ J}$$

which we round off to two figures and (using the work-kinetic energy theorem) conclude that the work done is $W = 5.3 \times 10^2$ J.

42. We solve the problem using the work-kinetic energy theorem, which states that the change in kinetic energy is equal to the work done by the applied force, $\Delta K = W$. In our

problem, the work done is $W = Fd$, where F is the tension in the cord and d is the length of the cord pulled as the cart slides from x_1 to x_2. From the figure, we have

$$d = \sqrt{x_1^2 + h^2} - \sqrt{x_2^2 + h^2} = \sqrt{(3.00\text{ m})^2 + (1.20\text{ m})^2} - \sqrt{(1.00\text{ m})^2 + (1.20\text{ m})^2}$$
$$= 3.23\text{ m} - 1.56\text{ m} = 1.67\text{ m}$$

which yields $\Delta K = Fd = (25.0\text{ N})(1.67\text{ m}) = 41.7\text{ J}.$

43. **THINK** This problem deals with the power and work done by a constant force.

EXPRESS The power done by a constant force F is given by $P = Fv$ and the work done by F from time t_1 to time t_2 is

$$W = \int_{t_1}^{t_2} P\, dt = \int_{t_1}^{t_2} Fv\, dt$$

Since F is the magnitude of the net force, the magnitude of the acceleration is $a = F/m$. Thus, if the initial velocity is $v_0 = 0$, then the velocity of the body as a function of time is given by $v = v_0 + at = (F/m)t$. Substituting the expression for v into the equation above, the work done during the time interval (t_1, t_2) becomes

$$W = \int_{t_1}^{t_2} (F^2/m)t\, dt = \frac{F^2}{2m}\left(t_2^2 - t_1^2\right).$$

ANALYZE (a) For $t_1 = 0$ and $t_2 = 1.0$ s, $W = \frac{1}{2}\left(\frac{(5.0\text{ N})^2}{15\text{ kg}}\right)[(1.0\text{ s})^2 - 0] = 0.83\text{ J}.$

(b) For $t_1 = 1.0$s, and $t_2 = 2.0$ s, $W = \frac{1}{2}\left(\frac{(5.0\text{ N})^2}{15\text{ kg}}\right)[(2.0\text{ s})^2 - (1.0\text{ s})^2] = 2.5\text{ J}.$

(c) For $t_1 = 2.0$ s and $t_2 = 3.0$ s, $W = \frac{1}{2}\left(\frac{(5.0\text{ N})^2}{15\text{ kg}}\right)[(3.0\text{ s})^2 - (2.0\text{ s})^2] = 4.2\text{ J}.$

(d) Substituting $v = (F/m)t$ into $P = Fv$ we obtain $P = F^2 t/m$ for the power at any time t. At the end of the third second, the instantaneous power is

$$P = \left(\frac{(5.0\text{ N})^2\ (3.0\text{ s})}{15\text{ kg}}\right) = 5.0\text{ W}.$$

LEARN The work done here is quadratic in t. Therefore, from the definition $P = dW/dt$ for the instantaneous power, we see that P increases linearly with t.

44. (a) Since constant speed implies $\Delta K = 0$, we require $W_a = -W_g$, by Eq. 7-15. Since W_g is the same in both cases (same weight and same path), then $W_a = 9.0\times10^2$ J just as it was in the first case.

(b) Since the speed of 1.0 m/s is constant, then 8.0 meters is traveled in 8.0 seconds. Using Eq. 7-42, and noting that average power is *the* power when the work is being done at a steady rate, we have

$$P = \frac{W}{\Delta t} = \frac{900\text{ J}}{8.0\text{ s}} = 1.1\times10^2\text{ W}.$$

(c) Since the speed of 2.0 m/s is constant, 8.0 meters is traveled in 4.0 seconds. Using Eq. 7-42, with *average power* replaced by *power*, we have

$$P = \frac{W}{\Delta t} = \frac{900\text{ J}}{4.0\text{ s}} = 225\text{ W} \approx 2.3\times10^2\text{ W}.$$

45. **THINK** A block is pulled at a constant speed by a force directed at some angle with respect to the direction of motion. The quantity we're interested in is the power, or the time rate at which work is done by the applied force.

EXPRESS The power associated with force $\vec{F}$ is given by $P = \vec{F}\cdot\vec{v} = Fv\cos\phi$, where $\vec{v}$ is the velocity of the object on which the force acts, and ϕ is the angle between $\vec{F}$ and $\vec{v}$.

ANALYZE With $F = 122\text{ N}$, $v = 5.0\text{ m/s}$ and $\phi = 37.0°$, we find the power to be

$$P = Fv\cos\phi = (122\text{ N})(5.0\text{ m/s})\cos 37.0° = 4.9\times10^2\text{ W}.$$

LEARN From the expression $P = Fv\cos\phi$, we see that the power is at a maximum when $\vec{F}$ and $\vec{v}$ are in the same direction ($\phi = 0$), and is zero when they are perpendicular of each other. In addition, we're told that the block moves at a constant speed, so $\Delta K = 0$, and the net work done on it must also be zero by the work-kinetic energy theorem. Thus, the applied force here must be compensating another force (e.g., friction) for the net rate to be zero.

46. Recognizing that the force in the cable must equal the total weight (since there is no acceleration), we employ Eq. 7-47:

$$P = Fv\cos\theta = mg\left(\frac{\Delta x}{\Delta t}\right)$$

where we have used the fact that $\theta = 0°$ (both the force of the cable and the elevator's motion are upward). Thus,

$$P = (3.0\times10^3\ \text{kg})(9.8\ \text{m/s}^2)\left(\frac{210\ \text{m}}{23\ \text{s}}\right) = 2.7\times10^5\ \text{W}.$$

47. (a) Equation 7-8 yields

$W = F_x\,\Delta x + F_y\,\Delta y + F_z\,\Delta z$
$\quad = (2.00\ \text{N})(7.5\ \text{m} - 0.50\ \text{m}) + (4.00\ \text{N})(12.0\ \text{m} - 0.75\ \text{m}) + (6.00\ \text{N})(7.2\text{m} - 0.20\ \text{m})$
$\quad = 101\ \text{J} \approx 1.0\times 10^2\ \text{J}.$

(b) Dividing this result by 12 s (see Eq. 7-42) yields P = 8.4 W.

48. (a) Since the force exerted by the spring on the mass is zero when the mass passes through the equilibrium position of the spring, the rate at which the spring is doing work on the mass at this instant is also zero.

(b) The rate is given by $P = \vec{F}\cdot\vec{v} = -Fv$, where the minus sign corresponds to the fact that $\vec{F}$ and $\vec{v}$ are anti-parallel to each other. The magnitude of the force is given by

$$F = kx = (500\ \text{N/m})(0.10\ \text{m}) = 50\ \text{N},$$

while v is obtained from conservation of energy for the spring-mass system:

$$E = K + U = 10\ \text{J} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}(0.30\ \text{kg})v^2 + \frac{1}{2}(500\ \text{N/m})(0.10\ \text{m})^2$$

which gives v = 7.1 m/s. Thus,

$$P = -Fv = -(50\ \text{N})(7.1\ \text{m/s}) = -3.5\times 10^2\ \text{W}.$$

49. **THINK** We have a loaded elevator moving upward at a constant speed. The forces involved are: gravitational force on the elevator, gravitational force on the counterweight, and the force by the motor via cable.

EXPRESS The total work is the sum of the work done by gravity on the elevator, the work done by gravity on the counterweight, and the work done by the motor on the system:

$$W = W_e + W_c + W_m.$$

Since the elevator moves at constant velocity, its kinetic energy does not change and according to the work-kinetic energy theorem the total work done is zero, i.e., $W = \Delta K = 0$.

ANALYZE The elevator moves *upward* through 54 m, so the work done by gravity on it is

$$W_e = -m_e gd = -(1200\text{ kg})(9.80\text{ m/s}^2)(54\text{ m}) = -6.35\times10^5\text{ J}.$$

The counterweight moves *downward* the same distance, so the work done by gravity on it is

$$W_c = m_c gd = (950\text{ kg})(9.80\text{ m/s}^2)(54\text{ m}) = 5.03\times10^5\text{ J}.$$

Since $W = 0$, the work done by the motor on the system is

$$W_m = -W_e - W_c = 6.35\times10^5\text{ J} - 5.03\times10^5\text{ J} = 1.32\times10^5\text{ J}.$$

This work is done in a time interval of $\Delta t = 3.0\text{ min} = 180\text{ s}$, so the power supplied by the motor to lift the elevator is

$$P = \frac{W_m}{\Delta t} = \frac{1.32\times10^5\text{ J}}{180\text{ s}} = 7.4\times10^2\text{ W}.$$

LEARN In general, the work done by the motor is $W_m = (m_e - m_c)gd$. So when the counterweight mass balances the total mass, $m_c = m_e$, no work is required by the motor.

50. (a) Using Eq. 7-48 and Eq. 3-23, we obtain

$$P = \vec{F}\cdot\vec{v} = (4.0\text{ N})(-2.0\text{ m/s}) + (9.0\text{ N})(4.0\text{ m/s}) = 28\text{ W}.$$

(b) We again use Eq. 7-48 and Eq. 3-23, but with a one-component velocity: $\vec{v} = v\hat{\text{j}}$.

$$P = \vec{F}\cdot\vec{v} \;\Rightarrow -12\text{ W} = (-2.0\text{ N})v.$$

which yields $v = 6$ m/s.

51. (a) The object's displacement is

$$\vec{d} = \vec{d}_f - \vec{d}_i = (-8.00\text{ m})\hat{\text{i}} + (6.00\text{ m})\hat{\text{j}} + (2.00\text{ m})\hat{\text{k}}\,.$$

Thus, Eq. 7-8 gives

$$W = \vec{F}\cdot\vec{d} = (3.00\text{ N})(-8.00\text{ m}) + (7.00\text{ N})(6.00\text{ m}) + (7.00\text{ N})(2.00\text{ m}) = 32.0\text{ J}.$$

(b) The average power is given by Eq. 7-42:

$$P_{\text{avg}} = \frac{W}{t} = \frac{32.0}{4.00} = 8.00\text{ W}.$$

(c) The distance from the coordinate origin to the initial position is

$$d_i = \sqrt{(3.00\text{ m})^2 + (-2.00\text{ m})^2 + (5.00\text{ m})^2} = 6.16\text{ m},$$

and the magnitude of the distance from the coordinate origin to the final position is

$$d_f = \sqrt{(-5.00\text{ m})^2 + (4.00\text{ m})^2 + (7.00\text{ m})^2} = 9.49\text{ m}.$$

Their scalar (dot) product is

$$\vec{d}_i \cdot \vec{d}_f = (3.00\text{ m})(-5.00\text{ m}) + (-2.00\text{ m})(4.00\text{ m}) + (5.00\text{ m})(7.00\text{ m}) = 12.0\text{ m}^2.$$

Thus, the angle between the two vectors is

$$\phi = \cos^{-1}\left(\frac{\vec{d}_i \cdot \vec{d}_f}{d_i d_f}\right) = \cos^{-1}\left(\frac{12.0}{(6.16)(9.49)}\right) = 78.2°.$$

52. According to the problem statement, the power of the car is

$$P = \frac{dW}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = mv\frac{dv}{dt} = \text{constant}.$$

The condition implies $dt = mvdv/P$, which can be integrated to give

$$\int_0^T dt = \int_0^{v_T} \frac{mvdv}{P} \quad \Rightarrow \quad T = \frac{mv_T^2}{2P}$$

where v_T is the speed of the car at $t = T$. On the other hand, the total distance traveled can be written as

$$L = \int_0^T vdt = \int_0^{v_T} v\frac{mvdv}{P} = \frac{m}{P}\int_0^{v_T} v^2 dv = \frac{mv_T^3}{3P}.$$

By squaring the expression for L and substituting the expression for T, we obtain

$$L^2 = \left(\frac{mv_T^3}{3P}\right)^2 = \frac{8P}{9m}\left(\frac{mv_T^2}{2P}\right)^3 = \frac{8PT^3}{9m}$$

which implies that

$$PT^3 = \frac{9}{8}mL^2 = \text{constant}.$$

Differentiating the above equation gives $dPT^3 + 3PT^2 dT = 0$, or $dT = -\dfrac{T}{3P}dP.$

53. (a) Noting that the x component of the third force is F_{3x} = (4.00 N)cos(60º), we apply Eq. 7-8 to the problem:

$$W = [5.00\text{ N} - 1.00\text{ N} + (4.00\text{ N})\cos 60^\circ](0.20\text{ m}) = 1.20\text{ J}.$$

(b) Equation 7-10 (along with Eq. 7-1) then yields $v = \sqrt{2W/m}$ = 1.10 m/s.

54. From Eq. 7-32, we see that the "area" in the graph is equivalent to the work done. We find the area in terms of rectangular [length × width] and triangular [$\frac{1}{2}$ base × height] areas and use the work-kinetic energy theorem appropriately. The initial point is taken to be x = 0, where v_0 = 4.0 m/s.

(a) With $K_i = \frac{1}{2}mv_0^2 = 16\text{ J},$ we have

$$K_3 - K_0 = W_{0<x<1} + W_{1<x<2} + W_{2<x<3} = -4.0\text{ J}$$

so that K_3 (the kinetic energy when x = 3.0 m) is found to equal 12 J.

(b) With SI units understood, we write $W_{3<x<x_f}$ as $F_x\Delta x = (-4.0\text{ N})(x_f - 3.0\text{ m})$ and apply the work-kinetic energy theorem:

$$K_{x_f} - K_3 = W_{3<x<x_f}$$
$$K_{x_f} - 12 = (-4)(x_f - 3.0)$$

so that the requirement $K_{xf} = 8.0\text{ J}$ leads to $x_f = 4.0\text{ m}.$

(c) As long as the work is positive, the kinetic energy grows. The graph shows this situation to hold until x = 1.0 m. At that location, the kinetic energy is

$$K_1 = K_0 + W_{0<x<1} = 16\text{ J} + 2.0\text{ J} = 18\text{ J}.$$

55. **THINK** A horse is doing work to pull a cart at a constant speed. We'd like to know the work done during a time interval and the corresponding average power.

EXPRESS The horse pulls with a force $\vec{F}$. As the cart moves through a displacement $\vec{d}$, the work done by $\vec{F}$ is $W = \vec{F}\cdot\vec{d} = Fd\cos\phi,$ where ϕ is the angle between $\vec{F}$ and $\vec{d}$.

ANALYZE (a) In 10 min the cart moves a distance

$$d = v\Delta t = \left(6.0\ \frac{\text{mi}}{\text{h}}\right)\left(\frac{5280\text{ ft/mi}}{60\text{ min/h}}\right)(10\text{ min}) = 5280\text{ ft}$$

so that Eq. 7-7 yields

$$W = Fd\cos\phi = (40 \text{ lb})(5280 \text{ ft}) \cos 30° = 1.8\times10^5 \text{ ft}\cdot\text{lb}.$$

(b) The average power is given by Eq. 7-42. With $\Delta t = 10 \text{ min} = 600 \text{ s}$, we obtain

$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{1.8\times10^5 \text{ ft}\cdot\text{lb}}{600 \text{ s}} = 305 \text{ ft}\cdot\text{lb/s},$$

which (using the conversion factor $1\,\text{hp} = 550\,\text{ft}\cdot\text{lb/s}$ found on the inside back cover) converts to P_{avg} = 0.55 hp.

LEARN The average power can also be calculate by using Eq. 7-48: $P_{\text{avg}} = Fv\cos\phi$. Converting the speed to $v = (6.0 \text{ mi/h})\left(\dfrac{5280 \text{ ft/mi}}{3600 \text{ s/h}}\right) = 8.8 \text{ ft/s}$, we get

$$P_{\text{avg}} = Fv\cos\phi = (40 \text{ lb})(8.8 \text{ ft/s})\cos 30° = 305 \text{ ft}\cdot\text{lb} = 0.55 \text{ hp}$$

which agrees with that found in (b).

56. The acceleration is constant, so we may use the equations in Table 2-1. We choose the direction of motion as $+x$ and note that the displacement is the same as the distance traveled, in this problem. We designate the force (assumed singular) along the x direction acting on the m = 2.0 kg object as F.

(a) With v_0 = 0, Eq. 2-11 leads to $a = v/t$. And Eq. 2-17 gives $\Delta x = \frac{1}{2}vt$. Newton's second law yields the force $F = ma$. Equation 7-8, then, gives the work:

$$W = F\Delta x = m\left(\frac{v}{t}\right)\left(\frac{1}{2}vt\right) = \frac{1}{2}mv^2$$

as we expect from the work-kinetic energy theorem. With v = 10 m/s, this yields $W = 1.0\times10^2$ J.

(b) Instantaneous power is defined in Eq. 7-48. With t = 3.0 s, we find

$$P = Fv = m\left(\frac{v}{t}\right)v = 67 \text{ W}.$$

(c) The velocity at $t' = 1.5\text{s}$ is $v' = at' = 5.0 \text{ m/s}$. Thus, $P' = Fv' = 33$ W.

57. (a) To hold the crate at equilibrium in the final situation, $\vec{F}$ must have the same magnitude as the horizontal component of the rope's tension $T \sin\theta$, where θ is the angle between the rope (in the final position) and vertical:

$$\theta = \sin^{-1}\left(\frac{4.00}{12.0}\right) = 19.5°.$$

But the vertical component of the tension supports against the weight: $T\cos\theta = mg$. Thus, the tension is

$$T = (230 \text{ kg})(9.80 \text{ m/s}^2)/\cos 19.5° = 2391 \text{ N}$$

and $F = (2391 \text{ N}) \sin 19.5° = 797 \text{ N}$.

An alternative approach based on drawing a vector triangle (of forces) in the final situation provides a quick solution.

(b) Since there is no change in kinetic energy, the net work on it is zero.

(c) The work done by gravity is $W_g = \vec{F}_g \cdot \vec{d} = -mgh$, where $h = L(1 - \cos\theta)$ is the vertical component of the displacement. With $L = 12.0$ m, we obtain $W_g = –1547$ J, which should be rounded to three significant figures: –1.55 kJ.

(d) The tension vector is everywhere perpendicular to the direction of motion, so its work is zero (since cos 90° = 0).

(e) The implication of the previous three parts is that the work due to $\vec{F}$ is $–W_g$ (so the net work turns out to be zero). Thus, $W_F = –W_g = 1.55$ kJ.

(f) Since $\vec{F}$ does not have constant magnitude, we cannot expect Eq. 7-8 to apply.

58. (a) The force of the worker on the crate is constant, so the work it does is given by $W_F = \vec{F} \cdot \vec{d} = Fd\cos\phi$, where $\vec{F}$ is the force, $\vec{d}$ is the displacement of the crate, and ϕ is the angle between the force and the displacement. Here $F = 210$ N, $d = 3.0$ m, and $\phi =$ 20°. Thus,

$$W_F = (210 \text{ N})(3.0 \text{ m}) \cos 20° = 590 \text{ J}.$$

(b) The force of gravity is downward, perpendicular to the displacement of the crate. The angle between this force and the displacement is 90° and cos 90° = 0, so the work done by the force of gravity is zero.

(c) The normal force of the floor on the crate is also perpendicular to the displacement, so the work done by this force is also zero.

(d) These are the only forces acting on the crate, so the total work done on it is 590 J.

59. The work done by the applied force $\vec{F}_a$ is given by $W = \vec{F}_a \cdot \vec{d} = F_a d \cos\phi$. From the figure, we see that $W = 25\text{ J}$ when $\phi = 0$ and $d = 5.0\text{ cm}$. This yields the magnitude of $\vec{F}_a$:

$$F_a = \frac{W}{d} = \frac{25\text{ J}}{0.050\text{ m}} = 5.0\times10^2\text{ N}.$$

(a) For $\phi = 64°$, we have $W = F_a d\cos\phi = (5.0\times10^2\text{ N})(0.050\text{ m})\cos 64° = 11\text{ J}.$

(b) For $\phi = 147°$, we have $W = F_a d\cos\phi = (5.0\times10^2\text{ N})(0.050\text{ m})\cos 147° = -21\text{ J}.$

60. (a) In the work-kinetic energy theorem, we include both the work due to an applied force W_a and work done by gravity W_g in order to find the latter quantity.

$$\Delta K = W_a + W_g \quad \Rightarrow \quad 30\text{ J} = (100\text{ N})(1.8\text{ m})\cos 180° + W_g$$

leading to $W_g = 2.1\times10^2\text{ J}$.

(b) The value of W_g obtained in part (a) still applies since the weight and the path of the child remain the same, so $\Delta K = W_g = 2.1\times10^2\text{ J}$.

61. One approach is to assume a "path" from $\vec{r}_i$ to $\vec{r}_f$ and do the line-integral accordingly. Another approach is to simply use Eq. 7-36, which we demonstrate:

$$W = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy = \int_2^{-4} (2x) dx + \int_3^{-3} (3)\, dy$$

with SI units understood. Thus, we obtain W = 12 J – 18 J = – 6 J.

62. (a) The compression of the spring is d = 0.12 m. The work done by the force of gravity (acting on the block) is, by Eq. 7-12,

$$W_1 = mgd = (0.25\text{ kg})\left(9.8\text{ m/s}^2\right)(0.12\text{ m}) = 0.29\text{ J}.$$

(b) The work done by the spring is, by Eq. 7-26,

$$W_2 = -\frac{1}{2}kd^2 = -\frac{1}{2}\,(250\text{ N/m})\,(0.12\text{ m})^2 = -1.8\text{ J}.$$

(c) The speed v_i of the block just before it hits the spring is found from the work-kinetic energy theorem (Eq. 7-15):

$$\Delta K = 0 - \frac{1}{2}mv_i^2 = W_1 + W_2$$

which yields

$$v_i = \sqrt{\frac{(-2)(W_1 + W_2)}{m}} = \sqrt{\frac{(-2)(0.29\text{ J} - 1.8\text{ J})}{0.25\text{ kg}}} = 3.5\text{ m/s}.$$

(d) If we instead had $v_i' = 7\,\text{m/s}$, we reverse the above steps and solve for d'. Recalling the theorem used in part (c), we have

$$0 - \frac{1}{2}mv_i'^2 = W_1' + W_2' = mgd' - \frac{1}{2}kd'^2$$

which (choosing the positive root) leads to

$$d' = \frac{mg + \sqrt{m^2g^2 + mkv_i'^2}}{k}$$

which yields $d' = 0.23$ m. In order to obtain this result, we have used more digits in our intermediate results than are shown above (so $v_i = \sqrt{12.048}\text{ m/s} = 3.471\text{ m/s}$ and v_i' = 6.942 m/s).

63. **THINK** A crate is being pushed up a frictionless inclined plane. The forces involved are: gravitational force on the crate, normal force on the crate, and the force applied by the worker.

EXPRESS The work done by a force $\vec{F}$ on an object as it moves through a displacement $\vec{d}$ is $W = \vec{F}\cdot\vec{d} = Fd\cos\phi,$ where ϕ is the angle between $\vec{F}$ and $\vec{d}$.

ANALYZE (a) The applied force is parallel to the inclined plane. Thus, using Eq. 7-6, the work done on the crate by the worker's applied force is

$$W_a = Fd\cos 0° = (209\text{ N})(1.50\text{ m}) \approx 314\text{ J}.$$

(b) Using Eq. 7-12, we find the work done by the gravitational force to be

$$\begin{aligned} W_g &= F_g d\cos(90° + 25°) = mgd\cos 115° \\ &= (25.0\,\text{kg})(9.8\,\text{m/s}^2)(1.50\text{ m})\cos 115° \\ &\approx -155\text{ J}. \end{aligned}$$

(c) The angle between the normal force and the direction of motion remains 90º at all times, so the work it does is zero:

$$W_N = F_N d\cos 90° = 0.$$

(d) The total work done on the crate is the sum of all three works:

$$W = W_a + W_g + W_N = 314 \text{ J} + (-155 \text{ J}) + 0 \text{ J} = 158 \text{ J}.$$

LEARN By work-kinetic energy theorem, if the crate is initially at rest ($K_i = 0$), then its kinetic energy after having moved 1.50 m up the incline would be $K_f = W = 158$ J, and the speed of the crate at that instant is

$$v = \sqrt{2K_f / m} = \sqrt{2(158 \text{ J})/25.0 \text{ kg}} = 3.56 \text{ m/s}.$$

64. (a) The force $\vec{F}$ of the incline is a combination of normal and friction force, which is serving to "cancel" the tendency of the box to fall downward (due to its 19.6 N weight). Thus, $\vec{F} = mg$ upward. In this part of the problem, the angle ϕ between the belt and $\vec{F}$ is 80°. From Eq. 7-47, we have

$$P = Fv \cos\phi = (19.6 \text{ N})(0.50 \text{ m/s}) \cos 80° = 1.7 \text{ W}.$$

(b) Now the angle between the belt and $\vec{F}$ is 90°, so that $P = 0$.

(c) In this part, the angle between the belt and $\vec{F}$ is 100°, so that

$$P = (19.6 \text{ N})(0.50 \text{ m/s}) \cos 100° = -1.7 \text{ W}.$$

65. There is no acceleration, so the lifting force is equal to the weight of the object. We note that the person's pull $\vec{F}$ is equal (in magnitude) to the tension in the cord.

(a) As indicated in the *hint*, tension contributes twice to the lifting of the canister: $2T = mg$. Since $|\vec{F}| = T$, we find $|\vec{F}| = 98$ N.

(b) To rise 0.020 m, two segments of the cord (see Fig. 7-47) must shorten by that amount. Thus, the amount of string pulled down at the left end (this is the magnitude of $\vec{d}$, the downward displacement of the hand) is $d = 0.040$ m.

(c) Since (at the left end) both $\vec{F}$ and $\vec{d}$ are downward, then Eq. 7-7 leads to

$$W = \vec{F} \cdot \vec{d} = (98 \text{ N})(0.040 \text{ m}) = 3.9 \text{ J}.$$

(d) Since the force of gravity $\vec{F}_g$ (with magnitude mg) is opposite to the displacement $\vec{d}_c = 0.020$ m (up) of the canister, Eq. 7-7 leads to

$$W = \vec{F}_g \cdot \vec{d}_c = -\ (196\ \text{N})(0.020\ \text{m}) = -3.9\ \text{J}.$$

This is consistent with Eq. 7-15 since there is no change in kinetic energy.

66. After converting the speed: $v = 120\ \text{km/h} = 33.33\ \text{m/s}$, we find

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1200\,\text{kg})(33.33\,\text{m/s})^2 = 6.67\times10^5\ \text{J}.$$

67. **THINK** In this problem we have packages hung from the spring. The extent of stretching can be determined from Hooke's law.

EXPRESS According to Hooke's law, the spring force is given by

$$F_x = -k(x - x_0) = -k\Delta x\,,$$

where Δx is the displacement from the equilibrium position. Thus, the first two situations in Fig. 7-48 can be written as

$$-110\ \text{N} = -k(40\ \text{mm} - x_0)$$
$$-240\ \text{N} = -k(60\ \text{mm} - x_0)$$

The two equations allow us to solve for *k*, the spring constant, as well as x_0, the relaxed position when no mass is hung.

ANALYZE (a) The two equations can be added to give

$$240\ \text{N} - 110\ \text{N}\ =\ k(60\ \text{mm} - 40\ \text{mm})$$

which yields *k* = 6.5 N/mm. Substituting the result into the first equation, we find

$$x_0 = 40\ \text{mm} - \frac{110\ \text{N}}{k} = 40\ \text{mm} - \frac{110\ \text{N}}{6.5\ \text{N/mm}} = 23\ \text{mm}.$$

(b) Using the results from part (a) to analyze that last picture, we find the weight to be

$$W = k(30\,\text{mm} - x_o) = (6.5\ \text{N/mm})(30\ \text{mm} - 23\ \text{mm}) = 45\ \text{N}.$$

LEARN An alternative method to calculate *W* in the third picture is to note that since the amount of stretching is proportional to the weight hung, we have $\frac{W}{W'} = \frac{\Delta x}{\Delta x'}$. Applying this relation to the second and the third pictures, the weight *W* is

$$W = \left(\frac{\Delta x_3}{\Delta x_2}\right)W_2 = \left(\frac{30\text{ mm} - 23\text{ mm}}{60\text{ mm} - 23\text{ mm}}\right)(240\text{ N}) = 45\text{ N},$$

in agreement with the result shown in (b).

68. Using Eq. 7-7, we have $W = Fd \cos \phi = 1504$ J. Then, by the work-kinetic energy theorem, we find the kinetic energy $K_f = K_i + W = 0 + 1504$ J. The answer is therefore 1.5 kJ.

69. The total weight is (100)(660 N) = 6.60×10^4 N, and the words "raises … at constant speed" imply zero acceleration, so the lift-force is equal to the total weight. Thus

$$P = Fv = (6.60 \times 10^4)(150\text{ m}/60.0\text{ s}) = 1.65 \times 10^5\text{ W}.$$

70. With SI units understood, Eq. 7-8 leads to $W = (4.0)(3.0) - c(2.0) = 12 - 2c$.

(a) If $W = 0$, then $c = 6.0$ N.

(b) If $W = 17$ J, then $c = -2.5$ N.

(c) If $W = -18$ J, then $c = 15$ N.

71. Using Eq. 7-8, we find

$$W = \vec{F} \cdot \vec{d} = (F\cos\theta\ \hat{\text{i}} + F\ \sin\theta\ \hat{\text{j}}) \cdot (x\hat{\text{i}} + y\hat{\text{j}}) = Fx\cos\theta + Fy\sin\theta$$

where $x = 2.0$ m, $y = -4.0$ m, $F = 10$ N, and $\theta = 150°$. Thus, we obtain $W = -37$ J. Note that the given mass value (2.0 kg) is not used in the computation.

72. (a) Eq. 7-10 (along with Eq. 7-1 and Eq. 7-7) leads to

$$v_f = (2\,\frac{d}{m}\,F\cos\theta)^{1/2} = (\cos\theta)^{1/2},$$

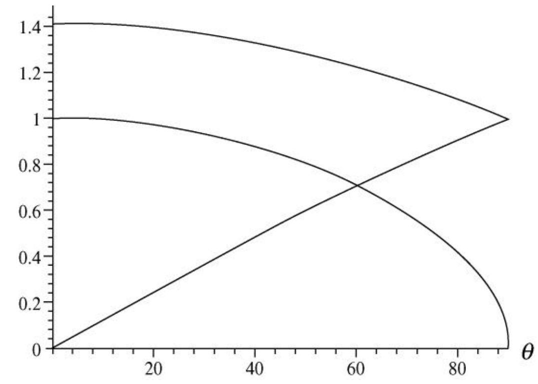
where we have substituted $F = 2.0$ N, $m = 4.0$ kg, and $d = 1.0$ m.

(b) With $v_i = 1$, those same steps lead to $v_f = (1 + \cos\theta)^{1/2}$.

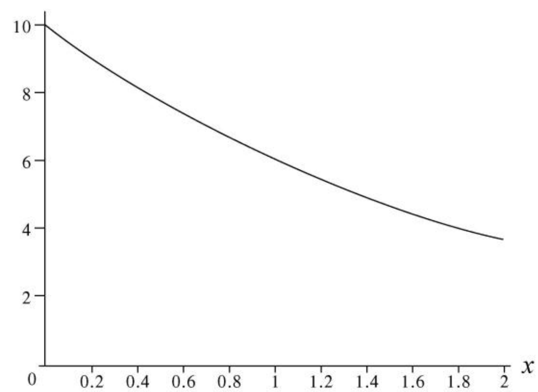
(c) Replacing θ with $180° - \theta$, and still using $v_i = 1$, we find

$$v_f = [1 + \cos(180° - \theta)]^{1/2} = (1 - \cos\theta)^{1/2}.$$

(d) The graphs are shown on the right. Note that as θ is increased in parts (a) and (b) the force provides less and less of a positive acceleration, whereas in part (c) the force provides less and less of a deceleration (as its θ value increases). The highest curve (which slowly decreases from 1.4 to 1) is the curve for part (b); the other decreasing curve (starting at 1 and ending at 0) is for part (a). The rising curve is for part (c); it is equal to 1 where $\theta = 90^{\circ}$.



73. (a) The plot of the function (with SI units understood) is shown below.



Estimating the area under the curve allows for a range of answers. Estimates from 11 J to 14 J are typical.

(b) Evaluating the work analytically (using Eq. 7-32), we have

$$W = \int_0^2 10e^{-x/2}dx = -20e^{-x/2}\Big|_0^2 = 12.6 \text{ J} \approx 13 \text{ J}.$$

74. (a) Using Eq. 7-8 and SI units, we find

$$W = \vec{F}\cdot\vec{d} = (2\,\hat{\text{i}} - 4\,\hat{\text{j}})\cdot(8\,\hat{\text{i}} + c\,\hat{\text{j}}) = 16 - 4c$$

which, if equal zero, implies $c = 16/4 = 4$ m.

(b) If $W > 0$ then $16 > 4c$, which implies $c < 4$ m.

(c) If $W < 0$ then $16 < 4c$, which implies $c > 4$ m.

75. **THINK** Power must be supplied in order to lift the elevator with load upward at a constant speed.

EXPRESS For the elevator-load system to move upward at a constant speed (zero acceleration), the applied force F must exactly balance the gravitational force on the system, i.e., $F = F_g = (m_{\text{elev}} + m_{\text{load}})g$. The power required can then be calculated using Eq. 17-48: $P = Fv$.

ANALYZE With $m_{\text{elev}} = 4500\,\text{kg}$, $m_{\text{load}} = 1800\,\text{kg}$ and $v = 3.80\,\text{m/s}$, we find the power to be

$$P = Fv = (m_{\text{elev}} + m_{\text{load}})gv = (4500\,\text{kg} + 1800\,\text{kg})(9.8\,\text{m/s}^2)(3.80\,\text{m/s}) = 235\,\text{kW}.$$

LEARN The power required is proportional to the speed at which the system moves; the greater the speed, the greater the power that must be supplied.

76. (a) The component of the force of gravity exerted on the ice block (of mass m) along the incline is $mg \sin\theta$, where $\theta = \sin^{-1}(0.91/1.5)$ gives the angle of inclination for the inclined plane. Since the ice block slides down with uniform velocity, the worker must exert a force $\vec{F}$ "uphill" with a magnitude equal to $mg \sin\theta$. Consequently,

$$F = mg\sin\theta = (45\text{ kg})(9.8\text{ m/s}^2)\left(\frac{0.91\text{m}}{1.5\text{m}}\right) = 2.7\times10^2\text{ N}.$$

(b) Since the "downhill" displacement is opposite to $\vec{F}$, the work done by the worker is

$$W_1 = -\left(2.7\times10^2\,\text{N}\right)(1.5\text{ m}) = -4.0\times10^2\,\text{J}.$$

(c) Since the displacement has a vertically downward component of magnitude 0.91 m (in the same direction as the force of gravity), we find the work done by gravity to be

$$W_2 = (45\text{ kg})\left(9.8\text{ m/s}^2\right)(0.91\text{ m}) = 4.0\times10^2\,\text{J}.$$

(d) Since $\vec{F}_N$ is perpendicular to the direction of motion of the block, and $\cos 90° = 0$, work done by the normal force is $W_3 = 0$ by Eq. 7-7.

(e) The resultant force $\vec{F}_{\text{net}}$ is zero since there is no acceleration. Thus, its work is zero, as can be checked by adding the above results $W_1 + W_2 + W_3 = 0$.

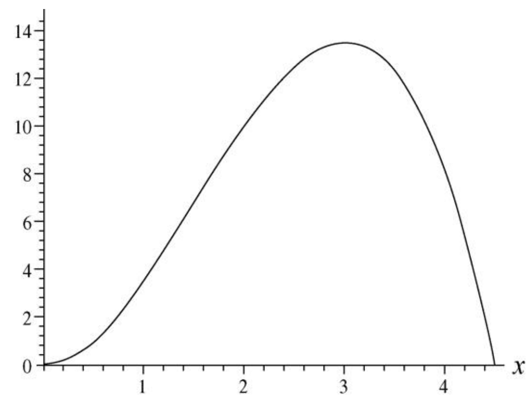
77. (a) To estimate the area under the curve between x = 1 m and x = 3 m (which should yield the value for the work done), one can try "counting squares" (or half-squares or thirds of squares) between the curve and the axis. Estimates between 5 J and 8 J are typical for this (crude) procedure.

(b) Equation 7-32 gives

$$\int_1^3 \frac{a}{x^2}\, dx = \frac{a}{3} - \frac{a}{1} = 6 \text{ J}$$

where a = –9 N·m^2 is given in the problem statement.

78. (a) Using Eq. 7-32, the work becomes $W = \frac{9}{2}x^2 - x^3$ (SI units understood). The plot is shown below:



(b) We see from the graph that its peak value occurs at x = 3.00 m. This can be verified by taking the derivative of W and setting equal to zero, or simply by noting that this is where the force vanishes.

(c) The maximum value is $W = \frac{9}{2}(3.00)^2 - (3.00)^3$ = 13.50 J.

(d) We see from the graph (or from our analytic expression) that W = 0 at x = 4.50 m.

(e) The case is at rest when $v = 0$. Since $W = \Delta K = mv^2/2$, the condition implies $W = 0$. This happens at x = 4.50 m.

79. **THINK** A box sliding in the +x-direction is slowed down by a steady wind in the –x-direction. The problem involves graphical analysis.

EXPRESS Fig. 7-51 represents $x(t)$, the position of the lunch box as a function of time. It is convenient to fit the curve to a concave-downward parabola:

$$x(t) = \frac{1}{10}t(10-t) = t - \frac{1}{10}t^2.$$

By taking one and two derivatives, we find the velocity and acceleration to be

$$v(t) = \frac{dx}{dt} = 1 - \frac{t}{5} \text{ (in m/s)}, \quad a = \frac{d^2x}{dt^2} = -\frac{1}{5} = -0.2 \text{ (in m/s}^2\text{)}.$$

The equations imply that the initial speed of the box is $v_i = v(0) = 1.0\text{ m/s}$, and the constant force by the wind is

$$F = ma = (2.0\text{ kg})(-0.2\text{ m/s}^2) = -0.40\text{ N}.$$

The corresponding work is given by (SI units understood)

$$W(t) = F\cdot x(t) = -0.04t(10-t).$$

The initial kinetic energy of the lunch box is

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(2.0\text{ kg})(1.0\text{ m/s})^2 = 1.0\text{ J}.$$

With $\Delta K = K_f - K_i = W$, the kinetic energy at a later time is given by (in SI units)

$$K(t) = K_i + W = 1 - 0.04t(10-t)$$

ANALYZE (a) When t = 1.0 s, the above expression gives

$$K(1\text{ s}) = 1 - 0.04(1)(10-1) = 1 - 0.36 = 0.64 \approx 0.6\text{ J}$$

where the second significant figure is not to be taken too seriously.

(b) At t = 5.0 s, the above method gives $K(5.0\text{ s}) = 1 - 0.04(5)(10-5) = 1 - 1 = 0.$

(c) The work done by the force from the wind from t = 1.0 s to t = 5.0 s is

$$W = K(5.0) - K(1.0\text{ s}) = 0 - 0.6 \approx -0.6\text{ J}.$$

LEARN The result in (c) can also be obtained by evaluating $W(t) = -0.04t(10-t)$ directly at t = 5.0 s and t = 1.0 s, and subtracting:

$$W(5) - W(1) = -0.04(5)(10-5) - \left[-0.04(1)(10-1)\right] = -1 - (-0.36) = -0.64 \approx -0.6\text{ J}.$$

Note that at t = 5.0 s, K = 0, the box comes to a stop and then reverses its direction subsequently (with x decreasing).

80. The problem indicates that SI units are understood, so the result (of Eq. 7-23) is in joules. Done numerically, using features available on many modern calculators, the result is roughly 0.47 J. For the interested student it might be worthwhile to quote the "exact" answer (in terms of the "error function"):

$$\int_{.15}^{1.2} e^{-2x^2}\,dx = \tfrac{1}{4}\sqrt{2\pi}\,[\text{erf}(6\sqrt{2}/5) - \text{erf}(3\sqrt{2}/20)]\,.$$

81. (a) The work done by the spring force is $W_s = \frac{1}{2}k(x_i^2 - x_f^2)$. By energy conservation, when the block is at $x_f = 0$, the energy stored in the spring is completed converted to the kinetic energy of the block: $W_s = K = \frac{1}{2}mv^2$. Solving for v, we obtain

$$\frac{1}{2}k(x_i^2 - x_f^2) = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{\frac{k}{m}}x_i = \sqrt{\frac{500\text{ N/m}}{4.00\text{ kg}}}\left(0.300\text{ m}\right) = 3.35\text{ m/s}\,.$$

(b) The work done by the spring is

$$W_s = \frac{1}{2}kx_i^2 = \frac{1}{2}(500\text{ N/m})(0.300\text{ m})^2 = 22.5\text{ J}\,.$$

(c) The instantaneous power due to the spring can be written as

$$P = Fv = (kx)\sqrt{\frac{k}{m}\left(x_i^2 - x^2\right)}$$

At the release point x_i, the power is zero.

(d) Similarly, at $x = 0$, we also have $P = 0$.

(e) The position where the power is maximum can be found by differentiating P with respect to x, setting $dP/dx = 0$:

$$\frac{dP}{dx} = \frac{k^2(x_i^2 - 2x^2)}{\sqrt{\dfrac{k(x_i^2 - x^2)}{m}}} = 0$$

which gives $x = \dfrac{x_i}{\sqrt{2}} = \dfrac{(0.300\text{ m})}{\sqrt{2}} = 0.212\text{ m}$.

82. (a) Applying Newton's second law to the x (directed uphill) and y (normal to the inclined plane) axes, we obtain

$$F - mg\sin\theta = ma$$
$$F_N - mg\cos\theta = 0.$$

The second equation allows us to solve for the angle the inclined plane makes with the horizontal:

$$\theta = \cos^{-1}\left(\frac{F_N}{mg}\right) = \cos^{-1}\left(\frac{13.41\text{ N}}{(4.00\text{ kg})(9.8\text{ m/s}^2)}\right) = 70.0°$$

From the equation for the x-axis, we find the acceleration of the block to be

$$a = \frac{F}{m} - g\sin\theta = \frac{50\ \text{N}}{4.00\ \text{kg}} - (9.8\ \text{m/s}^2)\sin 70.0° = 3.29\ \text{m/s}^2$$

Using the kinematic equation $v^2 = v_0^2 + 2ad$, the speed of the block when d = 3.00 m is

$$v = \sqrt{2ad} = \sqrt{2(3.29\ \text{m/s}^2)(3.00\ \text{m})} = 4.44\ \text{m/s}$$

83. (a) The work done by the spring force (with spring constant $k = 18\ \text{N/cm} = 1800\ \text{N/m}$) is

$$W_s = \frac{1}{2}k(x_i^2 - x_f^2) = -\frac{1}{2}kx_f^2 = -\frac{1}{2}(1800\ \text{N/m})(7.60\times10^{-3}\text{m})^2 = -5.20\times10^{-2}\,\text{J}$$

(b) For $x'_f = 2x_f$, the work done by the spring force is $W'_s = -\frac{1}{2}kx'^2_f = -\frac{1}{2}k(2x_f)^2$, so the additional work done is

$$\Delta W = W'_s - W_s = -\frac{1}{2}k(2x_f)^2 - \left(-\frac{1}{2}kx_f^2\right) = -\frac{3}{2}kx_f^2 = 3W_s = 3(-5.20\times10^{-2}\,\text{J}) = -0.156\ \text{J}$$

84. (a) The displacement of the object is

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (-4.10\hat{\text{i}} + 3.30\hat{\text{j}} + 5.40\,\hat{\text{k}}) - (2.70\hat{\text{i}} - 2.90\hat{\text{j}} + 5.50\,\hat{\text{k}}) = (-6.80\hat{\text{i}} + 6.20\hat{\text{j}} - 0.10\,\hat{\text{k}})$$

The work done by $\vec{F} = (2.00\hat{\text{i}} + 9.00\hat{\text{j}} + 5.30\,\hat{\text{k}})\text{N}$ is (in SI units)

$$W = \vec{F}\cdot\Delta\vec{r} = (2.00\hat{\text{i}} + 9.00\hat{\text{j}} + 5.30\,\hat{\text{k}})\cdot(-6.80\hat{\text{i}} + 6.20\hat{\text{j}} - 0.10\,\hat{\text{k}}) = 41.7\ \text{J}$$

(b) The average power due to the force during the time interval is

$$P = \frac{W}{\Delta t} = \frac{41.7\ \text{J}}{2.10\ \text{s}} = 19.8\ \text{W}$$

(c) The magnitudes of the position vectors are (in SI units)

$$r_1 = |\vec{r}_1| = \sqrt{(2.70)^2 + (-2.90)^2 + (5.50)^2} = 6.78$$
$$r_2 = |\vec{r}_2| = \sqrt{(-4.10)^2 + (3.30)^2 + (5.40)^2} = 7.54$$

and their dot product is

$$\vec{r}_1 \cdot \vec{r}_2 = (2.70\hat{\mathrm{i}} - 2.90\hat{\mathrm{j}} + 5.50\,\hat{\mathrm{k}}) \cdot (-4.10\hat{\mathrm{i}} + 3.30\hat{\mathrm{j}} + 5.40\,\hat{\mathrm{k}})$$
$$= (2.70)(-4.10) + (-2.90)(3.30) + (5.50)(5.40) = 9.06$$

Using $\vec{r}_1 \cdot \vec{r}_2 = r_1 r_2 \cos\theta,$, the angle between $\vec{r}_1$ and $\vec{r}_2$ is

$$\theta = \cos^{-1}\left(\frac{\vec{r}_1 \cdot \vec{r}_2}{r_1 r_2}\right) = \cos^{-1}\left(\frac{9.06}{(6.78)(7.54)}\right) = 79.8°$$

85. The work done by the force is (in SI units)

$$W = \vec{F} \cdot \vec{d} = (-5.00\hat{\mathrm{i}} + 5.00\hat{\mathrm{j}} + 4.00\,\hat{\mathrm{k}}) \cdot (2.00\hat{\mathrm{i}} + 2.00\hat{\mathrm{j}} + 7.00\,\hat{\mathrm{k}}) = 28\ \mathrm{J}$$

By energy conservation, $W = \Delta K = \frac{1}{2} m(v_f^2 - v_i^2).$ Thus, the final speed of the particle is

$$v_f = \sqrt{v_i^2 + \frac{2W}{m}} = \sqrt{(4.00\ \mathrm{m/s})^2 + \frac{2(28\ \mathrm{J})}{2.00\ \mathrm{kg}}} = 6.63\ \mathrm{m/s}\ .$$