Chapter 6

1. The greatest deceleration (of magnitude a) is provided by the maximum friction force (Eq. 6-1, with $F_N = mg$ in this case). Using Newton's second law, we find

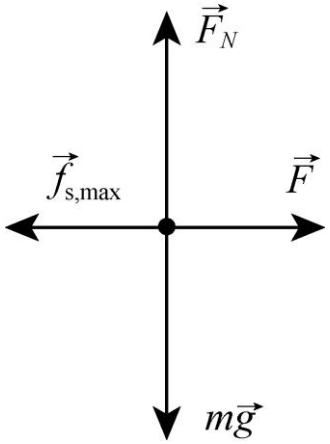
$$a = f_{s,max}/m = \mu_s g.$$

Eq. 2-16 then gives the shortest distance to stop: $|\Delta x| = v^2/2a = 36$ m. In this calculation, it is important to first convert v to 13 m/s.

2. Applying Newton's second law to the horizontal motion, we have $F - \mu_k m g = ma$, where we have used Eq. 6-2, assuming that $F_N = mg$ (which is equivalent to assuming that the vertical force from the broom is negligible). Eq. 2-16 relates the distance traveled and the final speed to the acceleration: $v^2 = 2a\Delta x$. This gives $a = 1.4$ m/s^2. Returning to the force equation, we find (with $F = 25$ N and $m = 3.5$ kg) that $\mu_k = 0.58$.

3. **THINK** In the presence of friction between the floor and the bureau, a minimum horizontal force must be applied before the bureau would begin to move.

EXPRESS The free-body diagram for the bureau is shown to the right. We denote $\vec{F}$ as the horizontal force of the person, $\vec{f}_s$ is the force of static friction (in the $-x$ direction), F_N is the vertical normal force exerted by the floor (in the $+y$ direction), and $m\vec{g}$ is the force of gravity. We do not consider the possibility that the bureau might tip, and treat this as a purely horizontal motion problem (with the person's push $\vec{F}$ in the $+x$ direction). Applying Newton's second law to the x and y axes, we obtain



$$F - f_{s,\,max} = ma$$
$$F_N - mg = 0$$

respectively.
The second equation yields the normal force $F_N = mg$, whereupon the maximum static friction is found to be (from Eq. 6-1) $f_{s,max} = \mu_s mg$. Thus, the first equation becomes

$$F - \mu_s mg = ma = 0$$

where we have set $a = 0$ to be consistent with the fact that the static friction is still (just barely) able to prevent the bureau from moving.
ANALYZE (a) With $\mu_s = 0.45$ and $m = 45$ kg, the equation above leads to

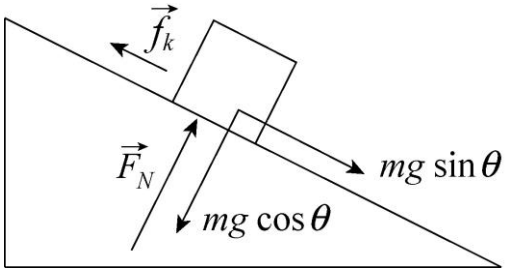
$$F = \mu_s mg = (0.45)(45\,\text{kg})(9.8\,\text{m/s}^2) = 198\,\text{N}.$$

To bring the bureau into a state of motion, the person should push with any force greater than this value. Rounding to two significant figures, we can therefore say the minimum required push is $F = 2.0 \times 10^2$ N.

(b) Replacing m = 45 kg with m = 28 kg, the reasoning above leads to roughly $F = 1.2 \times 10^2$ N.

LEARN The values found above represent the minimum force required to overcome the friction. Applying a force greater than $\mu_s mg$ results in a net force in the +x-direction, and hence, non-zero acceleration.

4. We first analyze the forces on the pig of mass m. The incline angle is θ.



The +x direction is "downhill.'' Application of Newton's second law to the x- and y-axes leads to

$$mg \sin\theta - f_k = ma$$
$$F_N - mg \cos\theta = 0.$$

Solving these along with Eq. 6-2 ($f_k = \mu_k F_N$) produces the following result for the pig's downhill acceleration:

$$a = g(\sin\theta - \mu_k \cos\theta).$$

To compute the time to slide from rest through a downhill distance ℓ, we use Eq. 2-15:

$$\ell = v_0 t + \frac{1}{2}at^2 \quad \Rightarrow \quad t = \sqrt{\frac{2\ell}{a}}.$$

We denote the frictionless ($\mu_k = 0$) case with a prime and set up a ratio:

$$\frac{t}{t'} = \frac{\sqrt{2\ell/a}}{\sqrt{2\ell/a'}} = \sqrt{\frac{a'}{a}}$$

which leads us to conclude that if $t/t' = 2$ then $a' = 4a$. Putting in what we found out above about the accelerations, we have

$$g\sin\theta = 4g\left(\sin\theta - \mu_k\cos\theta\right).$$

Using $\theta = 35°$, we obtain $\mu_k = 0.53$.

5. In addition to the forces already shown in Fig. 6-17, a free-body diagram would include an upward normal force $\vec{F}_N$ exerted by the floor on the block, a downward $m\vec{g}$ representing the gravitational pull exerted by Earth, and an assumed-leftward $\vec{f}$ for the kinetic or static friction. We choose $+x$ rightwards and $+y$ upwards. We apply Newton's second law to these axes:

$$F - f = ma$$
$$P + F_N - mg = 0$$

where F = 6.0 N and m = 2.5 kg is the mass of the block.

(a) In this case, P = 8.0 N leads to

$$F_N = (2.5\text{ kg})(9.8\text{ m/s}^2) - 8.0\text{ N} = 16.5\text{ N}.$$

Using Eq. 6-1, this implies $f_{s,\max} = \mu_s F_N = 6.6$ N, which is larger than the 6.0 N rightward force – so the block (which was initially at rest) does not move. Putting a = 0 into the first of our equations above yields a static friction force of $f = P$ = 6.0 N.

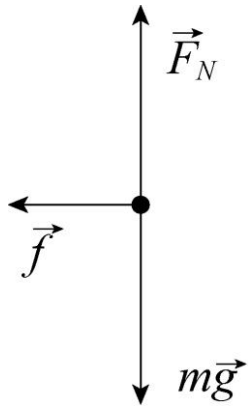
(b) In this case, P = 10 N, the normal force is

$$F_N = (2.5\text{ kg})(9.8\text{ m/s}^2) - 10\text{ N} = 14.5\text{ N}.$$

Using Eq. 6-1, this implies $f_{s,\max} = \mu_s F_N = 5.8$ N, which is less than the 6.0 N rightward force – so the block does move. Hence, we are dealing not with static but with kinetic friction, which Eq. 6-2 reveals to be $f_k = \mu_k F_N = 3.6$ N.

(c) In this last case, P = 12 N leads to F_N = 12.5 N and thus to $f_{s,\max} = \mu_s F_N = 5.0$ N, which (as expected) is less than the 6.0 N rightward force – so the block moves. The kinetic friction force, then, is $f_k = \mu_k F_N = 3.1$ N.

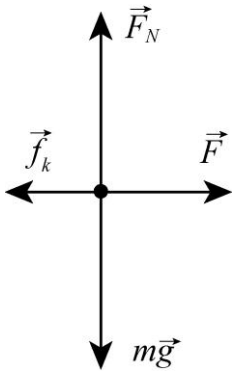
6. The free-body diagram for the player is shown to the right. $\vec{F}_N$ is the normal force of the ground on the player, $m\vec{g}$ is the force of gravity, and $\vec{f}$ is the force of friction. The force of friction is related to the normal force by $f = \mu_k F_N$. We use Newton's second law applied to the vertical axis to find the normal force. The vertical component of the acceleration is zero, so we obtain $F_N - mg = 0$; thus, $F_N = mg$. Consequently,



$$\mu_k = \frac{f}{F_N} = \frac{470\ \text{N}}{(79\ \text{kg})\ (9.8\ \text{m/s}^2)} = 0.61.$$

7. **THINK** A force is being applied to accelerate a crate in the presence of friction. We apply Newton's second law to solve for the acceleration.

EXPRESS The free-body diagram for the crate is shown to the right. We denote $\vec{F}$ as the horizontal force of the person exerted on the crate (in the $+x$ direction), $\vec{f}_k$ is the force of kinetic friction (in the $-x$ direction), F_N is the vertical normal force exerted by the floor (in the $+y$ direction), and $m\vec{g}$ is the force of gravity. The magnitude of the force of friction is given by Eq. 6-2: $f_k = \mu_k F_N$. Applying Newton's second law to the x and y axes, we obtain



$$F - f_k = ma$$
$$F_N - mg = 0$$

respectively.

ANALYZE (a) The second equation above yields the normal force $F_N = mg$, so that the friction is

$$f_k = \mu_k F_N = \mu_k mg = (0.35)(55\ \text{kg})(9.8\ \text{m/s}^2) = 1.9 \times 10^2\ \text{N}.$$

(b) The first equation becomes

$$F - \mu_k mg = ma$$

which (with $F = 220$ N) we solve to find

$$a = \frac{F}{m} - \mu_k g = \frac{220\ \text{N}}{55\ \text{kg}} - (0.35)(9.8\ \text{m/s}^2) = 0.56\ \text{m/s}^2.$$

LEARN For the crate to accelerate, the condition $F > f_k = \mu_k mg$ must be met. As can be seen from the equation above, the greater the value of μ_k, the smaller the acceleration under the same applied force.

8. To maintain the stone's motion, a horizontal force (in the $+x$ direction) is needed that cancels the retarding effect due to kinetic friction. Applying Newton's second to the x and y axes, we obtain

$$F - f_k = ma$$
$$F_N - mg = 0$$

respectively. The second equation yields the normal force $F_N = mg$, so that (using Eq. 6-2) the kinetic friction becomes $f_k = \mu_k mg$. Thus, the first equation becomes

$$F - \mu_k mg = ma = 0$$

where we have set $a = 0$ to be consistent with the idea that the horizontal velocity of the stone should remain constant. With $m = 20$ kg and $\mu_k = 0.80$, we find $F = 1.6 \times 10^2$ N.

9. We choose $+x$ horizontally rightwards and $+y$ upwards and observe that the 15 N force has components $F_x = F \cos\theta$ and $F_y = -F \sin\theta$.

(a) We apply Newton's second law to the y axis:

$$F_N - F\sin\theta - mg = 0 \;\Rightarrow\; F_N = (15\text{ N})\sin 40° + (3.5\text{ kg})(9.8\text{ m/s}^2) = 44\text{ N}.$$

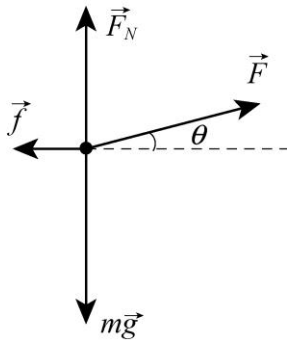
With $\mu_k = 0.25$, Eq. 6-2 leads to $f_k = 11$ N.

(b) We apply Newton's second law to the x axis:

$$F\cos\theta - f_k = ma \;\Rightarrow\; a = \frac{(15\text{ N})\cos 40° - 11\text{ N}}{3.5\text{ kg}} = 0.14\text{ m/s}^2.$$

Since the result is positive-valued, then the block is accelerating in the $+x$ (rightward) direction.

10. (a) The free-body diagram for the block is shown below, with $\vec{F}$ being the force applied to the block, $\vec{F}_N$ the normal force of the floor on the block, $m\vec{g}$ the force of gravity, and $\vec{f}$ the force of friction.



We take the $+x$ direction to be horizontal to the right and the $+y$ direction to be up. The equations for the x and the y components of the force according to Newton's second law are:

$$F_x = F\cos\theta - f = ma$$
$$F_y = F\sin\theta + F_N - mg = 0$$

Now $f = \mu_k F_N$, and the second equation gives $F_N = mg - F\sin\theta$, which yields $f = \mu_k(mg - F\sin\theta)$. This expression is substituted for f in the first equation to obtain

$$F\cos\theta - \mu_k(mg - F\sin\theta) = ma,$$

so the acceleration is

$$a = \frac{F}{m}(\cos\theta + \mu_k \sin\theta) - \mu_k g.$$

(a) If $\mu_s = 0.600$ and $\mu_k = 0.500$, then the magnitude of $\vec{f}$ has a maximum value of

$$f_{s,\max} = \mu_s F_N = (0.600)(mg - 0.500mg\sin 20°) = 0.497mg.$$

On the other hand, $F\cos\theta = 0.500mg\cos 20° = 0.470mg$. Therefore, $F\cos\theta < f_{s,\max}$ and the block remains stationary with $a = 0$.

(b) If $\mu_s = 0.400$ and $\mu_k = 0.300$, then the magnitude of $\vec{f}$ has a maximum value of

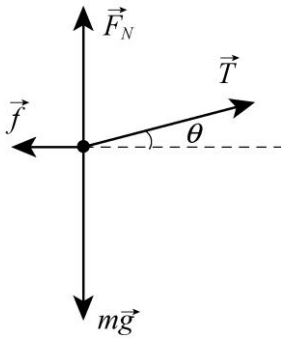
$$f_{s,\max} = \mu_s F_N = (0.400)(mg - 0.500mg\sin 20°) = 0.332mg.$$

In this case, $F\cos\theta = 0.500mg\cos 20° = 0.470mg > f_{s,\max}$. Therefore, the acceleration of the block is

$$\begin{aligned} a &= \frac{F}{m}(\cos\theta + \mu_k \sin\theta) - \mu_k g \\ &= (0.500)(9.80\ \text{m/s}^2)\left[\cos 20° + (0.300)\sin 20°\right] - (0.300)(9.80\ \text{m/s}^2) \\ &= 2.17\ \text{m/s}^2. \end{aligned}$$

11. **THINK** Since the crate is being pulled by a rope at an angle with the horizontal, we need to analyze the force components in both the *x* and *y*-directions.

EXPRESS The free-body diagram for the crate is shown to the right. Here $\vec{T}$ is the tension force of the rope on the crate, $\vec{F}_N$ is the normal force of the floor on the crate, $m\vec{g}$ is the force of gravity, and $\vec{f}$ is the force of friction. We take the $+x$ direction to be horizontal to the right and the $+y$ direction to be up. We assume the crate is motionless.



The equations for the *x* and the *y* components of the force according to Newton's second law are:

$$T\cos\theta - f = 0, \quad T\sin\theta + F_N - mg = 0$$

where $\theta = 15°$ is the angle between the rope and the horizontal. The first equation gives $f = T\cos\theta$ and the second gives $F_N = mg - T\sin\theta$. If the crate is to remain at rest, f must be less than $\mu_s F_N$, or $T\cos\theta < \mu_s(mg - T\sin\theta)$. When the tension force is sufficient to just start the crate moving, we must have $T\cos\theta = \mu_s(mg - T\sin\theta)$.

ANALYZE (a) We solve for the tension:

$$T = \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta} = \frac{(0.50)\,(68\text{ kg})\,(9.8\text{ m/s}^2)}{\cos 15° + 0.50 \sin 15°} = 304\text{ N} \approx 3.0\times10^2\text{ N}.$$

(b) The second law equations for the moving crate are

$$T\cos\theta - f = ma, \quad T\sin\theta + F_N - mg = 0.$$

Now $f = \mu_k F_N$, and the second equation above gives $F_N = mg - T\sin\theta$, which then yields $f = \mu_k(mg - T\sin\theta)$. This expression is substituted for f in the first equation to obtain

$$T\cos\theta - \mu_k(mg - T\sin\theta) = ma,$$

so the acceleration is

$$a = \frac{T(\cos\theta + \mu_k \sin\theta)}{m} - \mu_k g$$
$$= \frac{(304\text{ N})(\cos 15° + 0.35\sin 15°)}{68\text{ kg}} - (0.35)(9.8\text{ m/s}^2) = 1.3\text{ m/s}^2.$$

LEARN Let's check the limit where $\theta = 0$. In this case, we recover the familiar expressions: $T = \mu_s mg$ and $a = (T - \mu_k mg)/m$.

12. There is no acceleration, so the (upward) static friction forces (there are four of them, one for each thumb and one for each set of opposing fingers) equals the magnitude of the (downward) pull of gravity. Using Eq. 6-1, we have

$$4\mu_s F_N = mg = (79\text{ kg})(9.8\text{ m/s}^2)$$

which, with $\mu_s = 0.70$, yields $F_N = 2.8 \times 10^2$ N.

13. We denote the magnitude of 110 N force exerted by the worker on the crate as F. The magnitude of the static frictional force can vary between zero and $f_{s,\max} = \mu_s F_N$.

(a) In this case, application of Newton's second law in the vertical direction yields $F_N = mg$. Thus,

$$f_{s,\max} = \mu_s F_N = \mu_s mg = (0.37)(35\text{kg})(9.8\text{m/s}^2) = 1.3\times10^2\text{ N}$$

which is greater than F.

(b) The block, which is initially at rest, stays at rest since $F < f_{s,\max}$. Thus, it does not move.

(c) By applying Newton's second law to the horizontal direction, that the magnitude of the frictional force exerted on the crate is $f_s = 1.1\times10^2$ N.

(d) Denoting the upward force exerted by the second worker as F_2, then application of Newton's second law in the vertical direction yields $F_N = mg - F_2$, which leads to

$$f_{s,\max} = \mu_s F_N = \mu_s (mg - F_2) .$$

In order to move the crate, F must satisfy the condition $F > f_{s,\max} = \mu_s (mg - F_2)$, or

$$110\,\text{N} > (0.37)\left[(35\,\text{kg})(9.8\,\text{m/s}^2) - F_2\right].$$

The minimum value of F_2 that satisfies this inequality is a value slightly bigger than 45.7 N, so we express our answer as $F_{2,\,\min} = 46$ N.

(e) In this final case, moving the crate requires a greater horizontal push from the worker than static friction (as computed in part (a)) can resist. Thus, Newton's law in the horizontal direction leads to

$$F + F_2 > f_{s,\,\max} \quad \Rightarrow \quad 110\ \text{N} + F_2 > 126.9\ \text{N}$$

which leads (after appropriate rounding) to $F_{2,\,\min} = 17$ N.

14. (a) Using the result obtained in Sample Problem – "Friction, applied force at an angle," the maximum angle for which static friction applies is

$$\theta_{\max} = \tan^{-1}\mu_s = \tan^{-1} 0.63 \approx 32^\circ .$$

This is greater than the dip angle in the problem, so the block does not slide.

(b) Applying Newton's second law, we have

$$F + mg\sin\theta - f_{s,\,\max} = ma = 0$$
$$F_N - mg\cos\theta = 0.$$

Along with Eq. 6-1 ($f_{s,\,\max} = \mu_s F_N$) we have enough information to solve for F. With $\theta = 24^\circ$ and $m = 1.8 \times 10^7$ kg, we find

$$F = mg\left(\mu_s\cos\theta - \sin\theta\right) = 3.0\times10^7\ \text{N}.$$

15. An excellent discussion and equation development related to this problem is given in Sample Problem – "Friction, applied force at an angle." We merely quote (and apply) their main result:

$$\theta = \tan^{-1}\mu_s = \tan^{-1} 0.04 \approx 2^\circ .$$

16. (a) In this situation, we take $\vec{f}_s$ to point uphill and to be equal to its maximum value, in which case $f_{s,\max} = \mu_s F_N$ applies, where $\mu_s = 0.25$. Applying Newton's second law to the block of mass $m = W/g = 8.2$ kg, in the x and y directions, produces

$$F_{\min 1} - mg\sin\theta + f_{s,\max} = ma = 0$$
$$F_N - mg\cos\theta = 0$$

which (with $\theta = 20°$) leads to

$$F_{\min 1} - mg\left(\sin\theta + \mu_s\cos\theta\right) = 8.6 \text{ N}.$$

(b) Now we take $\vec{f}_s$ to point downhill and to be equal to its maximum value, in which case $f_{s,\max} = \mu_s F_N$ applies, where $\mu_s = 0.25$. Applying Newton's second law to the block of mass $m = W/g = 8.2$ kg, in the x and y directions, produces

$$F_{\min 2} = mg\sin\theta - f_{s,\max} = ma = 0$$
$$F_N - mg\cos\theta = 0$$

which (with $\theta = 20°$) leads to

$$F_{\min 2} = mg\left(\sin\theta + \mu_s\cos\theta\right) = 46 \text{ N}.$$

A value slightly larger than the "exact" result of this calculation is required to make it accelerate uphill, but since we quote our results here to two significant figures, 46 N is a "good enough" answer.

(c) Finally, we are dealing with kinetic friction (pointing downhill), so that

$$0 = F - mg\sin\theta - f_k = ma$$
$$0 = F_N - mg\cos\theta$$

along with $f_k = \mu_k F_N$ (where $\mu_k = 0.15$) brings us to

$$F = mg\left(\sin\theta + \mu_k\cos\theta\right) = 39 \text{ N}.$$

17. If the block is sliding then we compute the kinetic friction from Eq. 6-2; if it is not sliding, then we determine the extent of static friction from applying Newton's law, with zero acceleration, to the x axis (which is parallel to the incline surface). The question of whether or not it is sliding is therefore crucial, and depends on the maximum static friction force, as calculated from Eq. 6-1. The forces are resolved in the incline plane coordinate system in Figure 6-5 in the textbook. The acceleration, if there is any, is along the x axis, and we are taking uphill as $+x$. The net force along the y axis, then, is certainly zero, which provides the following relationship:

$$\sum \vec{F}_y = 0 \Rightarrow F_N = W\cos\theta$$

where $W = mg = 45$ N is the weight of the block, and $\theta = 15°$ is the incline angle. Thus, $F_N = 43.5$ N, which implies that the maximum static friction force should be

$$f_{s,\max} = (0.50)\,(43.5\text{ N}) = 21.7\text{ N}.$$

(a) For $\vec{P} = (-5.0\text{ N})\hat{\mathrm{i}}$, Newton's second law, applied to the x axis becomes

$$f - |P| - mg\sin\theta = ma\,.$$

Here we are assuming $\vec{f}$ is pointing uphill, as shown in Figure 6-5, and if it turns out that it points downhill (which *is* a possibility), then the result for f_s will be negative. If $f = f_s$ then $a = 0$, we obtain

$$f_s = |P| + mg\sin\theta = 5.0\text{ N} + (43.5\text{ N})\sin 15° = 17\text{ N},$$

or $\vec{f}_s = (17\text{ N})\hat{\mathrm{i}}$. This is clearly allowed since f_s is less than $f_{s,\max}$.

(b) For $\vec{P} = (-8.0\text{ N})\hat{\mathrm{i}}$, we obtain (from the same equation) $\vec{f}_s = (20\text{ N})\hat{\mathrm{i}}$, which is still allowed since it is less than $f_{s,\max}$.

(c) But for $\vec{P} = (-15\text{ N})\hat{\mathrm{i}}$, we obtain (from the same equation) $f_s = 27$ N, which is not allowed since it is larger than $f_{s,\max}$. Thus, we conclude that it is the kinetic friction instead of the static friction that is relevant in this case. The result is

$$\vec{f}_k = \mu_k F_N \hat{\mathrm{i}} = (0.34)(43.5\text{ N})\,\hat{\mathrm{i}} = (15\text{ N})\,\hat{\mathrm{i}}\,.$$

18. (a) We apply Newton's second law to the "downhill" direction:

$$mg\sin\theta - f = ma,$$

where, using Eq. 6-11,

$$f = f_k = \mu_k F_N = \mu_k mg\cos\theta\,.$$

Thus, with $\mu_k = 0.600$, we have

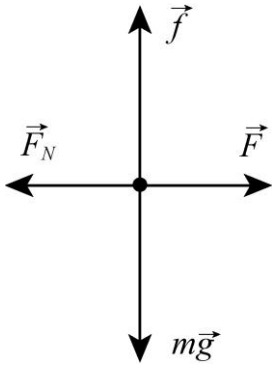
$$a = g\sin\theta - \mu_k\cos\theta = -3.72\text{ m/s}^2$$

which means, since we have chosen the positive direction in the direction of motion (down the slope) then the acceleration vector points "uphill"; it is decelerating. With $v_0 = 18.0$ m/s and $\Delta x = d = 24.0$ m, Eq. 2-16 leads to

$$v = \sqrt{v_0^2 + 2ad} = 12.1 \text{ m/s}.$$

(b) In this case, we find $a = +1.1$ m/s^2, and the speed (when impact occurs) is 19.4 m/s.

19. (a) The free-body diagram for the block is shown below. $\vec{F}$ is the applied force, $\vec{F}_N$ is the normal force of the wall on the block, $\vec{f}$ is the force of friction, and $m\vec{g}$ is the force of gravity. To determine if the block falls, we find the magnitude f of the force of friction required to hold it without accelerating and also find the normal force of the wall on the block. We compare f and $\mu_s F_N$. If $f < \mu_s F_N$, the block does not slide on the wall but if $f > \mu_s F_N$, the block does slide. The horizontal component of Newton's second law is $F - F_N = 0$, so $F_N = F = 12$ N and



$$\mu_s F_N = (0.60)(12 \text{ N}) = 7.2 \text{ N}.$$

The vertical component is $f - mg = 0$, so $f = mg = 5.0$ N. Since $f < \mu_s F_N$ the block does not slide.

(b) Since the block does not move $f = 5.0$ N and $F_N = 12$ N. The force of the wall on the block is

$$\vec{F}_w = -F_N \hat{\text{i}} + f \hat{\text{j}} = -(12\text{N})\hat{\text{i}} + (5.0\text{N})\hat{\text{j}}$$

where the axes are as shown on Fig. 6-26 of the text.

20. Treating the two boxes as a single system of total mass $m_C + m_W = 1.0 + 3.0 = 4.0$ kg, subject to a total (leftward) friction of magnitude 2.0 N + 4.0 N = 6.0 N, we apply Newton's second law (with $+x$ rightward):

$$F - f_{\text{total}} = m_{\text{total}}\, a \quad \Rightarrow \quad 12.0 \text{ N} - 6.0 \text{ N} = (4.0 \text{ kg})a$$

which yields the acceleration $a = 1.5$ m/s^2. We have treated F as if it were known to the nearest tenth of a Newton so that our acceleration is "good" to two significant figures. Turning our attention to the larger box (the Wheaties box of mass $m_W = 3.0$ kg) we apply Newton's second law to find the contact force F' exerted by the Cheerios box on it.

$$F' - f_W = m_W a \quad \Rightarrow \quad F' - 4.0 \text{ N} = (3.0 \text{ kg})(1.5 \text{ m/s}^2).$$

From the above equation, we find the contact force to be $F' = 8.5$ N.

21. Fig. 6-4 in the textbook shows a similar situation (using ϕ for the unknown angle) along with a free-body diagram. We use the same coordinate system as in that figure.

(a) Thus, Newton's second law leads to

$$x: \quad T\cos\phi - f = ma$$
$$y: \; T\sin\phi + F_N - mg = 0$$

Setting $a = 0$ and $f = f_{s,\max} = \mu_s F_N$, we solve for the mass of the box-and-sand (as a function of angle):

$$m = \frac{T}{g}\left(\sin\phi + \frac{\cos\phi}{\mu_s}\right)$$

which we will solve with calculus techniques (to find the angle ϕ_m corresponding to the maximum mass that can be pulled).

$$\frac{dm}{dt} = \frac{T}{g}\left(\cos\phi_m - \frac{\sin\phi_m}{\mu_s}\right) = 0$$

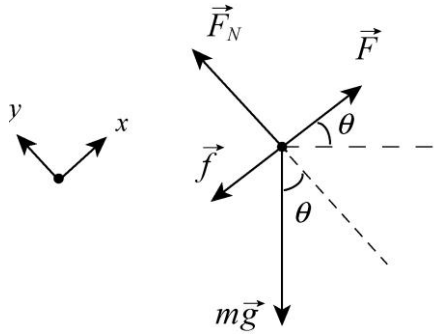
This leads to tan $\phi_m = \mu_s$ which (for $\mu_s = 0.35$) yields $\phi_m = 19°$.

(b) Plugging our value for ϕ_m into the equation we found for the mass of the box-and-sand yields $m = 340$ kg. This corresponds to a weight of $mg = 3.3 \times 10^3$ N.

22. The free-body diagram for the sled is shown below, with $\vec{F}$ being the force applied to the sled, $\vec{F}_N$ the normal force of the inclined plane on the sled, $m\vec{g}$ the force of gravity, and $\vec{f}$ the force of friction.

We take the $+x$ direction to be along the inclined plane and the $+y$ direction to be in its normal direction. The equations for the x and the y components of the force according to Newton's second law are:

$$F_x = F - f - mg\sin\theta = ma = 0$$
$$F_y = F_N - mg\cos\theta = 0$$



Now $f = \mu F_N$, and the second equation gives $F_N = mg\cos\theta$, which yields $f = \mu mg\cos\theta$. This expression is substituted for f in the first equation to obtain

$$F = mg(\sin\theta + \mu\cos\theta)$$

From the figure, we see that $F = 2.0$ N when $\mu = 0$. This implies $mg\sin\theta = 2.0$ N. Similarly, we also find $F = 5.0$ N when $\mu = 0.5$:

$$5.0 \text{ N} = mg(\sin\theta + 0.50\cos\theta) = 2.0 \text{ N} + 0.50mg\cos\theta$$

which yields $mg\cos\theta = 6.0$ N. Combining the two results, we get

$$\tan\theta = \frac{2}{6} = \frac{1}{3} \quad \Rightarrow \quad \theta = 18°.$$

23. Let the tensions on the strings connecting m_2 and m_3 be $T_{23,}$ and that connecting m_2 and m_1 be T_{12}, respectively. Applying Newton's second law (and Eq. 6-2, with $F_N = m_2 g$ in this case) to the *system* we have

$$\begin{aligned} m_3 g - T_{23} &= m_3 a \\ T_{23} - \mu_k m_2 g - T_{12} &= m_2 a \\ T_{12} - m_1 g &= m_1 a \end{aligned}$$

Adding up the three equations and using $m_1 = M, m_2 = m_3 = 2M$, we obtain

$$2Mg - 2\mu_k Mg - Mg = 5Ma\ .$$

With a = 0.500 m/s^2 this yields μ_k = 0.372. Thus, the coefficient of kinetic friction is roughly μ_k = 0.37.

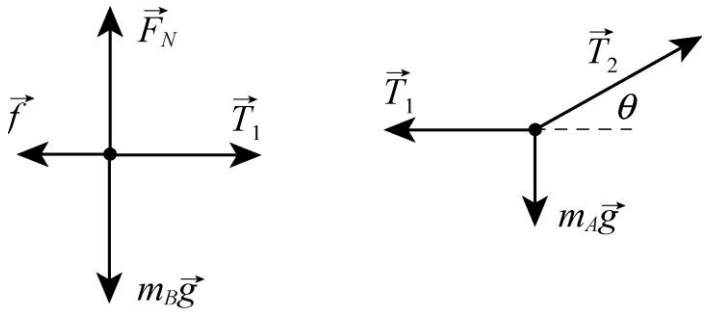
24. We find the acceleration from the slope of the graph (recall Eq. 2-11): a = 4.5 m/s^2. Thus, Newton's second law leads to

$$F - \mu_k mg = ma,$$

where F = 40.0 N is the constant horizontal force applied. With m = 4.1 kg, we arrive at μ_k =0.54.

25. **THINK** In order that the two blocks remain in equilibrium, friction must be present between block *B* and the surface.

EXPRESS The free-body diagrams for block *B* and for the knot just above block *A* are shown below. $\vec{T}_1$ is the tension force of the rope pulling on block *B* or pulling on the knot (as the case may be), $\vec{T}_2$ is the tension force exerted by the second rope (at angle θ = 30°) on the knot, $\vec{f}$ is the force of static friction exerted by the horizontal surface on block *B*, $\vec{F}_N$ is normal force exerted by the surface on block *B*, W_A is the weight of block *A* (W_A is the magnitude of $m_A\vec{g}$), and W_B is the weight of block *B* (W_B = 711 N is the magnitude of $m_B\vec{g}$).



For each object we take $+x$ horizontally rightward and $+y$ upward. Applying Newton's second law in the x and y directions for block B and then doing the same for the knot results in four equations:

$$\begin{aligned} T_1 - f_{s,\max} &= 0 \\ F_N - W_B &= 0 \\ T_2 \cos\theta - T_1 &= 0 \\ T_2 \sin\theta - W_A &= 0 \end{aligned}$$

where we assume the static friction to be at its maximum value (permitting us to use Eq. 6-1). The above equations yield $T_1 = \mu_s F_N$, $F_N = W_B$ and $T_1 = T_2 \cos\theta$.

ANALYZE Solving these equations with $\mu_s = 0.25$, we obtain

$$\begin{aligned} W_A &= T_2 \sin\theta = T_1 \tan\theta = \mu_s F_N \tan\theta = \mu_s W_B \tan\theta \\ &= (0.25)(711\text{ N})\tan 30° = 1.0\times 10^2\text{ N} \end{aligned}$$

LEARN As expected, the maximum weight of A is proportional to the weight of B, as well as the coefficient of static friction. In addition, we see that W_A is proportional to $\tan\theta$ (the larger the angle, the greater the vertical component of T_2 that supports its weight).

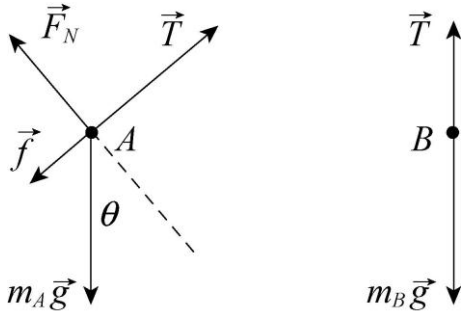
26. (a) Applying Newton's second law to the *system* (of total mass M = 60.0 kg) and using Eq. 6-2 (with $F_N = Mg$ in this case) we obtain

$$F - \mu_k Mg = Ma \;\Rightarrow\; a = 0.473\text{ m/s}^2.$$

Next, we examine the forces just on m_3 and find $F_{32} = m_3(a + \mu_k g) = 147$ N. If the algebra steps are done more systematically, one ends up with the interesting relationship: $F_{32} = (m_3 / M)F$ (which is independent of the friction!).

(b) As remarked at the end of our solution to part (a), the result does not depend on the frictional parameters. The answer here is the same as in part (a).

27. First, we check to see if the bodies start to move. We assume they remain at rest and compute the force of (static) friction which holds them there, and compare its magnitude with the maximum value $\mu_s F_N$. The free-body diagrams are shown below.



T is the magnitude of the tension force of the string, f is the magnitude of the force of friction on body A, F_N is the magnitude of the normal force of the plane on body A, $m_A\vec{g}$ is the force of gravity on body A (with magnitude W_A = 102 N), and $m_B\vec{g}$ is the force of gravity on body B (with magnitude W_B = 32 N). θ = 40° is the angle of incline. We are told the direction of $\vec{f}$ but we assume it is downhill. If we obtain a negative result for f, then we know the force is actually up the plane.

(a) For A we take the $+x$ to be uphill and $+y$ to be in the direction of the normal force. The x and y components of Newton's second law become

$$T - f - W_A \sin\theta = 0$$
$$F_N - W_A \cos\theta = 0.$$

Taking the positive direction to be *downward* for body B, Newton's second law leads to $W_B - T = 0$. Solving these three equations leads to

$$f = W_B - W_A \sin\theta = 32\text{ N} - (102\text{ N})\sin 40° = -34\text{ N}$$

(indicating that the force of friction is *uphill*) and to

$$F_N = W_A \cos\theta = (102\text{ N})\cos 40° = 78\text{N}$$

which means that

$$f_{s,\max} = \mu_s F_N = (0.56)\ (78\text{ N}) = 44\text{ N}.$$

Since the magnitude f of the force of friction that holds the bodies motionless is less than $f_{s,\max}$ the bodies remain at rest. The acceleration is zero.

(b) Since A is moving up the incline, the force of friction is downhill with magnitude $f_k = \mu_k F_N$. Newton's second law, using the same coordinates as in part (a), leads to

$$\begin{aligned} T - f_k - W_A \sin\theta &= m_A a \\ F_N - W_A \cos\theta &= 0 \\ W_B - T &= m_B a \end{aligned}$$

for the two bodies. We solve for the acceleration:

$$a = \frac{W_B - W_A \sin\theta - \mu_k W_A \cos\theta}{m_B + m_A} = \frac{32\text{N} - (102\text{N})\sin 40° - (0.25)(102\text{N})\cos 40°}{(32\text{N} + 102\text{N}) / (9.8\ \text{m/s}^2)}$$

$$= -3.9\ \text{m/s}^2.$$

The acceleration is down the plane, i.e., $\vec{a} = (-3.9\ \text{m/s}^2)\hat{\text{i}}$, which is to say (since the initial velocity was uphill) that the objects are slowing down. We note that $m = W/g$ has been used to calculate the masses in the calculation above.

(c) Now body *A* is initially moving down the plane, so the force of friction is uphill with magnitude $f_k = \mu_k F_N$. The force equations become

$$\begin{aligned} T + f_k - W_A \sin\theta &= m_A a \\ F_N - W_A \cos\theta &= 0 \\ W_B - T &= m_B a \end{aligned}$$

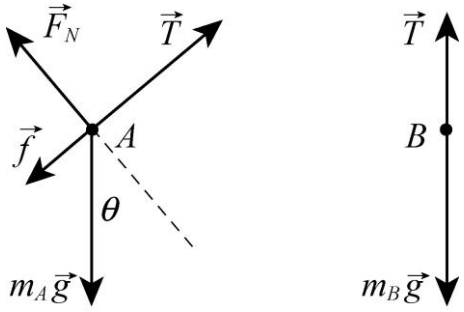
which we solve to obtain

$$a = \frac{W_B - W_A \sin\theta + \mu_k W_A \cos\theta}{m_B + m_A} = \frac{32\text{N} - (102\text{N})\sin 40° + (0.25)(102\text{N})\cos 40°}{(32\text{N} + 102\text{N}) / (9.8\ \text{m/s}^2)}$$

$$= -1.0\ \text{m/s}^2.$$

The acceleration is again downhill the plane, i.e., $\vec{a} = (-1.0\ \text{m/s}^2)\hat{\text{i}}$. In this case, the objects are speeding up.

28. The free-body diagrams are shown to the right, where *T* is the magnitude of the tension force of the string, *f* is the magnitude of the force of friction on block *A*, F_N is the magnitude of the normal force of the plane on block *A*, $m_A\vec{g}$ is the force of gravity on body *A* (where m_A = 10 kg), and $m_B\vec{g}$ is the force of gravity on block *B*. θ = 30° is the angle of incline. For *A* we take the +*x* to be uphill and +*y* to be in the direction of the normal force; the positive direction is chosen *downward* for block *B*.



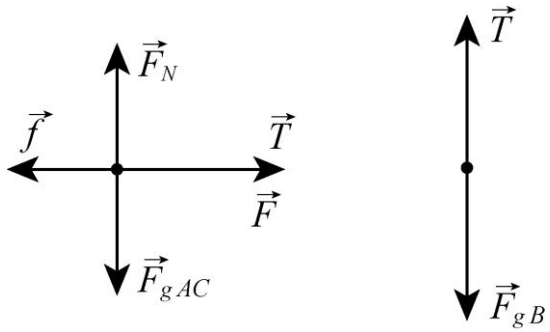
Since *A* is moving down the incline, the force of friction is uphill with magnitude $f_k = \mu_k F_N$ (where $\mu_k = 0.20$). Newton's second law leads to

$$T - f_k + m_A g \sin\theta = m_A a = 0$$
$$F_N - m_A g \cos\theta = 0$$
$$m_B g - T = m_B a = 0$$

for the two bodies (where $a = 0$ is a consequence of the velocity being constant). We solve these for the mass of block *B*.

$$m_B = m_A\left(\sin\theta - \mu_k \cos\theta\right) = 3.3 \text{ kg}.$$

29. (a) Free-body diagrams for the blocks *A* and *C*, considered as a single object, and for the block *B* are shown below.



T is the magnitude of the tension force of the rope, F_N is the magnitude of the normal force of the table on block *A*, f is the magnitude of the force of friction, W_{AC} is the combined weight of blocks *A* and *C* (the magnitude of force $\vec{F}_{g\,AC}$ shown in the figure), and W_B is the weight of block *B* (the magnitude of force $\vec{F}_{g\,B}$ shown). Assume the blocks are not moving. For the blocks on the table we take the *x* axis to be to the right and the *y* axis to be upward. From Newton's second law, we have

$$x \text{ component:} \qquad T - f = 0$$
$$y \text{ component:} \qquad F_N - W_{AC} = 0.$$

For block *B* take the downward direction to be positive. Then Newton's second law for that block is $W_B - T = 0$. The third equation gives $T = W_B$ and the first gives $f = T = W_B$. The second equation gives $F_N = W_{AC}$. If sliding is not to occur, f must be less than $\mu_s F_N$, or $W_B < \mu_s W_{AC}$. The smallest that W_{AC} can be with the blocks still at rest is

$$W_{AC} = W_B/\mu_s = (22 \text{ N})/(0.20) = 110 \text{ N}.$$

Since the weight of block *A* is 44 N, the least weight for *C* is (110 – 44) N = 66 N.

(b) The second law equations become

$$T - f = (W_A/g)a$$

$$F_N - W_A = 0$$
$$W_B - T = (W_B/g)a.$$

In addition, $f = \mu_k F_N$. The second equation gives $F_N = W_A$, so $f = \mu_k W_A$. The third gives $T = W_B - (W_B/g)a$. Substituting these two expressions into the first equation, we obtain

$$W_B - (W_B/g)a - \mu_k W_A = (W_A/g)a.$$

Therefore,

$$a = \frac{g\left(W_B - \mu_k W_A\right)}{W_A + W_B} = \frac{(9.8\ \text{m/s}^2)\left(22\ \text{N} - (0.15)(44\ \text{N})\right)}{44\ \text{N} + 22\ \text{N}} = 2.3\ \text{m/s}^2.$$

30. We use the familiar horizontal and vertical axes for x and y directions, with rightward and upward positive, respectively. The rope is assumed massless so that the force exerted by the child $\vec{F}$ is identical to the tension uniformly through the rope. The x and y components of $\vec{F}$ are $F\cos\theta$ and $F\sin\theta$, respectively. The static friction force points leftward.

(a) Newton's Law applied to the y-axis, where there is presumed to be no acceleration, leads to

$$F_N + F\sin\theta - mg = 0$$

which implies that the maximum static friction is $\mu_s(mg - F\sin\theta)$. If $f_s = f_{s,\,\max}$ is assumed, then Newton's second law applied to the x axis (which also has $a = 0$ even though it is "verging" on moving) yields

$$F\cos\theta - f_s = ma \quad \Rightarrow \quad F\cos\theta - \mu_s(mg - F\sin\theta) = 0$$

which we solve, for $\theta = 42°$ and $\mu_s = 0.42$, to obtain $F = 74$ N.

(b) Solving the above equation algebraically for F, with W denoting the weight, we obtain

$$F = \frac{\mu_s W}{\cos\theta + \mu_s \sin\theta} = \frac{(0.42)(180\ \text{N})}{\cos\theta + (0.42)\sin\theta} = \frac{76\ \text{N}}{\cos\theta + (0.42)\sin\theta}.$$

(c) We minimize the above expression for F by working through the condition:

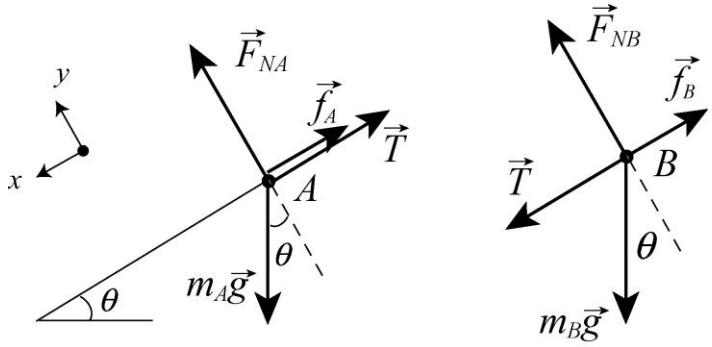
$$\frac{dF}{d\theta} = \frac{\mu_s W(\sin\theta - \mu_s\cos\theta)}{(\cos\theta + \mu_s \sin\theta)^2} = 0$$

which leads to the result $\theta = \tan^{-1}\mu_s = 23°$.

(d) Plugging $\theta = 23°$ into the above result for F, with $\mu_s = 0.42$ and $W = 180$ N, yields $F = 70$ N.

31. **THINK** In this problem we have two blocks connected by a string sliding down an inclined plane; the blocks have different coefficient of kinetic friction.

EXPRESS The free-body diagrams for the two blocks are shown below. T is the magnitude of the tension force of the string, $\vec{F}_{NA}$ is the normal force on block A (the leading block), $\vec{F}_{NB}$ is the normal force on block B, $\vec{f}_A$ is kinetic friction force on block A, $\vec{f}_B$ is kinetic friction force on block B. Also, m_A is the mass of block A (where $m_A = W_A/g$ and $W_A = 3.6$ N), and m_B is the mass of block B (where $m_B = W_B/g$ and $W_B = 7.2$ N). The angle of the incline is $\theta = 30°$.



For each block we take $+x$ downhill (which is toward the lower-left in these diagrams) and $+y$ in the direction of the normal force. Applying Newton's second law to the x and y directions of both blocks A and B, we arrive at four equations:

$$W_A \sin\theta - f_A - T = m_A a$$
$$F_{NA} - W_A \cos\theta = 0$$
$$W_B \sin\theta - f_B + T = m_B a$$
$$F_{NB} - W_B \cos\theta = 0$$

which, when combined with Eq. 6-2 ($f_A = \mu_{kA} F_{NA}$ where $\mu_{k\,A}$ = 0.10 and $f_B = \mu_{kB} F_{NB} f_B$ where $\mu_{k\,B}$ = 0.20), fully describe the dynamics of the system so long as the blocks have the same acceleration and $T > 0$.

ANALYZE (a) From these equations, we find the acceleration to be

$$a = g\left(\sin\theta - \left(\frac{\mu_{kA} W_A + \mu_{kB} W_B}{W_A + W_B}\right)\cos\theta\right) = 3.5 \text{ m/s}^2.$$

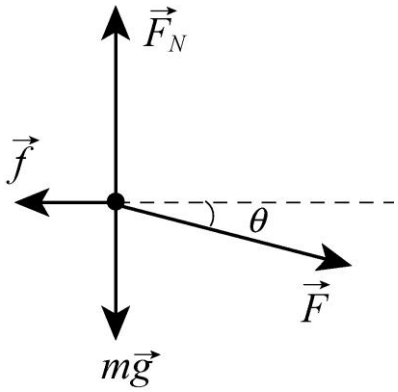
(b) We solve the above equations for the tension and obtain

$$T = \left(\frac{W_A W_B}{W_A + W_B}\right)(\mu_{kB} - \mu_{kA})\cos\theta = \frac{(3.6\text{ N})(7.2\text{ N})}{3.6\text{ N} + 7.2\text{ N}}(0.20 - 0.10)\cos 30° = 0.21\text{ N}.$$

LEARN The tension in the string is proportional to $\mu_{kB} - \mu_{kA}$, the difference in coefficients of kinetic friction for the two blocks. When the coefficients are equal ($\mu_{kB} = \mu_{kA}$), the two blocks can be viewed as moving independent of one another and the tension is zero. Similarly, when $\mu_{kB} < \mu_{kA}$ (the leading block *A* has larger coefficient than the *B*), the string is slack, so the tension is also zero.

32. The free-body diagram for the block is shown below, with $\vec{F}$ being the force applied to the block, $\vec{F}_N$ the normal force of the floor on the block, $m\vec{g}$ the force of gravity, and $\vec{f}$ the force of friction. We take the $+x$ direction to be horizontal to the right and the $+y$ direction to be up. The equations for the x and the y components of the force according to Newton's second law are:

$$F_x = F\cos\theta - f = ma$$
$$F_y = F_N - F\sin\theta - mg = 0$$



Now $f = \mu_k F_N$, and the second equation gives $F_N = mg + F\sin\theta$, which yields

$$f = \mu_k(mg + F\sin\theta).$$

This expression is substituted for f in the first equation to obtain

$$F\cos\theta - \mu_k(mg + F\sin\theta) = ma,$$

so the acceleration is

$$a = \frac{F}{m}(\cos\theta - \mu_k\sin\theta) - \mu_k g.$$

From the figure, we see that $a = 3.0\ \text{m/s}^2$ when $\mu_k = 0$. This implies

$$3.0\ \text{m/s}^2 = \frac{F}{m}\cos\theta.$$

We also find $a = 0$ when $\mu_k = 0.20$:

$$0 = \frac{F}{m}(\cos\theta - (0.20)\sin\theta) - (0.20)(9.8\ \text{m/s}^2) = 3.00\ \text{m/s}^2 - 0.20\frac{F}{m}\sin\theta - 1.96\ \text{m/s}^2$$
$$= 1.04\ \text{m/s}^2 - 0.20\frac{F}{m}\sin\theta$$

which yields $5.2\ \text{m/s}^2 = \frac{F}{m}\sin\theta$. Combining the two results, we get

$$\tan\theta = \left(\frac{5.2\ \text{m/s}^2}{3.0\ \text{m/s}^2}\right) = 1.73 \ \Rightarrow \ \theta = 60°.$$

33. **THINK** In this problem, the frictional force is not a constant, but instead proportional to the speed of the boat. Integration is required to solve for the speed.

EXPRESS We denote the magnitude of the frictional force as αv, where $\alpha = 70\ \text{N}\cdot\text{s/m}$. We take the direction of the boat's motion to be positive. Newton's second law gives

$$-\alpha v = m\frac{dv}{dt} \ \Rightarrow \ \frac{dv}{v} = -\frac{\alpha}{m}dt.$$

Integrating the equation gives

$$\int_{v_0}^{v}\frac{dv}{v} = -\frac{\alpha}{m}\int_0^t dt$$

where v_0 is the velocity at time zero and v is the velocity at time t. Solving the integral allows us to calculate the time it takes for the boat to slow down to 45 km/h, or $v = v_0/2$, where $v_0 = 90\ \text{km/h}$.

ANALYZE The integrals are evaluated with the result

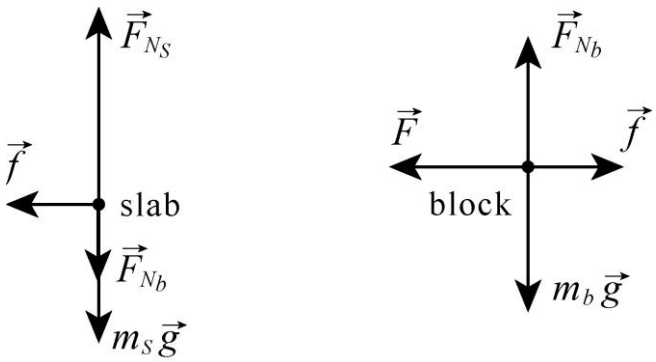
$$\ln\left(\frac{v}{v_0}\right) = -\frac{\alpha t}{m}$$

With $v = v_0/2$ and $m = 1000$ kg, we find the time to be

$$t = -\frac{m}{\alpha}\ln\left(\frac{v}{v_0}\right) = -\frac{m}{\alpha}\ln\left(\frac{1}{2}\right) = -\frac{1000\ \text{kg}}{70\ \text{N}\cdot\text{s/m}}\ln\left(\frac{1}{2}\right) = 9.9\ \text{s}.$$

LEARN The speed of the boat is given by $v(t) = v_0 e^{-\alpha t/m}$, showing exponential decay with time. The greater the value of α, the more rapidly the boat slows down.

34. The free-body diagrams for the slab and block are shown below.



$\vec{F}$ is the 100 N force applied to the block, $\vec{F}_{Ns}$ is the normal force of the floor on the slab, F_{Nb} is the magnitude of the normal force between the slab and the block, $\vec{f}$ is the force of friction between the slab and the block, m_s is the mass of the slab, and m_b is the mass of the block. For both objects, we take the $+x$ direction to be to the right and the $+y$ direction to be up.

Applying Newton's second law for the x and y axes for (first) the slab and (second) the block results in four equations:

$$\begin{aligned} -f &= m_s a_s \\ F_{Ns} - F_{Ns} - m_s g &= 0 \\ f - F &= m_b a_b \\ F_{Nb} - m_b g &= 0 \end{aligned}$$

from which we note that the maximum possible static friction magnitude would be

$$\mu_s F_{Nb} = \mu_s m_b g = (0.60)(10\text{ kg})(9.8\text{ m/s}^2) = 59\text{ N}\ .$$

We check to see if the block slides on the slab. Assuming it does not, then $a_s = a_b$ (which we denote simply as a) and we solve for f:

$$f = \frac{m_s F}{m_s + m_b} = \frac{(40\text{ kg})(100\text{ N})}{40\text{ kg} + 10\text{ kg}} = 80\text{ N}$$

which is greater than $f_{s,\max}$ so that we conclude the block is sliding across the slab (their accelerations are different).

(a) Using $f = \mu_k F_{Nb}$ the above equations yield

$$a_b = \frac{\mu_k m_b g - F}{m_b} = \frac{(0.40)(10\text{ kg})(9.8\text{ m/s}^2) - 100\text{ N}}{10\text{ kg}} = -6.1\text{ m/s}^2.$$

The negative sign means that the acceleration is leftward. That is, $\vec{a}_b = (-6.1\text{ m/s}^2)\hat{\text{i}}$

(b) We also obtain

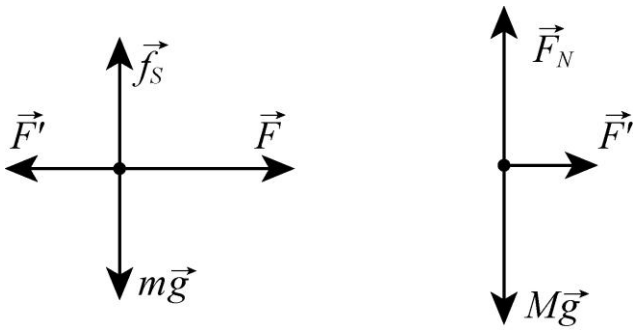
$$a_s = -\frac{\mu_k m_b g}{m_s} = -\frac{(0.40)(10\text{ kg})(9.8\text{ m/s}^2)}{40\text{ kg}} = -0.98\text{ m/s}^2.$$

As mentioned above, this means it accelerates to the left. That is, $\vec{a}_s = (-0.98\text{ m/s}^2)\hat{\text{i}}$

35. The free-body diagrams for the two blocks, treated individually, are shown below (first *m* and then *M*). *F′* is the contact force between the two blocks, and the static friction force $\vec{f}_s$ is at its maximum value (so Eq. 6-1 leads to $f_s = f_{s,\max} = \mu_s F'$ where $\mu_s = 0.38$).

Treating the two blocks together as a single system (sliding across a frictionless floor), we apply Newton's second law (with +*x* rightward) to find an expression for the acceleration:

$$F = m_{\text{total}}\, a \quad \Rightarrow a = \frac{F}{m+M}$$



This is equivalent to having analyzed the two blocks individually and then combined their equations. Now, when we analyze the small block individually, we apply Newton's second law to the *x* and *y* axes, substitute in the above expression for *a*, and use Eq. 6-1.

$$F - F' = ma \quad \Rightarrow \quad F' = F - m\left(\frac{F}{m+M}\right)$$
$$f_s - mg = 0 \quad \Rightarrow \quad \mu_s F' - mg = 0$$

These expressions are combined (to eliminate *F′*) and we arrive at

$$F = \frac{mg}{\mu_s\left(1 - \dfrac{m}{m+M}\right)} = 4.9\times10^2 \text{ N}.$$

36. Using Eq. 6-16, we solve for the area $A\dfrac{2m\,g}{C\rho\, v_t^2}$ which illustrates the inverse proportionality between the area and the speed-squared. Thus, when we set up a ratio of areas – of the slower case to the faster case – we obtain

$$\frac{A_{\text{slow}}}{A_{\text{fast}}} = \left(\frac{310 \text{ km/h}}{160 \text{ km/h}}\right)^2 = 3.75.$$

37. In the solution to exercise 4, we found that the force provided by the wind needed to equal F = 157 N (where that last figure is not "significant'').

(a) Setting $F = D$ (for Drag force) we use Eq. 6-14 to find the wind speed v along the ground (which actually is relative to the moving stone, but we assume the stone is moving slowly enough that this does not invalidate the result):

$$v = \sqrt{\frac{2F}{C\rho A}} = \sqrt{\frac{2(157\text{ N})}{(0.80)(1.21\text{ kg/m}^3)(0.040\text{ m}^2)}} = 90\text{ m/s} = 3.2\times10^2\text{ km/h.}$$

(b) Doubling our previous result, we find the reported speed to be 6.5×10^2 km/h.

(c) The result is not reasonable for a terrestrial storm. A category 5 hurricane has speeds on the order of 2.6×10^2 m/s.

38. (a) From Table 6-1 and Eq. 6-16, we have

$$v_t = \sqrt{\frac{2F_g}{C\rho A}} \Rightarrow C\rho A = 2\frac{mg}{v_t^2}$$

where v_t = 60 m/s. We estimate the pilot's mass at about m = 70 kg. Now, we convert v = 1300(1000/3600) ≈ 360 m/s and plug into Eq. 6-14:

$$D = \frac{1}{2}C\rho A v^2 = \frac{1}{2}\left(2\frac{mg}{v_t^2}\right)v^2 = mg\left(\frac{v}{v_t}\right)^2$$

which yields D = (70 kg)(9.8 m/s^2)(360/60)^2 ≈ 2×10^4 N.

(b) We assume the mass of the ejection seat is roughly equal to the mass of the pilot. Thus, Newton's second law (in the horizontal direction) applied to this system of mass 2m gives the magnitude of acceleration:

$$|a| = \frac{D}{2m} = \frac{g}{2}\left(\frac{v}{v_t}\right)^2 = 18g\ .$$

39. For the passenger jet $D_j = \frac{1}{2}C\rho_1 A v_j^2$, and for the prop-driven transport $D_t = \frac{1}{2}C\rho_2 A v_t^2$, where ρ_1 and ρ_2 represent the air density at 10 km and 5.0 km, respectively. Thus the ratio in question is

$$\frac{D_j}{D_t} = \frac{\rho_1 v_j^2}{\rho_2 v_t^2} = \frac{\left(0.38\text{ kg/m}^3\right)\left(1000\text{ km/h}\right)^2}{\left(0.67\text{ kg/m}^3\right)\left(500\text{ km/h}\right)^2} = 2.3.$$

40. This problem involves Newton's second law for motion along the slope.

(a) The force along the slope is given by

$$\begin{aligned} F_g &= mg\sin\theta - \mu F_N = mg\sin\theta - \mu mg\cos\theta = mg(\sin\theta - \mu\cos\theta) \\ &= (85.0\text{ kg})(9.80\text{ m/s}^2)\left[\sin 40.0^\circ - (0.04000)\cos 40.0^\circ\right] \\ &= 510\text{ N}. \end{aligned}$$

Thus, the terminal speed of the skier is

$$v_t = \sqrt{\frac{2F_g}{C\rho A}} = \sqrt{\frac{2(510\text{ N})}{(0.150)(1.20\text{ kg/m}^3)(1.30\text{ m}^2)}} = 66.0\text{ m/s}.$$

(b) Differentiating v_t with respect to C, we obtain

$$\begin{aligned} dv_t &= -\frac{1}{2}\sqrt{\frac{2F_g}{\rho A}}\,C^{-3/2}dC = -\frac{1}{2}\sqrt{\frac{2(510\text{ N})}{(1.20\text{ kg/m}^3)(1.30\text{ m}^2)}}(0.150)^{-3/2}dC \\ &= -(2.20\times 10^2\text{ m/s})dC. \end{aligned}$$

41. Perhaps surprisingly, the equations pertaining to this situation are exactly those in Sample Problem – "Car in flat circular turn," although the logic is a little different. In the Sample Problem, the car moves along a (stationary) road, whereas in this problem the cat is stationary relative to the merry-go-around platform. But the static friction plays the same role in both cases since the bottom-most point of the car tire is instantaneously at rest with respect to the race track, just as static friction applies to the contact surface between cat and platform. Using Eq. 6-23 with Eq. 4-35, we find

$$\mu_s = (2\pi R/T)^2/gR = 4\pi^2 R/gT^2.$$

With $T = 6.0$ s and $R = 5.4$ m, we obtain $\mu_s = 0.60$.

42. The magnitude of the acceleration of the car as it rounds the curve is given by v^2/R, where v is the speed of the car and R is the radius of the curve. Since the road is horizontal, only the frictional force of the road on the tires makes this acceleration possible. The horizontal component of Newton's second law is $f = mv^2/R$. If F_N is the normal force of the road on the car and m is the mass of the car, the vertical component of Newton's second law leads to $F_N = mg$. Thus, using Eq. 6-1, the maximum value of static friction is

$$f_{s,\max} = \mu_s F_N = \mu_s mg.$$

If the car does not slip, $f \le \mu_s mg$. This means

$$\frac{v^2}{R} \le \mu_s g \quad \Rightarrow \quad v \le \sqrt{\mu_s R g}.$$

Consequently, the maximum speed with which the car can round the curve without slipping is

$$v_{\max} = \sqrt{\mu_s R g} = \sqrt{(0.60)(30.5\text{ m})(9.8\text{ m/s}^2)} = 13\text{ m/s} \approx 48\text{ km/h}.$$

43. The magnitude of the acceleration of the cyclist as it rounds the curve is given by v^2/R, where v is the speed of the cyclist and R is the radius of the curve. Since the road is horizontal, only the frictional force of the road on the tires makes this acceleration possible. The horizontal component of Newton's second law is $f = mv^2/R$. If F_N is the normal force of the road on the bicycle and m is the mass of the bicycle and rider, the vertical component of Newton's second law leads to $F_N = mg$. Thus, using Eq. 6-1, the maximum value of static friction is

$$f_{s,\max} = \mu_s F_N = \mu_s mg.$$

If the bicycle does not slip, $f \le \mu_s mg$. This means

$$\frac{v^2}{R} \le \mu_s g \quad \Rightarrow \quad R \ge \frac{v^2}{\mu_s g}.$$

Consequently, the minimum radius with which a cyclist moving at 29 km/h = 8.1 m/s can round the curve without slipping is

$$R_{\min} = \frac{v^2}{\mu_s g} = \frac{(8.1\text{ m/s})^2}{(0.32)(9.8\text{ m/s}^2)} = 21\text{ m}.$$

44. With v = 96.6 km/h = 26.8 m/s, Eq. 6-17 readily yields

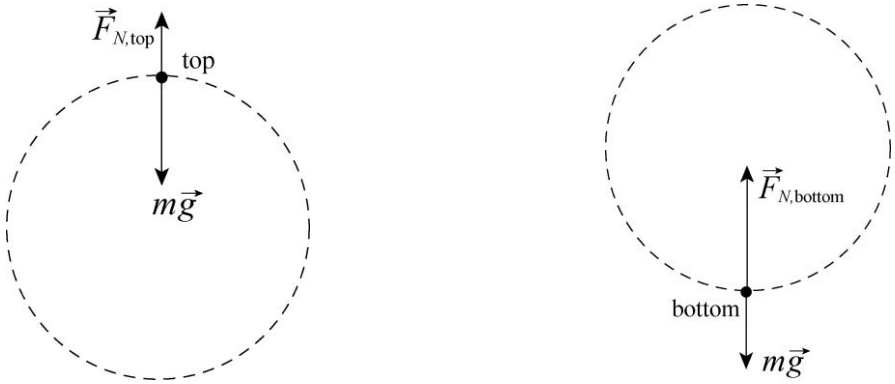
$$a = \frac{v^2}{R} = \frac{(26.8\text{ m/s})^2}{7.6\text{ m}} = 94.7\text{ m/s}^2$$

which we express as a multiple of g:

$$a = \left(\frac{a}{g}\right) g = \left(\frac{94.7\text{ m/s}^2}{9.80\text{ m/s}^2}\right) g = 9.7g.$$

45. **THINK** Ferris wheel ride is a vertical circular motion. The apparent weight of the rider varies with his position.

EXPRESS The free-body diagrams of the student at the top and bottom of the Ferris wheel are shown next:



At the top (the highest point in the circular motion) the seat pushes up on the student with a force of magnitude $F_{N,\text{top}}$, while the Earth pulls down with a force of magnitude mg. Newton's second law for the radial direction gives

$$mg - F_{N,\text{top}} = \frac{mv^2}{R}.$$

At the bottom of the ride, $F_{N,\text{bottom}}$ is the magnitude of the upward force exerted by the seat. The net force toward the center of the circle is (choosing upward as the positive direction):

$$F_{N,\text{bottom}} - mg = \frac{mv^2}{R}.$$

The Ferris wheel is "steadily rotating" so the value $F_c = mv^2/R$ is the same everywhere. The apparent weight of the student is given by F_N.

ANALYZE (a) At the top, we are told that $F_{N,\text{top}}$ = 556 N and mg = 667 N. This means that the seat is pushing up with a force that is smaller than the student's weight, and we say the student experiences a decrease in his "apparent weight" at the highest point. Thus, he feels "light."

(b) From (a), we find the centripetal force to be

$$F_c = \frac{mv^2}{R} = mg - F_{N,\text{top}} = 667\text{ N} - 556\text{ N} = 111\text{ N}.$$

Thus, the normal force at the bottom is

$$F_{N,\text{bottom}} = \frac{mv^2}{R} + mg = F_c + mg = 111\text{ N} + 667\text{ N} = 778\text{ N}.$$

(c) If the speed is doubled,

$$F_c' = \frac{m(2v)^2}{R} = 4(111\text{ N}) = 444\text{ N}.$$

Therefore, at the highest point we have

$$F'_{N,\text{top}} = mg - F'_c = 667\text{ N} - 444\text{ N} = 223\text{ N}.$$

(d) Similarly, the normal force at the lowest point is now found to be

$$F'_{N,\text{bottom}} = F'_c + mg = 444\text{ N} + 667\text{ N} = 1111\text{ N}.$$

LEARN The apparent weight of the student is the greatest at the bottom and smallest at the top of the ride. The speed $v = \sqrt{gR}$ would result in $F_{N,\text{top}} = 0$, giving the student a sudden sensation of "weightlessness" at the top of the ride.

46. (a) We note that the speed 80.0 km/h in SI units is roughly 22.2 m/s. The horizontal force that keeps her from sliding must equal the centripetal force (Eq. 6-18), and the upward force on her must equal *mg*. Thus,

$$F_{\text{net}} = \sqrt{(mg)^2 + (mv^2/R)^2} = 547\text{ N}.$$

(b) The angle is

$$\tan^{-1}[(mv^2/R)/(mg)] = \tan^{-1}(v^2/gR) = 9.53^\circ$$

as measured from a vertical axis.

47. (a) Eq. 4-35 gives $T = 2\pi R/v = 2\pi(10\text{ m})/(6.1\text{ m/s}) = 10$ s.

(b) The situation is similar to that of Sample Problem – "Vertical circular loop, Diavolo," but with the normal force direction reversed. Adapting Eq. 6-19, we find

$$F_N = m(g - v^2/R) = 486\text{ N} \approx 4.9 \times 10^2\text{ N}.$$

(c) Now we reverse both the normal force direction and the acceleration direction (from what is shown in Sample Problem – "Vertical circular loop, Diavolo") and adapt Eq. 6-19 accordingly. Thus we obtain

$$F_N = m(g + v^2/R) = 1081\text{ N} \approx 1.1\text{ kN}.$$

48. We will start by assuming that the normal force (on the car from the rail) points up. Note that gravity points down, and the *y* axis is chosen positive upwards. Also, the direction to the center of the circle (the direction of centripetal acceleration) is down. Thus, Newton's second law leads to

$$F_N - mg = m\left(-\frac{v^2}{r}\right).$$

(a) When $v = 11$ m/s, we obtain $F_N = 3.7 \times 10^3$ N.

(b) $\vec{F}_N$ points upward.

(c) When v = 14 m/s, we obtain $F_N = -1.3 \times 10^3$ N, or $|F_N| = 1.3 \times 10^3$ N.

(d) The fact that this answer is negative means that $\vec{F}_N$ points opposite to what we had assumed. Thus, the magnitude of $\vec{F}_N$ is $|\vec{F}_N| = 1.3$ kN and its direction is *down*.

49. At the top of the hill, the situation is similar to that of Sample Problem – "Vertical circular loop, Diavolo," but with the normal force direction reversed. Adapting Eq. 6-19, we find

$$F_N = m(g - v^2/R).$$

Since $F_N = 0$ there (as stated in the problem) then $v^2 = gR$. Later, at the bottom of the valley, we reverse both the normal force direction and the acceleration direction (from what is shown in the Sample Problem) and adapt Eq. 6-19 accordingly. Thus we obtain

$$F_N = m(g + v^2/R) = 2mg = 1372 \text{ N} \approx 1.37 \times 10^3 \text{ N}.$$

50. The centripetal force on the passenger is $F = mv^2 / r$.

(a) The slope of the plot at $v = 8.30$ m/s is

$$\left.\frac{dF}{dv}\right|_{v=8.30\text{ m/s}} = \left.\frac{2mv}{r}\right|_{v=8.30\text{ m/s}} = \frac{2(85.0\text{ kg})(8.30\text{ m/s})}{3.50\text{ m}} = 403\text{ N}\cdot\text{s/m}.$$

(b) The period of the circular ride is $T = 2\pi r / v$. Thus,

$$F = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 mr}{T^2},$$

and the variation of F with respect to T while holding r constant is

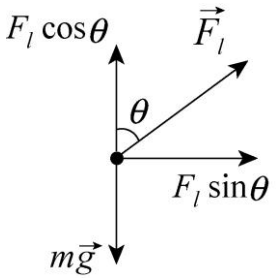
$$dF = -\frac{8\pi^2 mr}{T^3}\,dT.$$

The slope of the plot at $T = 2.50$ s is

$$\left.\frac{dF}{dT}\right|_{T=2.50\text{ s}} = -\left.\frac{8\pi^2 mr}{T^3}\right|_{T=2.50\text{ s}} = \frac{8\pi^2(85.0\text{ kg})(3.50\text{ m})}{(2.50\text{ s})^3} = -1.50\times10^3\text{ N/s}.$$

51. **THINK** An airplane with its wings tilted at an angle is in a circular motion. Centripetal force is involved in this problem.

EXPRESS The free-body diagram for the airplane of mass *m* is shown to the right. We note that $\vec{F}_l$ is the force of aerodynamic lift and $\vec{a}$ points rightwards in the figure. We also note that $|\vec{a}| = v^2 / R$. Applying Newton's law to the axes of the problem (+*x* rightward and +*y* upward) we obtain



$$F_l \sin\theta = m\frac{v^2}{R}$$
$$F_l \cos\theta = mg$$

Eliminating mass from these equations leads to $\tan\theta = \frac{v^2}{gR}$. The equation allows us to solve for the radius *R*.

ANALYZE With *v* = 480 km/h = 133 m/s and $\theta = 40°$, we find

$$R = \frac{v^2}{g\tan\theta} = \frac{(133\,\text{m/s})^2}{(9.8\,\text{m/s}^2)\tan 40°} = 2151\ \text{m} \approx 2.2\times10^3\ \text{m}.$$

LEARN Our approach to solving this problem is identical to that discussed in the Sample Problem – "Car in banked circular turn." Do you see the similarities?

52. The situation is somewhat similar to that shown in the "loop-the-loop" example done in the textbook (see Figure 6-10) except that, instead of a downward normal force, we are dealing with the force of the boom $\vec{F}_B$ on the car – which is capable of pointing any direction. We will assume it to be upward as we apply Newton's second law to the car (of total weight 5000 N): $F_B - W = ma$ where $m = W/g$ and $a = -v^2/r$. Note that the centripetal acceleration is downward (our choice for negative direction) for a body at the top of its circular trajectory.

(a) If *r* = 10 m and *v* = 5.0 m/s, we obtain $F_B = 3.7 \times 10^3$ N = 3.7 kN.

(b) The direction of $\vec{F}_B$ is up.

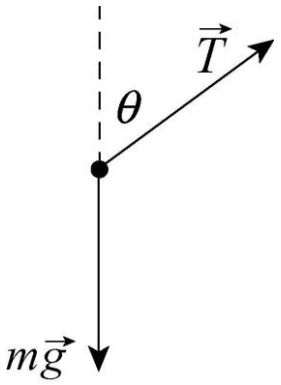
(c) If *r* = 10 m and *v* = 12 m/s, we obtain $F_B = -2.3 \times 10^3$ N = – 2.3 kN, or $|F_B| = 2.3$ kN.

(d) The minus sign indicates that $\vec{F}_B$ points downward.

53. The free-body diagram (for the hand straps of mass *m*) is the view that a passenger might see if she was looking forward and the streetcar was curving towards the right (so $\vec{a}$ points rightwards in the figure). We note that $|\vec{a}| = v^2 / R$ where *v* = 16 km/h = 4.4 m/s.

Applying Newton's law to the axes of the problem (+*x* is rightward and +*y* is upward) we obtain

$$T\sin\theta = m\frac{v^2}{R}$$
$$T\cos\theta = mg.$$



We solve these equations for the angle:

$$\theta = \tan^{-1}\left(\frac{v^2}{Rg}\right)$$

which yields $\theta = 12°$.

54. The centripetal force on the passenger is $F = mv^2/r$.

(a) The variation of F with respect to r while holding v constant is $dF = -\frac{mv^2}{r^2}dr$.

(b) The variation of F with respect to v while holding r constant is $dF = \frac{2mv}{r}dv$.

(c) The period of the circular ride is $T = 2\pi r/v$. Thus,

$$F = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 mr}{T^2},$$

and the variation of F with respect to T while holding r constant is

$$dF = -\frac{8\pi^2 mr}{T^3}dT = -8\pi^2 mr\left(\frac{v}{2\pi r}\right)^3 dT = -\left(\frac{mv^3}{\pi r^2}\right)dT.$$

55. We note that the period T is eight times the time between flashes ($\frac{1}{2000}$ s), so T = 0.0040 s. Combining Eq. 6-18 with Eq. 4-35 leads to

$$F = \frac{4m\pi^2 R}{T^2} = \frac{4(0.030\text{ kg})\pi^2(0.035\text{ m})}{(0.0040\text{ s})^2} = 2.6\times 10^3\text{ N}.$$

56. We refer the reader to Sample Problem – "Car in banked circular turn," and use the result Eq. 6-26:

$$\theta = \tan^{-1}\left(\frac{v^2}{gR}\right)$$

with v = 60(1000/3600) = 17 m/s and R = 200 m. The banking angle is therefore $\theta = 8.1°$. Now we consider a vehicle taking this banked curve at v' = 40(1000/3600) = 11 m/s. Its

(horizontal) acceleration is $a' = v'^2/R$, which has components parallel the incline and perpendicular to it:

$$a_{||} = a'\cos\theta = \frac{v'^2\cos\theta}{R}$$
$$a_{\perp} = a'\sin\theta = \frac{v'^2\sin\theta}{R}.$$

These enter Newton's second law as follows (choosing downhill as the +*x* direction and away-from-incline as +*y*):

$$mg\sin\theta - f_s = ma_{||}$$
$$F_N - mg\cos\theta = ma_{\perp}$$

and we are led to

$$\frac{f_s}{F_N} = \frac{mg\sin\theta - mv'^2\cos\theta/R}{mg\cos\theta + mv'^2\sin\theta/R}.$$

We cancel the mass and plug in, obtaining $f_s/F_N = 0.078$. The problem implies we should set $f_s = f_{s,\max}$ so that, by Eq. 6-1, we have $\mu_s = 0.078$.

57. For the puck to remain at rest the magnitude of the tension force *T* of the cord must equal the gravitational force *Mg* on the cylinder. The tension force supplies the centripetal force that keeps the puck in its circular orbit, so $T = mv^2/r$. Thus $Mg = mv^2/r$. We solve for the speed:

$$v = \sqrt{\frac{Mgr}{m}} = \sqrt{\frac{(2.50\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m})}{1.50\text{ kg}}} = 1.81\text{ m/s}.$$

58. (a) Using the kinematic equation given in Table 2-1, the deceleration of the car is

$$v^2 = v_0^2 + 2ad \quad \Rightarrow \quad 0 = (35\text{ m/s})^2 + 2a(107\text{ m})$$

which gives $a = -5.72\text{ m/s}^2$. Thus, the force of friction required to stop by car is

$$f = m|a| = (1400\text{ kg})(5.72\text{ m/s}^2) \approx 8.0\times10^3\text{ N}.$$

(b) The maximum possible static friction is

$$f_{s,\max} = \mu_s mg = (0.50)(1400\text{ kg})(9.80\text{ m/s}^2) \approx 6.9\times10^3\text{ N}.$$

(c) If $\mu_k = 0.40$, then $f_k = \mu_k mg$ and the deceleration is $a = -\mu_k g$. Therefore, the speed of the car when it hits the wall is

$$v = \sqrt{v_0^2 + 2ad} = \sqrt{(35\text{ m/s})^2 - 2(0.40)(9.8\text{ m/s}^2)(107\text{ m})} \approx 20\text{ m/s}.$$

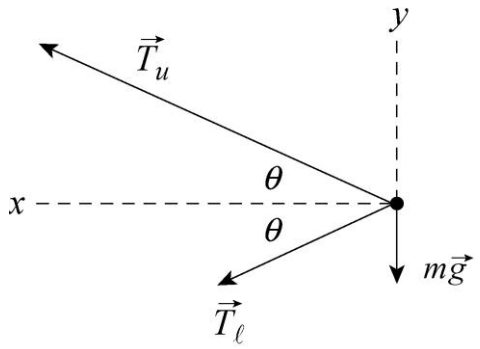
(d) The force required to keep the motion circular is

$$F_r = \frac{mv_0^2}{r} = \frac{(1400\text{ kg})(35.0\text{ m/s})^2}{107\text{ m}} = 1.6\times10^4\text{ N}.$$

(e) Since $F_r > f_{s,\max}$, no circular path is possible.

59. **THINK** As illustrated in Fig. 6-45, our system consists of a ball connected by two strings to a rotating rod. The tensions in the strings provide the source of centripetal force.

EXPRESS The free-body diagram for the ball is shown below. $\vec{T}_u$ is the tension exerted by the upper string on the ball, $\vec{T}_\ell$ is the tension in the lower string, and *m* is the mass of the ball. Note that the tension in the upper string is greater than the tension in the lower string. It must balance the downward pull of gravity and the force of the lower string.



We take the +*x* direction to be leftward (toward the center of the circular orbit) and +*y* upward. Since the magnitude of the acceleration is $a = v^2/R$, the *x* component of Newton's second law is

$$T_u\cos\theta + T_\ell\cos\theta = \frac{mv^2}{R},$$

where *v* is the speed of the ball and *R* is the radius of its orbit. The *y* component is

$$T_u\sin\theta - T_\ell\sin\theta - mg = 0.$$

The second equation gives the tension in the lower string: $T_\ell = T_u - mg/\sin\theta$.

ANALYZE (a) Since the triangle is equilateral, the angle is $\theta = 30.0°$. Thus

$$T_\ell = T_u - \frac{mg}{\sin\theta} = 35.0\text{ N} - \frac{(1.34\text{ kg})(9.80\text{ m/s}^2)}{\sin 30.0°} = 8.74\text{ N}.$$

(b) The net force in the *y*-direction is zero. In the *x*-direction, the net force has magnitude

$$F_{\text{net,str}} = \left(T_u + T_\ell\right)\cos\theta = (35.0\text{ N} + 8.74\text{ N})\cos 30.0° = 37.9\text{ N}.$$

(c) The radius of the path is

$$R = L\cos\theta = (1.70\text{ m})\cos 30° = 1.47\text{ m}.$$

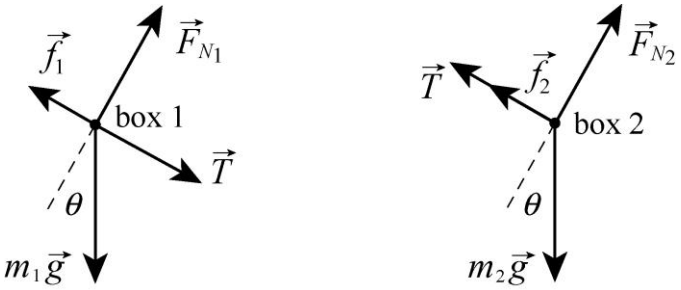
Using $F_{\text{net,str}} = mv^2/R$, we find the speed of the ball to be

$$v = \sqrt{\frac{RF_{\text{net,str}}}{m}} = \sqrt{\frac{(1.47\text{ m})(37.9\text{ N})}{1.34\text{ kg}}} = 6.45\text{ m/s}.$$

(d) The direction of $\vec{F}_{\text{net,str}}$ is leftward ("radially inward'').

LEARN The upper string, with a tension about 4 times that in the lower string ($T_u \approx 4T_\ell$), will break more easily than the lower one.

60. The free-body diagrams for the two boxes are shown below. T is the magnitude of the force in the rod (when $T > 0$ the rod is said to be in tension and when $T < 0$ the rod is under compression), $\vec{F}_{N2}$ is the normal force on box 2 (the uncle box), $\vec{F}_{N1}$ is the the normal force on the aunt box (box 1), $\vec{f}_1$ is kinetic friction force on the aunt box, and $\vec{f}_2$ is kinetic friction force on the uncle box. Also, $m_1 = 1.65$ kg is the mass of the aunt box and $m_2 = 3.30$ kg is the mass of the uncle box (which is a lot of ants!).



For each block we take $+x$ downhill (which is toward the lower-right in these diagrams) and $+y$ in the direction of the normal force. Applying Newton's second law to the x and y directions of first box 2 and next box 1, we arrive at four equations:

$$m_2 g\sin\theta - f_2 - T = m_2\, a$$
$$F_{N2} - m_2 g\cos\theta = 0$$
$$m_1 g\sin\theta - f_1 + T = m_1 a$$
$$F_{N1} - m_1 g\cos\theta = 0$$

which, when combined with Eq. 6-2 ($f_1 = \mu_1 F_{N1}$ where $\mu_1 = 0.226$ and $f_2 = \mu_2 F_{N2}$ where $\mu_2 = 0.113$), fully describe the dynamics of the system.

(a) We solve the above equations for the tension and obtain

$$T = \left(\frac{m_2 m_1 g}{m_2 + m_1} \right) (\mu_1 - \mu_2) \cos\theta = 1.05 \text{ N}.$$

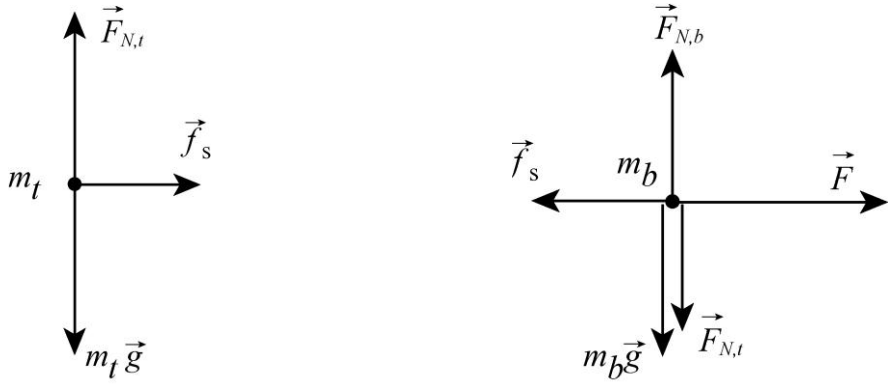
(b) These equations lead to an acceleration equal to

$$a = g\left(\sin\theta - \left(\frac{\mu_2 m_2 + \mu_1 m_1}{m_2 + m_1} \right) \cos\theta \right) = 3.62 \text{ m/s}^2.$$

(c) Reversing the blocks is equivalent to switching the labels. We see from our algebraic result in part (a) that this gives a negative value for T (equal in magnitude to the result we got before). Thus, the situation is as it was before except that the rod is now in a state of compression.

61. **THINK** Our system consists of two blocks, one on top of the other. If we pull the bottom block too hard, the top block will slip on the bottom one. We're interested in the maximum force that can be applied such that the two will move together.

EXPRESS The free-body diagrams for the two blocks are shown below.



We first calculate the coefficient of static friction for the surface between the two blocks. When the force applied is at a maximum, the frictional force between the two blocks must also be a maximum. Since $F_t = 12$ N of force has to be applied to the top block for slipping to take place, using $F_t = f_{s,\max} = \mu_s F_{N,t} = \mu_s m_t g$, we have

$$\mu_s = \frac{F_t}{m_t g} = \frac{12 \text{ N}}{(4.0 \text{ kg})(9.8 \text{ m/s}^2)} = 0.31.$$

Using the same reasoning, for the two masses to move together, the maximum applied force would be

$$F = \mu_s(m_t + m_b)g$$

ANALYZE (a) Substituting the value of μ_s found above, the maximum horizontal force has a magnitude

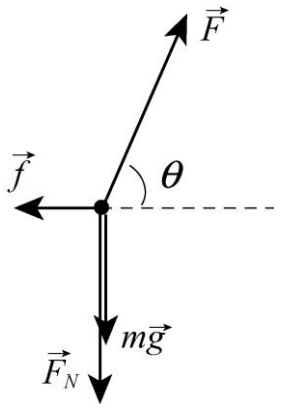
$$F = \mu_s(m_t + m_b)g = (0.31)(4.0\,\text{kg} + 5.0\,\text{kg})(9.8\,\text{m/s}^2) = 27\,\text{N}$$

(b) The maximum acceleration is

$$a_{\max} = \frac{F}{m_t + m_b} = \mu_s g = (0.31)(9.8\,\text{m/s}^2) = 3.0\,\text{m/s}^2.$$

LEARN Slipping will occur if the applied force exceeds 27.3 N. In the absence of friction ($\mu_s = 0$) between the two blocks, any amount of force would cause the top block to slip.

62. The free-body diagram for the stone is shown to the right, with $\vec{F}$ being the force applied to the stone, $\vec{F}_N$ the *downward* normal force of the ceiling on the stone, $m\vec{g}$ the force of gravity, and $\vec{f}$ the force of friction. We take the +*x* direction to be horizontal to the right and the +*y* direction to be up. The equations for the *x* and the *y* components of the force according to Newton's second law are:



$$F_x = F\cos\theta - f = ma$$
$$F_y = F\sin\theta - F_N - mg = 0$$

Now $f = \mu_k F_N$, and the second equation gives $F_N = F\sin\theta - mg$, which yields $f = \mu_k(F\sin\theta - mg)$. This expression is substituted for *f* in the first equation to obtain

$$F\cos\theta - \mu_k(F\sin\theta - mg) = ma.$$

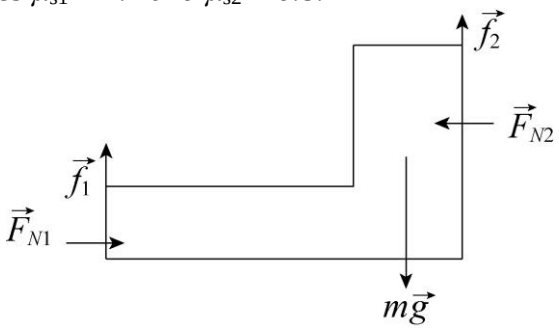
For $a = 0$, the force is

$$F = \frac{-\mu_k mg}{\cos\theta - \mu_k \sin\theta}.$$

With $\mu_k = 0.65$, $m = 5.0$ kg, and $\theta = 70^\circ$, we obtain $F = 118$ N.

63. (a) The free-body diagram for the person (shown as an L-shaped block) is shown below. The force that she exerts on the rock slabs is not directly shown (since the diagram should only show forces exerted on her), but it is related by Newton's third law) to the normal forces $\vec{F}_{N1}$ and $\vec{F}_{N2}$ exerted horizontally by the slabs onto her shoes and

back, respectively. We will show in part (b) that $F_{N1} = F_{N2}$ so that we there is no ambiguity in saying that the magnitude of her push is F_{N2}. The total upward force due to (maximum) static friction is $\vec{f} = \vec{f}_1 + \vec{f}_2$ where $f_1 = \mu_{s1} F_{N1}$ and $f_2 = \mu_{s2} F_{N2}$. The problem gives the values $\mu_{s1} = 1.2$ and $\mu_{s2} = 0.8$.



(b) We apply Newton's second law to the x and y axes (with $+x$ rightward and $+y$ upward and there is no acceleration in either direction).

$$F_{N1} - F_{N2} = 0$$
$$f_1 + f_2 - mg = 0$$

The first equation tells us that the normal forces are equal $F_{N1} = F_{N2} = F_N$. Consequently, from Eq. 6-1,

$$f_1 = \mu_{s1} F_N$$
$$f_2 = \mu_{s2} F_N$$

we conclude that

$$f_1 = \left(\frac{\mu_{s1}}{\mu_{s2}} \right) f_2 \ .$$

Therefore, $f_1 + f_2 - mg = 0$ leads to

$$\left(\frac{\mu_{s1}}{\mu_{s2}} + 1 \right) f_2 = mg$$

which (with $m = 49$ kg) yields $f_2 = 192$ N. From this we find $F_N = f_2 / \mu_{s2} = 240$ N. This is equal to the magnitude of the push exerted by the rock climber.

(c) From the above calculation, we find $f_1 = \mu_{s1} F_N = 288$ N which amounts to a fraction

$$\frac{f_1}{W} = \frac{288}{(49)(9.8)} = 0.60$$

or 60% of her weight.

64. (a) The upward force exerted by the car on the passenger is equal to the downward force of gravity (W = 500 N) on the passenger. So the *net* force does not have a vertical contribution; it only has the contribution from the horizontal force (which is necessary for maintaining the circular motion). Thus $\left|\vec{F}_{\text{net}}\right| = F = 210$ N.

(b) Using Eq. 6-18, we have

$$v = \sqrt{\frac{FR}{m}} = \sqrt{\frac{(210\text{ N})(470\text{ m})}{51.0\text{ kg}}} = 44.0\text{ m/s}.$$

65. The layer of ice has a mass of

$$m_{\text{ice}} = \left(917\text{ kg/m}^3\right)\ (400\text{ m}\times 500\text{ m}\times 0.0040\text{ m}) = 7.34\times 10^5\text{ kg}.$$

This added to the mass of the hundred stones (at 20 kg each) comes to $m = 7.36 \times 10^5$ kg.

(a) Setting $F = D$ (for Drag force) we use Eq. 6-14 to find the wind speed v along the ground (which actually is relative to the moving stone, but we assume the stone is moving slowly enough that this does not invalidate the result):

$$v = \sqrt{\frac{\mu_k mg}{4C_{\text{ice}}\rho A_{\text{ice}}}} = \sqrt{\frac{(0.10)\left(7.36\times 10^5\text{ kg}\right)\left(9.8\text{ m/s}^2\right)}{4(0.002)\left(1.21\text{ kg/m}^3\right)\left(400\times 500\text{ m}^2\right)}} = 19\text{ m/s}\ \approx 69\text{ km/h}.$$

(b) Doubling our previous result, we find the reported speed to be 139 km/h.

(c) The result is reasonable for storm winds. (A category-5 hurricane has speeds on the order of 2.6×10^2 m/s.)

66. Note that since no static friction coefficient is mentioned, we assume f_s is not relevant to this computation. We apply Newton's second law to each block's x axis, which for m_1 is positive rightward and for m_2 is positive downhill:

$$\begin{aligned} T - f_k &= m_1 a \\ m_2 g\sin\theta - T &= m_2 a \end{aligned}$$

Adding the equations, we obtain the acceleration:

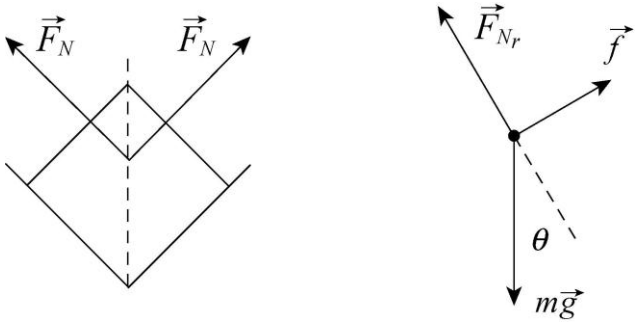
$$a = \frac{m_2 g\sin\theta - f_k}{m_1 + m_2}$$

For $f_k = \mu_k F_N = \mu_k m_1 g$, we obtain

$$a = \frac{(3.0\text{ kg})(9.8\text{ m/s}^2)\sin 30° - (0.25)(2.0\text{ kg})(9.8\text{ m/s}^2)}{3.0\text{ kg} + 2.0\text{ kg}} = 1.96\text{ m/s}^2 .$$

Returning this value to either of the above two equations, we find T = 8.8 N.

67. Each side of the trough exerts a normal force on the crate. The first diagram shows the view looking in toward a cross section.



The net force is along the dashed line. Since each of the normal forces makes an angle of 45° with the dashed line, the magnitude of the resultant normal force is given by

$$F_{Nr} = 2F_N \cos 45° = \sqrt{2}F_N .$$

The second diagram is the free-body diagram for the crate (from a "side" view, similar to that shown in the first picture in Fig. 6-51). The force of gravity has magnitude mg, where m is the mass of the crate, and the magnitude of the force of friction is denoted by f. We take the $+x$ direction to be down the incline and $+y$ to be in the direction of $\vec{F}_{Nr}$. Then the x and the y components of Newton's second law are

$$x: \quad mg \sin\theta - f = ma$$
$$y: \quad F_{Nr} - mg\cos\theta = 0.$$

Since the crate is moving, each side of the trough exerts a force of kinetic friction, so the total frictional force has magnitude

$$f = 2\mu_k F_N = 2\mu_k F_{Nr} / \sqrt{2} = \sqrt{2}\mu_k F_{Nr}$$

Combining this expression with $F_{Nr} = mg\cos\theta$ and substituting into the x component equation, we obtain

$$mg\sin\theta - \sqrt{2}mg\cos\theta = ma .$$

Therefore $a = g(\sin\theta - \sqrt{2}\mu_k \cos\theta)$.

68. (a) To be on the verge of sliding out means that the force of static friction is acting "down the bank" (in the sense explained in the problem statement) with maximum

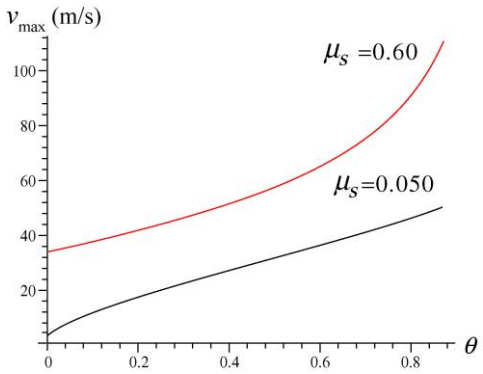
possible magnitude. We first consider the vector sum $\vec{F}$ of the (maximum) static friction force and the normal force. Due to the facts that they are perpendicular and their magnitudes are simply proportional (Eq. 6-1), we find $\vec{F}$ is at angle (measured from the *vertical* axis) $\phi = \theta + \theta_s$, where $\tan\theta_s = \mu_s$ (compare with Eq. 6-13), and θ is the bank angle (as stated in the problem). Now, the vector sum of $\vec{F}$ and the vertically downward pull (mg) of gravity must be equal to the (horizontal) centripetal force (mv^2/R), which leads to a surprisingly simple relationship:

$$\tan\phi = \frac{mv^2/R}{mg} = \frac{v^2}{Rg} \quad .$$

Writing this as an expression for the maximum speed, we have

$$v_{\max} = \sqrt{Rg\tan(\theta + \tan^{-1}\mu_s)} = \sqrt{\frac{Rg(\tan\theta + \mu_s)}{1 - \mu_s\tan\theta}}$$

(b) The graph is shown below (with θ in radians):



(c) Either estimating from the graph (μ_s = 0.60, upper curve) or calculated it more carefully leads to v = 41.3 m/s = 149 km/h when θ = 10° = 0.175 radian.

(d) Similarly (for μ_s = 0.050, the lower curve) we find v = 21.2 m/s = 76.2 km/h when θ = 10° = 0.175 radian.

69. For simplicity, we denote the 70° angle as θ and the magnitude of the push (80 N) as P. The vertical forces on the block are the downward normal force exerted on it by the ceiling, the downward pull of gravity (of magnitude mg) and the vertical component of $\vec{P}$ (which is upward with magnitude $P \sin\theta$). Since there is no acceleration in the vertical direction, we must have

$$F_N = P\sin\theta - mg$$

in which case the leftward-pointed kinetic friction has magnitude

$$f_k = \mu_k (P\sin\theta - mg).$$

Choosing +*x* rightward, Newton's second law leads to

$$P\cos\theta - f_k = ma \;\Rightarrow\; a = \frac{P\cos\theta - u_k(P\sin\theta - mg)}{m}$$

which yields $a = 3.4$ m/s^2 when $\mu_k = 0.40$ and $m = 5.0$ kg.

70. (a) We note that R (the horizontal distance from the bob to the axis of rotation) is the circumference of the circular path divided by 2π; therefore, $R = 0.94/2\pi = 0.15$ m. The angle that the cord makes with the horizontal is now easily found:

$$\theta = \cos^{-1}(R/L) = \cos^{-1}(0.15\text{ m}/0.90\text{ m}) = 80^\circ.$$

The vertical component of the force of tension in the string is $T\sin\theta$ and must equal the downward pull of gravity (mg). Thus,

$$T = \frac{mg}{\sin\theta} = 0.40\text{ N}.$$

Note that we are using T for tension (not for the period).

(b) The horizontal component of that tension must supply the centripetal force (Eq. 6-18), so we have $T\cos\theta = mv^2/R$. This gives speed $v = 0.49$ m/s. This divided into the circumference gives the time for one revolution: 0.94/0.49 = 1.9 s.

71. (a) To be "on the verge of sliding" means the applied force is equal to the maximum possible force of static friction (Eq. 6-1, with $F_N = mg$ in this case):

$$f_{s,\max} = \mu_s mg = 35.3\text{ N}.$$

(b) In this case, the applied force $\vec{F}$ indirectly decreases the maximum possible value of friction (since its *y* component causes a reduction in the normal force) as well as directly opposing the friction force itself (because of its *x* component). The normal force turns out to be

$$F_N = mg - F\sin\theta$$

where $\theta = 60^\circ$, so that the horizontal equation (the *x* application of Newton's second law) becomes

$$F\cos\theta - f_{s,\max} = F\cos\theta - \mu_s(mg - F\sin\theta) = 0 \quad \Rightarrow F = 39.7\text{ N}.$$

(c) Now, the applied force $\vec{F}$ indirectly increases the maximum possible value of friction (since its *y* component causes a reduction in the normal force) as well as directly opposing the friction force itself (because of its *x* component). The normal force in this case turns out to be

$$F_N = mg + F\sin\theta,$$

where $\theta = 60^\circ$, so that the horizontal equation becomes

$$F\cos\theta - f_{s,\max} = F\cos\theta - \mu_s(mg + F\sin\theta) = 0 \quad \Rightarrow F = 320 \text{ N}.$$

72. With $\theta = 40^\circ$, we apply Newton's second law to the "downhill" direction:

$$mg\sin\theta - f = ma,$$
$$f = f_k = \mu_k F_N = \mu_k mg\cos\theta$$

using Eq. 6-12. Thus,

$$a = 0.75 \text{ m/s}^2 = g(\sin\theta - \mu_k\cos\theta)$$

determines the coefficient of kinetic friction: $\mu_k = 0.74$.

73. (a) With $\theta = 60^\circ$, we apply Newton's second law to the "downhill" direction:

$$mg\sin\theta - f = ma$$
$$f = f_k = \mu_k F_N = \mu_k mg\cos\theta.$$

Thus,

$$a = g(\sin\theta - \mu_k\cos\theta) = 7.5 \text{ m/s}^2.$$

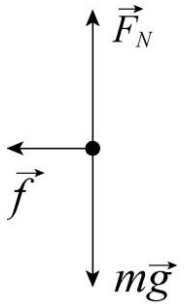
(b) The direction of the acceleration $\vec{a}$ is down the slope.

(c) Now the friction force is in the "downhill" direction (which is our positive direction) so that we obtain

$$a = g(\sin\theta + \mu_k\cos\theta) = 9.5 \text{ m/s}^2.$$

(d) The direction is down the slope.

74. The free-body diagram for the puck is shown on the right. $\vec{F}_N$ is the normal force of the ice on the puck, $\vec{f}$ is the force of friction (in the $-x$ direction), and $m\vec{g}$ is the force of gravity.



(a) The horizontal component of Newton's second law gives $-f = ma$, and constant acceleration kinematics (Table 2-1) can be used to find the acceleration.

Since the final velocity is zero, $v^2 = v_0^2 + 2ax$ leads to $a = -v_0^2/2x$. This is substituted into the Newton's law equation to obtain

$$f = \frac{mv_0^2}{2x} = \frac{(0.110\text{ kg})(6.0\text{ m/s})^2}{2(15\text{ m})} = 0.13\text{ N}.$$

(b) The vertical component of Newton's second law gives $F_N - mg = 0$, so $F_N = mg$ which implies (using Eq. 6-2) $f = \mu_k mg$. We solve for the coefficient:

$$\mu_k = \frac{f}{mg} = \frac{0.13\text{ N}}{(0.110\text{ kg})(9.8\text{ m/s}^2)} = 0.12\,.$$

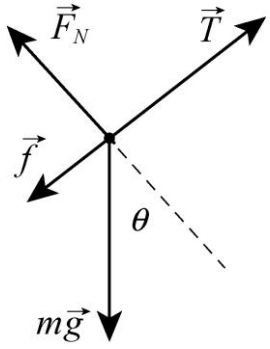
75. We may treat all 25 cars as a single object of mass $m = 25 \times 5.0 \times 10^4$ kg and (when the speed is 30 km/h = 8.3 m/s) subject to a friction force equal to

$$f = 25 \times 250 \times 8.3 = 5.2 \times 10^4\text{ N}.$$

(a) Along the level track, this object experiences a "forward" force T exerted by the locomotive, so that Newton's second law leads to

$$T - f = ma \;\Rightarrow\; T = 5.2\times10^4 + (1.25\times10^6)(0.20) = 3.0\times10^5\text{ N}\,.$$

(b) The free-body diagram is shown next, with θ as the angle of the incline. The $+x$ direction (which is the only direction to which we will be applying Newton's second law) is uphill (to the upper right in our sketch). Thus, we obtain



$$T - f - mg\sin\theta = ma$$

where we set $a = 0$ (implied by the problem statement) and solve for the angle. We obtain $\theta = 1.2°$.

76. An excellent discussion and equation development related to this problem is given in Sample Problem – "Friction, applied force at an angle." Using the result, we obtain

$$\theta = \tan^{-1}\mu_s = \tan^{-1}0.50 = 27°$$

which implies that the angle through which the slope should be *reduced* is

$$\phi = 45° - 27° \approx 20°.$$

77. We make use of Eq. 6-16 which yields

$$\sqrt{\frac{2mg}{C\rho\pi R^2}} = \sqrt{\frac{2(6)(9.8)}{(1.6)(1.2)\pi(0.03)^2}} = 147\text{ m/s}.$$

78. (a) The coefficient of static friction is $\mu_s = \tan(\theta_{\text{slip}}) = 0.577 \approx 0.58$.

(b) Using

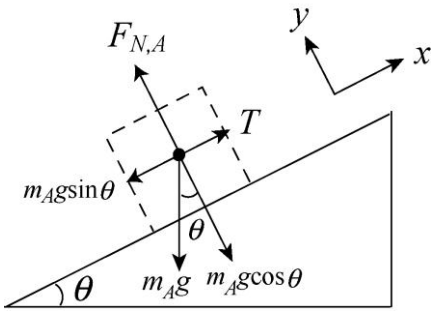
$$mg\sin\theta - f = ma$$

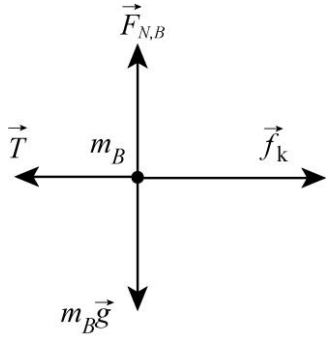
$$f = f_k = \mu_k F_N = \mu_k mg\cos\theta$$

and $a = 2d/t^2$ (with $d = 2.5$ m and $t = 4.0$ s), we obtain $\mu_k = 0.54$.

79. **THINK** We have two blocks connected by a cord, as shown in Fig. 6-56. As block *A* slides down the frictionless inclined plane, it pulls block *B*, so there's a tension in the cord.

EXPRESS The free-body diagrams for blocks *A* and *B* are shown below:





Newton's law gives

$$m_A g\sin\theta - T = m_A a$$

for block *A* (where $\theta = 30^\circ$). For block *B*, we have

$$T - f_k = m_B a$$

Now the frictional force is given by $f_k = \mu_k F_{N,B} = \mu_k m_B g$. The equations allow us to solve for the tension *T* and the acceleration *a*.

ANALYZE (a) Combining the above equations to solve for *T*, we obtain

$$T = \frac{m_A m_B}{m_A + m_B}\left(\sin\theta + \mu_k\right)g = \frac{(4.0\text{ kg})(2.0\text{ kg})}{4.0\text{ kg} + 2.0\text{ kg}}\left(\sin 30^\circ + 0.50\right)(9.80\text{ m/s}^2) = 13\text{ N}.$$

(b) Similarly, the acceleration of the two-block system is

$$a = \left(\frac{m_A\sin\theta - \mu_k m_B}{m_A + m_B}\right)g = \frac{(4.0\text{ kg})\sin 30^\circ - (0.50)(2.0\text{ kg})}{4.0\text{ kg} + 2.0\text{ kg}}(9.80\text{ m/s}^2) = 1.6\text{ m/s}^2.$$

LEARN In the case where $\theta = 90^\circ$ and $\mu_k = 0$, we have

$$T = \frac{m_A m_B}{m_A + m_B} g, \quad a = \frac{m_A}{m_A + m_B} g$$

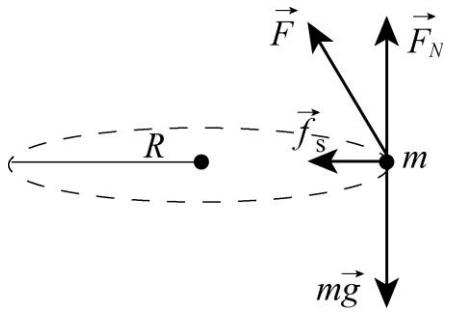
which correspond to the Sample Problem – "Block on table, block hanging," discussed in Chapter 5.

80. We use Eq. 6-14, $D = \frac{1}{2} C\rho A v^2$, where ρ is the air density, A is the cross-sectional area of the missile, v is the speed of the missile, and C is the drag coefficient. The area is given by $A = \pi R^2$, where $R = 0.265$ m is the radius of the missile. Thus

$$D = \frac{1}{2}(0.75)\left(1.2 \text{ kg/m}^3\right)\pi\left(0.265 \text{ m}\right)^2\left(250 \text{ m/s}\right)^2 = 6.2 \times 10^3 \text{ N}.$$

81. **THINK** How can a cyclist move in a circle? It is the force of friction that provides the centripetal force required for the circular motion.

EXPRESS The free-body diagram is shown below. The magnitude of the acceleration of the cyclist as it moves along the horizontal circular path is given by v^2/R, where v is the speed of the cyclist and R is the radius of the curve.



The horizontal component of Newton's second law is $f_s = mv^2/R$, where f_s is the static friction exerted horizontally by the ground on the tires. Similarly, if F_N is the vertical force of the ground on the bicycle and m is the mass of the bicycle and rider, the vertical component of Newton's second law leads to $F_N = mg = 833 \text{ N}$.

ANALYZE (a) The frictional force is $f_s = \frac{mv^2}{R} = \frac{\left(85.0 \text{ kg}\right)\left(9.00 \text{ m/s}\right)^2}{25.0 \text{ m}} = 275 \text{ N}.$

(b) Since the frictional force $\vec{f}_s$ and $\vec{F}_N$, the normal force exerted by the road, are perpendicular to each other, the magnitude of the force exerted by the ground on the bicycle is

$$F = \sqrt{f_s^2 + F_N^2} = \sqrt{(275 \text{ N})^2 + (833 \text{ N})^2} = 877 \text{ N}.$$

LEARN The force exerted by the ground on the bicycle is at an angle $\theta = \tan^{-1}(275\text{ N}/833\text{ N}) = 18.3°$ with respect to the *vertical* axis.

82. At the top of the hill the vertical forces on the car are the upward normal force exerted by the ground and the downward pull of gravity. Designating +*y* downward, we have

$$mg - F_N = \frac{mv^2}{R}$$

from Newton's second law. To find the greatest speed without leaving the hill, we set $F_N = 0$ and solve for *v*:

$$v = \sqrt{gR} = \sqrt{(9.8\text{ m/s}^2)(250\text{ m})} = 49.5\text{ m/s} = 49.5(3600/1000)\text{ km/h} = 178\text{ km/h}.$$

83. (a) The push (to get it moving) must be at least as big as $f_{s,max} = \mu_s F_N$ (Eq. 6-1, with $F_N = mg$ in this case), which equals (0.51)(165 N) = 84.2 N.

(b) While in motion, constant velocity (zero acceleration) is maintained if the push is equal to the kinetic friction force $f_k = \mu_k F_N = \mu_k mg = 52.8$ N.

(c) We note that the mass of the crate is 165/9.8 = 16.8 kg. The acceleration, using the push from part (a), is

$$a = (84.2\text{ N} - 52.8\text{ N})/(16.8\text{ kg}) \approx 1.87\text{ m/s}^2.$$

84. (a) The *x* component of $\vec{F}$ tries to move the crate while its *y* component indirectly contributes to the inhibiting effects of friction (by increasing the normal force). Newton's second law implies

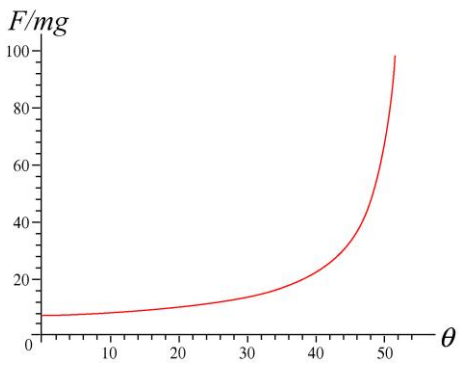
$$x\text{ direction: } F\cos\theta - f_s = 0$$

$$y\text{ direction: } F_N - F\sin\theta - mg = 0.$$

To be "on the verge of sliding" means $f_s = f_{s,max} = \mu_s F_N$ (Eq. 6-1). Solving these equations for *F* (actually, for the ratio of *F* to *mg*) yields

$$\frac{F}{mg} = \frac{\mu_s}{\cos\theta - \mu_s \sin\theta}\ .$$

This is plotted on the right (θ in degrees).



(b) The denominator of our expression (for *F/mg*) vanishes when

$$\cos\theta - \mu_s \sin\theta = 0 \quad \Rightarrow \quad \theta_{\text{inf}} = \tan^{-1}\left(\frac{1}{\mu_s}\right)$$

For $\mu_s = 0.70$, we obtain $\theta_{\text{inf}} = \tan^{-1}\left(\frac{1}{\mu_s}\right) = 55°$.

(c) Reducing the coefficient means increasing the angle by the condition in part (b).

(d) For $\mu_s = 0.60$ we have $\theta_{\text{inf}} = \tan^{-1}\left(\frac{1}{\mu_s}\right) = 59°$.

85. The car is in "danger of sliding" down when

$$\mu_s = \tan\theta = \tan 35.0° = 0.700.$$

This value represents a 3.4% decrease from the given 0.725 value.

86. (a) The tension will be the greatest at the lowest point of the swing. Note that there is no substantive difference between the tension T in this problem and the normal force F_N in Sample Problem – "Vertical circular loop, Diavolo." Eq. 6-19 of that Sample Problem examines the situation at the top of the circular path (where F_N is the least), and rewriting that for the bottom of the path leads to

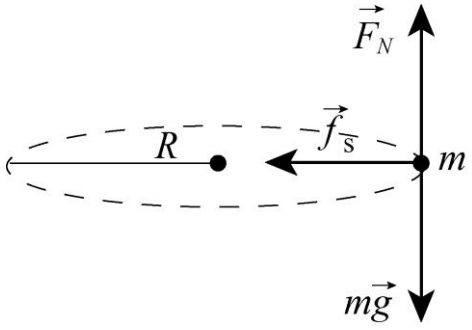
$$T = mg + mv^2/r$$

where F_N is at its greatest value.

(b) At the breaking point T = 33 N = $m(g + v^2/r)$ where m = 0.26 kg and r = 0.65 m. Solving for the speed, we find that the cord should break when the speed (at the lowest point) reaches 8.73 m/s.

87. **THINK** A car is making a turn on an unbanked curve. Friction is what provides the centripetal force needed for this circular motion.

EXPRESS The free-body diagram is shown to the right. The mass of the car is m = (10700/9.80) kg = 1.09×10^3 kg. We choose "inward" (horizontally toward the center of the circular path) as the positive direction. The normal force is $F_N = mg$ in this situation, and the required frictional force is $f_s = mv^2 / R$.



ANALYZE (a) With a speed of v = 13.4 m/s and a radius R = 61 m, Newton's second law (using Eq. 6-18) leads to

$$f_s = \frac{mv^2}{R} = \frac{(1.09\times10^3\ \text{kg})(13.4\ \text{m/s})^2}{61.0\ \text{m}} = 3.21\times10^3\ \text{N}.$$

(b) The maximum possible static friction is found to be

$$f_{s,\max} = \mu_s mg = (0.35)(10700\ \text{N}) = 3.75\times10^3\ \text{N}$$

using Eq. 6-1. We see that the static friction found in part (a) is less than this, so the car rolls (no skidding) and successfully negotiates the curve.

LEARN From the above expressions, we see that with a coefficient of friction μ_s, the maximum speed of the car negotiating a curve of radius R is $v_{\max} = \sqrt{\mu_s gR}$. So in this case, the car can go up to a maximum speed of

$$v_{\max} = \sqrt{(0.35)(9.8\ \text{m/s}^2)(61\ \text{m})} = 14.5\ \text{m/s}$$

without skidding.

88. For the m_2 = 1.0 kg block, application of Newton's laws result in

$$\begin{aligned} F\cos\theta - T - f_k &= m_2 a \quad && x \text{ axis} \\ F_N - F\sin\theta - m_2 g &= 0 && y \text{ axis} \end{aligned}$$

Since $f_k = \mu_k F_N$, these equations can be combined into an equation to solve for a:

$$F(\cos\theta - \mu_k \sin\theta) - T - \mu_k m_2 g = m_2 a$$

Similarly (but without the applied push) we analyze the m_1= 2.0 kg block:

$$\begin{aligned} T - f_k' &= m_1 a \quad && x \text{ axis} \\ F_N' - m_1 g &= 0 && y \text{ axis} \end{aligned}$$

Using $f_k = \mu_k F_N'$, the equations can be combined:

$$T - \mu_k m_1 g = m_1 a$$

Subtracting the two equations for a and solving for the tension, we obtain

$$T = \frac{m_1(\cos\theta - \mu_k \sin\theta)}{m_1 + m_2} F = \frac{(2.0\ \text{kg})[\cos 35° - (0.20)\sin 35°]}{2.0\ \text{kg} + 1.0\ \text{kg}}(20\ \text{N}) = 9.4\ \text{N}.$$

89. **THINK** In order to move a filing cabinet, the force applied must be able to overcome the frictional force.

EXPRESS We apply Newton's second law (as $F_{\text{push}} - f = ma$). If we find the applied force F_{push} to be less than $f_{s,\max}$, the maximum static frictional force, our conclusion would then be "no, the cabinet does not move" (which means a is actually 0 and the frictional force is simply $f = F_{\text{push}}$). On the other hand, if we obtain $a > 0$ then the cabinet moves (so $f = f_k$). For $f_{s,\max}$ and f_k we use Eq. 6-1 and Eq. 6-2 (respectively), and in those formulas we set the magnitude of the normal force to the weight of the cabinet: $F_N = mg = 556\ \text{N}$. Thus, the maximum static frictional force is

$$f_{s,\max} = \mu_s F_N = \mu_s mg = (0.68)(556\ \text{N}) = 378\ \text{N}.$$

and the kinetic frictional force is

$$f_k = \mu_k F_N = \mu_k mg = (0.56)(556\ \text{N}) = 311\ \text{N}.$$

ANALYZE (a) Here we find $F_{\text{push}} < f_{s,\max}$ which leads to $f = F_{\text{push}} = 222$ N. The cabinet does not move.

(b) Again we find $F_{\text{push}} < f_{s,\max}$ which leads to $f = F_{\text{push}} = 334$ N. The cabinet does not move.

(c) Now we have $F_{\text{push}} > f_{s,\max}$ which means the cabinet moves and $f = f_k = 311$ N.

(d) Again we have $F_{\text{push}} > f_{s,\max}$ which means the cabinet moves and $f = f_k = 311$ N.

(e) The cabinet moves in (c) and (d).

LEARN In summary, in order to make the cabinet move, the minimum applied force is equal to the maximum static frictional force $f_{s,\max}$.

90. Analysis of forces in the horizontal direction (where there can be no acceleration) leads to the conclusion that $F = F_N$; the magnitude of the normal force is 60 N. The maximum possible static friction force is therefore $\mu_s F_N = 33$ N, and the kinetic friction force (when applicable) is $\mu_k F_N = 23$ N.

(a) In this case, $\vec{P} = 34$ N upward. Assuming $\vec{f}$ points down, then Newton's second law for the y leads to

$$P - mg - f = ma.$$

if we assume $f = f_s$ and $a = 0$, we obtain $f = (34 - 22)$ N $= 12$ N. This is less than $f_{s,\max}$, which shows the consistency of our assumption. The answer is: $\vec{f}_s = 12$ N down.

(b) In this case, $\vec{P}$ = 12 N upward. The above equation, with the same assumptions as in part (a), leads to f = (12 – 22) N = –10 N. Thus, $|f_s| < f_{s,\,max}$, justifying our assumption that the block is stationary, but its negative value tells us that our initial assumption about the direction of $\vec{f}$ is incorrect in this case. Thus, the answer is: $\vec{f}_s$ = 10 N up.

(c) In this case, $\vec{P}$ = 48 N upward. The above equation, with the same assumptions as in part (a), leads to f = (48 – 22) N = 26 N. Thus, we again have $f_s < f_{s,\,max}$, and our answer is: $\vec{f}_s$ = 26 N down.

(d) In this case, $\vec{P}$ = 62 N upward. The above equation, with the same assumptions as in part (a), leads to f = (62 – 22) N = 40 N, which is larger than $f_{s,\,max,}$ -- invalidating our assumptions. Therefore, we take $f = f_k$ and $a \neq 0$ in the above equation; if we wished to find the value of a we would find it to be positive, as we should expect. The answer is: $\vec{f}_k$ = 23 N down.

(e) In this case, $\vec{P}$ = 10 N downward. The above equation (but with P replaced with $-P$) with the same assumptions as in part (a), leads to f = (–10 – 22) N = –32 N. Thus, we have $|f_s| < f_{s,\,max}$, justifying our assumption that the block is stationary, but its negative value tells us that our initial assumption about the direction of $\vec{f}$ is incorrect in this case. Thus, the answer is: $\vec{f}_s$ = 32 N up.

(f) In this case, $\vec{P}$ = 18 N downward. The above equation (but with P replaced with $-P$) with the same assumptions as in part (a), leads to f = (–18 – 22) N = –40 N, which is larger (in absolute value) than $f_{s,\,max}$, -- invalidating our assumptions. Therefore, we take $f = f_k$ and $a \neq 0$ in the above equation; if we wished to find the value of a we would find it to be negative, as we should expect. The answer is: $\vec{f}_k$ = 23 N up.

(g) The block moves up the wall in case (d) where $a > 0$.

(h) The block moves down the wall in case (f) where $a < 0$.

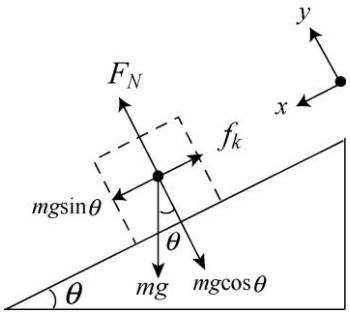
(i) The frictional force $\vec{f}_s$ is directed down in cases (a), (c) and (d).

91. **THINK** Whether the block is sliding down or up the incline, there is a frictional force in the opposite direction of the motion.

EXPRESS The free-body diagram for the first part of this problem (when the block is sliding downhill with zero acceleration) is shown next.

Newton's second law gives

$$mg\sin\theta - f_k = mg\sin\theta - \mu_k F_N = ma_x = 0$$
$$mg\cos\theta - F_N = ma_y = 0$$

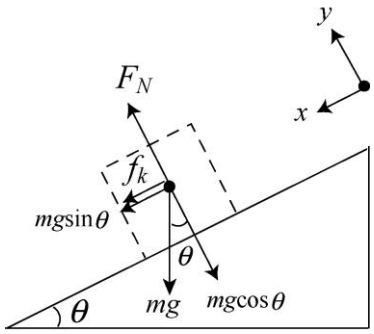


The two equations can be combined to give

$$\mu_k = \tan\theta.$$

Now (for the second part of the problem, with the block projected uphill) the friction direction is reversed (see figure to the right). Newton's second law for the uphill motion (and Eq. 6-12) leads to

$$mg\sin\theta + f_k = mg\sin\theta + \mu_k F_N = ma_x$$
$$mg\cos\theta - F_N = ma_y = 0$$



Note that by our convention, $a_x > 0$ means that the acceleration is downhill, and therefore, the speed of the block will decrease as it moves up the incline.

ANALYZE (a) Using $\mu_k = \tan\theta$ and $F_N = mg\cos\theta$, we find the *x*-component of the acceleration to be

$$a_x = g\sin\theta + \frac{\mu_k F_N}{m} = g\sin\theta + \frac{(\tan\theta)(mg\cos\theta)}{m} = 2g\sin\theta.$$

The distance the block travels before coming to a stop can be found by using Eq. 2-16: $v_f^2 = v_0^2 - 2a_x\Delta x$, which yields

$$\Delta x = \frac{v_0^2}{2a_x} = \frac{v_0^2}{2(2g\sin\theta)} = \frac{v_0^2}{4g\sin\theta}.$$

(b) We usually expect $\mu_s > \mu_k$ (see the discussion in Section 6-1). The "angle of repose" (the minimum angle necessary for a stationary block to start sliding downhill) is $\mu_s = \tan(\theta_{\text{repose}})$. Therefore, we expect $\theta_{\text{repose}} > \theta$ found in part (a). Consequently, when the block comes to rest, the incline is not steep enough to cause it to start slipping down the incline again.

LEARN An alternative way to see that the block will not slide down again is to note that the downward force of gravitation is not large enough to overcome the force of friction, i.e., $mg\sin\theta = f_k < f_{s,\max}$.

92. Consider that the car is "on the verge of sliding out" – meaning that the force of static friction is acting "down the bank" (or "downhill" from the point of view of an ant on the banked curve) with maximum possible magnitude. We first consider the vector sum $\vec{F}$ of the (maximum) static friction force and the normal force. Due to the facts that they are perpendicular and their magnitudes are simply proportional (Eq. 6-1), we find $\vec{F}$ is at angle (measured from the vertical axis) $\phi = \theta + \theta_s$ where $\tan\theta_s = \mu_s$ (compare with Eq. 6-13), and θ is the bank angle. Now, the vector sum of $\vec{F}$ and the vertically downward pull (mg) of gravity must be equal to the (horizontal) centripetal force (mv^2/R), which leads to a surprisingly simple relationship:

$$\tan\phi = \frac{mv^2/R}{mg} = \frac{v^2}{Rg} .$$

Writing this as an expression for the maximum speed, we have

$$v_{\max} = \sqrt{Rg\tan(\theta + \tan^{-1}\mu_s)} = \sqrt{\frac{Rg(\tan\theta + \mu_s)}{1 - \mu_s\tan\theta}} .$$

(a) We note that the given speed is (in SI units) roughly 17 m/s. If we do not want the cars to "depend" on the static friction to keep from sliding out (that is, if we want the component "down the back" of gravity to be sufficient), then we can set $\mu_s = 0$ in the above expression and obtain $v = \sqrt{Rg\tan\theta}$. With $R = 150$ m, this leads to $\theta = 11°$.

(b) If, however, the curve is not banked (so $\theta = 0$) then the above expression becomes

$$v = \sqrt{Rg\tan(\tan^{-1}\mu_s)} = \sqrt{Rg\mu_s}$$

Solving this for the coefficient of static friction $\mu_s = 0.19$.

93. (a) The box doesn't move until $t = 2.8$ s, which is when the applied force $\vec{F}$ reaches a magnitude of $F = (1.8)(2.8) = 5.0$ N, implying therefore that $f_{s,\max} = 5.0$ N. Analysis of the vertical forces on the block leads to the observation that the normal force magnitude equals the weight $F_N = mg = 15$ N. Thus,

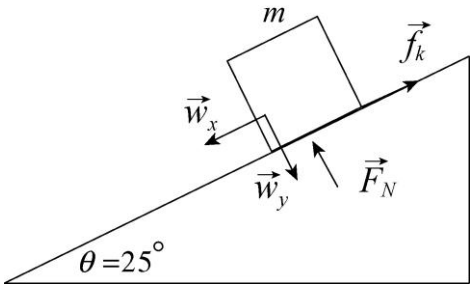
$$\mu_s = f_{s,\max}/F_N = 0.34.$$

(b) We apply Newton's second law to the horizontal x axis (positive in the direction of motion):

$$F - f_k = ma \;\Rightarrow\; 1.8t - f_k = (1.5)(1.2t - 2.4)$$

Thus, we find $f_k = 3.6$ N. Therefore, $\mu_k = f_k / F_N = 0.24$.

94. In the figure below, m = 140/9.8 = 14.3 kg is the mass of the child. We use $\vec{w}_x$ and $\vec{w}_y$ as the components of the gravitational pull of Earth on the block; their magnitudes are $w_x = mg \sin\theta$ and $w_y = mg \cos\theta$.



(a) With the x axis directed up along the incline (so that a = –0.86 m/s^2), Newton's second law leads to

$$f_k - 140\sin 25° = m(-0.86)$$

which yields f_k = 47 N. We also apply Newton's second law to the y axis (perpendicular to the incline surface), where the acceleration-component is zero:

$$F_N - 140\cos 25° = 0 \quad \Rightarrow \quad F_N = 127 \text{ N}.$$

Therefore, $\mu_k = f_k/F_N = 0.37$.

(b) Returning to our first equation in part (a), we see that if the downhill component of the weight force were insufficient to overcome static friction, the child would not slide at all. Therefore, we require 140 sin 25° > $f_{s,\max} = \mu_s F_N$, which leads to tan 25° = 0.47 > μ_s. The minimum value of μ_s equals μ_k and is more subtle; reference to §6-1 is recommended. If μ_k exceeded μ_s then when static friction were overcome (as the incline is raised) then it should start to move – which is impossible if f_k is large enough to cause deceleration! The bounds on μ_s are therefore given by 0.47 > μ_s > 0.37.

95. (a) The x component of $\vec{F}$ contributes to the motion of the crate while its y component indirectly contributes to the inhibiting effects of friction (by increasing the normal force). Along the y direction, we have $F_N - F\cos\theta - mg = 0$ and along the x direction we have $F\sin\theta - f_k = 0$ (since it is not accelerating, according to the problem). Also, Eq. 6-2 gives $f_k = \mu_k F_N$. Solving these equations for F yields

$$F = \frac{\mu_k mg}{\sin\theta - \mu_k \cos\theta} \ .$$

(b) When $\theta < \theta_0 = \tan^{-1}\mu_s$, F will not be able to move the mop head.

96. (a) The distance traveled in one revolution is $2\pi R = 2\pi(4.6\text{ m}) = 29\text{ m}$. The (constant) speed is consequently $v = (29\text{ m})/(30\text{ s}) = 0.96\text{ m/s}$.

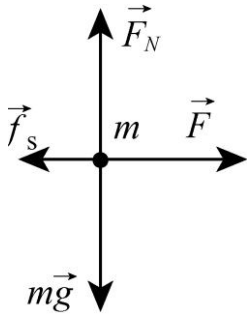
(b) Newton's second law (using Eq. 6-17 for the magnitude of the acceleration) leads to

$$f_s = m\left(\frac{v^2}{R}\right) = m(0.20)$$

in SI units. Noting that $F_N = mg$ in this situation, the maximum possible static friction is $f_{s,\max} = \mu_s mg$ using Eq. 6-1. Equating this with $f_s = m(0.20)$ we find the mass m cancels and we obtain $\mu_s = 0.20/9.8 = 0.021$.

97. **THINK** In this problem a force is applied to accelerate a box. From the distance traveled and the speed at that instant, we can calculate the coefficient of kinetic friction between the box and the floor.

EXPRESS The free-body diagram is shown to the right. We adopt the familiar axes with $+x$ rightward and $+y$ upward, and refer to the 85 N horizontal push of the worker as F (and assume it to be rightward). Applying Newton's second law to the x axis and y axis, respectively, produces



$$F - f_k = ma_x, \qquad F_N - mg = 0.$$

On the other hand, using Eq. 2-16 ($v^2 = v_0^2 + 2a_x\Delta x$), we find the acceleration to be

$$a_x = \frac{v^2 - v_0^2}{2\Delta x} = \frac{(1.0\text{ m/s})^2 - 0}{2(1.4\text{ m})} = 0.357\text{ m/s}^2.$$

The above equations can be combined to give μ_k.

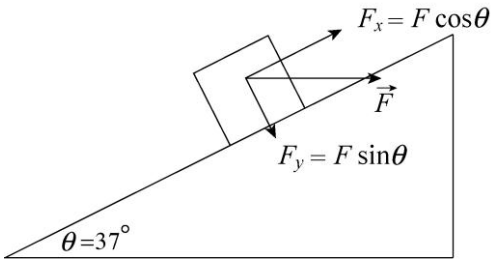
ANALYZE Using $f_k = \mu_k F_N$, we find the coefficient of kinetic friction between the box and the floor to be

$$\mu_k = \frac{f_k}{F_N} = \frac{F - ma_x}{mg} = \frac{85\text{ N} - (40\text{ kg})(0.357\text{ m/s}^2)}{(40\text{ kg})(9.8\text{ m/s}^2)} = 0.18.$$

LEARN In general, the acceleration can be written as $a_x = (F/m) - \mu_k g$. We see that the smaller the value of μ_k, the greater the acceleration. In the limit $\mu_k = 0$, we simply have $a_x = F/m$.

98. We resolve this horizontal force into appropriate components.

(a) Applying Newton's second law to the x (directed uphill) and y (directed away from the incline surface) axes, we obtain



$$F\cos\theta - f_k - mg\sin\theta = ma$$
$$F_N - F\sin\theta - mg\cos\theta = 0.$$

Using $f_k = \mu_k F_N$, these equations lead to

$$a = \frac{F}{m}(\cos\theta - \mu_k \sin\theta) - g(\sin\theta + \mu_k \cos\theta)$$

which yields $a = -2.1$ m/s^2, or $|a| = 2.1$ m/s^2, for $\mu_k = 0.30$, $F = 50$ N and $m = 5.0$ kg.

(b) The direction of $\vec{a}$ is down the plane.

(c) With $v_0 = +4.0$ m/s and $v = 0$, Eq. 2-16 gives $\Delta x = -\dfrac{(4.0\text{ m/s})^2}{2(-2.1\text{ m/s}^2)} = 3.9$ m.

(d) We expect $\mu_s \geq \mu_k$; otherwise, an object started into motion would immediately start decelerating (before it gained any speed)! In the minimal expectation case, where $\mu_s = 0.30$, the maximum possible (downhill) static friction is, using Eq. 6-1,

$$f_{s,\max} = \mu_s F_N = \mu_s (F\sin\theta + mg\cos\theta)$$

which turns out to be 21 N. But in order to have no acceleration along the x axis, we must have

$$f_s = F\cos\theta - mg\sin\theta = 10\text{ N}$$

(the fact that this is positive reinforces our suspicion that $\vec{f}_s$ points downhill). Since the f_s needed to remain at rest is less than $f_{s,\max}$ then it stays at that location.

99. (a) We note that $F_N = mg$ in this situation, so

$$f_{s,\max} = \mu_s mg = (0.52)(11\text{ kg})(9.8\text{ m/s}^2) = 56\text{ N}.$$

Consequently, the horizontal force $\vec{F}$ needed to initiate motion must be (at minimum) slightly more than 56 N.

(b) Analyzing vertical forces when $\vec{F}$ is at nonzero θ yields

$$F\sin\theta + F_N = mg \;\Rightarrow\; f_{s,\max} = \mu_s(mg - F\sin\theta).$$

Now, the horizontal component of $\vec{F}$ needed to initiate motion must be (at minimum) slightly more than this, so

$$F\cos\theta = \mu_s(mg - F\sin\theta) \Rightarrow F = \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta}$$

which yields F = 59 N when $\theta = 60°$.

(c) We now set $\theta = -60°$ and obtain

$$F = \frac{(0.52)(11\,\text{kg})(9.8\,\text{m/s}^2)}{\cos(-60°) + (0.52)\sin(-60°)} = 1.1\times10^3\ \text{N}.$$

100. (a) If the skier covers a distance L during time t with zero initial speed and a constant acceleration a, then $L = at^2/2$, which gives the acceleration a_1 for the first (old) pair of skis:

$$a_1 = \frac{2L}{t_1^2} = \frac{2(200\,\text{m})}{(61\,\text{s})^2} = 0.11\ \text{m/s}^2.$$

(b) The acceleration a_2 for the second (new) pair is

$$a_2 = \frac{2L}{t_2^2} = \frac{2(200\,\text{m})}{(42\,\text{s})^2} = 0.23\ \text{m/s}^2\ .$$

(c) The net force along the slope acting on the skier of mass m is

$$F_{\text{net}} = mg\sin\theta - f_k = mg(\sin\theta - \mu_k\cos\theta) = ma$$

which we solve for μ_{k1} for the first pair of skis:

$$\mu_{k1} = \tan\theta - \frac{a_1}{g\cos\theta} = \tan 3.0° - \frac{0.11\,\text{m/s}^2}{(9.8\,\text{m/s}^2)\cos 3.0°} = 0.041$$

(d) For the second pair, we have

$$\mu_{k2} = \tan\theta - \frac{a_2}{g\cos\theta} = \tan 3.0° - \frac{0.23\,\text{m/s}^2}{(9.8\,\text{m/s}^2)\cos 3.0°} = 0.029\ .$$

101. If we choose “downhill” positive, then Newton’s law gives

$$mg\sin\theta - f_k = ma$$

for the sliding child. Now using Eq. 6-12

$$f_k = \mu_k F_N = \mu_k m g,$$

so we obtain $a = g(\sin\theta - \mu_k \cos\theta) = -0.5$ m/s^2 (note that the problem gives the direction of the acceleration vector as uphill, even though the child is sliding downhill, so it is a deceleration). With $\theta = 35°$, we solve for the coefficient and find $\mu_k = 0.76$.

102. (a) Our $+x$ direction is horizontal and is chosen (as we also do with $+y$) so that the components of the 100 N force $\vec{F}$ are non-negative. Thus, $F_x = F \cos\theta = 100$ N, which the textbook denotes F_h in this problem.

(b) Since there is no vertical acceleration, application of Newton's second law in the y direction gives

$$F_N + F_y = mg \Rightarrow F_N = mg - F\sin\theta$$

where $m = 25.0$ kg. This yields $F_N = 245$ N in this case ($\theta = 0°$).

(c) Now, $F_x = F_h = F \cos\theta = 86.6$ N for $\theta = 30.0°$.

(d) And $F_N = mg - F \sin\theta = 195$ N.

(e) We find $F_x = F_h = F \cos\theta = 50.0$ N for $\theta = 60.0°$.

(f) And $F_N = mg - F \sin\theta = 158$ N.

(g) The condition for the chair to slide is

$$F_x > f_{s,\max} = \mu_s F_N \quad \text{where} \quad \mu_s = 0.42.$$

For $\theta = 0°$, we have

$$F_x = 100 \text{ N} < f_{s,\max} = (0.42)(245 \text{ N}) = 103 \text{ N}$$

so the crate remains at rest.

(h) For $\theta = 30.0°$, we find $F_x = 86.6 \text{ N} > f_{s,\max} = (0.42)(195 \text{ N}) = 81.9 \text{ N}$, so the crate slides.

(i) For $\theta = 60°$, we get $F_x = 50.0 \text{ N} < f_{s,\max} = (0.42)(158 \text{ N}) = 66.4 \text{ N}$, which means the crate must remain at rest.

103. (a) The intuitive conclusion, that the tension is greatest at the bottom of the swing, is certainly supported by application of Newton's second law there:

$$T - mg = \frac{mv^2}{R} \Rightarrow T = m\left(g + \frac{v^2}{R}\right)$$

where Eq. 6-18 has been used. Increasing the speed eventually leads to the tension at the bottom of the circle reaching that breaking value of 40 N.

(b) Solving the above equation for the speed, we find

$$v = \sqrt{R\left(\frac{T}{m} - g\right)} = \sqrt{(0.91\text{ m})\left(\frac{40\text{ N}}{0.37\text{ kg}} - 9.8\text{ m/s}^2\right)}$$

which yields v = 9.5 m/s.

104. (a) The component of the weight along the incline (with downhill understood as the positive direction) is $mg\sin\theta$ where m = 630 kg and θ = 10.2°. With f = 62.0 N, Newton's second law leads to $mg\sin\theta - f = ma$, which yields a = 1.64 m/s^2. Using Eq. 2-15, we have

$$80.0\text{ m} = \left(6.20\ \frac{\text{m}}{\text{s}}\right)t + \frac{1}{2}\left(1.64\ \frac{\text{m}}{\text{s}^2}\right)t^2 .$$

This is solved using the quadratic formula. The positive root is t = 6.80 s.

(b) Running through the calculation of part (a) with f = 42.0 N instead of f = 62 N results in t = 6.76 s.

105. Except for replacing f_s with f_k, Fig 6-5 in the textbook is appropriate. With that figure in mind, we choose uphill as the +x direction. Applying Newton's second law to the x axis, we have

$$f_k - W\sin\theta = ma \quad \text{where} \quad m = \frac{W}{g},$$

and where W = 40 N, a = +0.80 m/s^2 and θ = 25°. Thus, we find f_k = 20 N. Along the y-axis, we have

$$\sum \vec{F}_y = 0 \Rightarrow F_N = W\cos\theta$$

so that $\mu_k = f_k / F_N$ = 0.56.