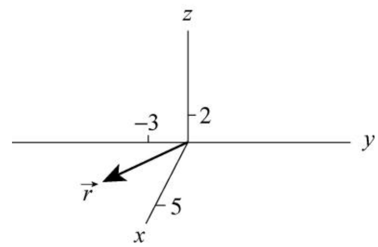
Chapter 4

1. (a) The magnitude of $\vec{r}$ is

$$|\vec{r}| = \sqrt{(5.0\text{ m})^2 + (-3.0\text{ m})^2 + (2.0\text{ m})^2} = 6.2\text{ m}.$$



(b) A sketch is shown. The coordinate values are in meters.

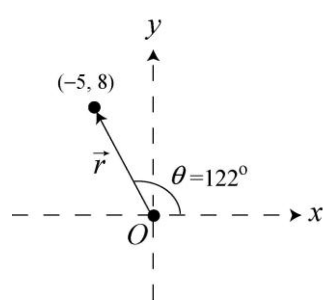
2. (a) The position vector, according to Eq. 4-1, is $\vec{r} = (-5.0\text{ m})\,\hat{\text{i}} + (8.0\text{ m})\hat{\text{j}}$.

(b) The magnitude is $|\vec{r}| = \sqrt{x^2 + y^2 + z^2} = \sqrt{(-5.0\text{ m})^2 + (8.0\text{ m})^2 + (0\text{ m})^2} = 9.4\text{ m}.$

(c) Many calculators have polar ↔ rectangular conversion capabilities that make this computation more efficient than what is shown below. Noting that the vector lies in the *xy* plane and using Eq. 3-6, we obtain:

$$\theta = \tan^{-1}\left(\frac{8.0\text{ m}}{-5.0\text{ m}}\right) = -58° \text{ or } 122°$$

where the latter possibility (122° measured counterclockwise from the +*x* direction) is chosen since the signs of the components imply the vector is in the second quadrant.



(d) The sketch is shown to the right. The vector is 122° counterclockwise from the +*x* direction.

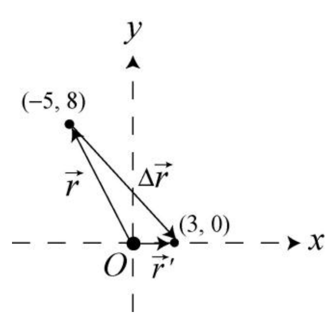
(e) The displacement is $\Delta\vec{r} = \vec{r}\,' - \vec{r}$ where $\vec{r}$ is given in part (a) and $\vec{r}\,' = (3.0\text{ m})\hat{\text{i}}$. Therefore, $\Delta\vec{r} = (8.0\text{ m})\hat{\text{i}} - (8.0\text{ m})\hat{\text{j}}$.

(f) The magnitude of the displacement is

$$|\Delta\vec{r}| = \sqrt{(8.0\text{ m})^2 + (-8.0\text{ m})^2} = 11\text{ m}.$$

(g) The angle for the displacement, using Eq. 3-6, is

$$\tan^{-1}\left(\frac{8.0\text{ m}}{-8.0\text{ m}}\right) = -45° \text{ or } 135°$$



where we choose the former possibility (–45°, or 45° measured *clockwise* from +*x*) since the signs of the components imply the vector is in the fourth quadrant. A sketch of $\Delta\vec{r}$ is shown on the right.

3. The initial position vector $\vec{r}_o$ satisfies $\vec{r} - \vec{r}_o = \Delta\vec{r}$, which results in

$$\vec{r}_o = \vec{r} - \Delta\vec{r} = (3.0\hat{j} - 4.0\hat{k})\text{m} - (2.0\hat{i} - 3.0\hat{j} + 6.0\hat{k})\text{m} = (-2.0\text{ m})\hat{i} + (6.0\text{ m})\hat{j} + (-10\text{ m})\hat{k}.$$

4. We choose a coordinate system with origin at the clock center and +*x* rightward (toward the "3:00" position) and +*y* upward (toward "12:00").

(a) In unit-vector notation, we have $\vec{r}_1 = (10\text{ cm})\hat{i}$ and $\vec{r}_2 = (-10\text{ cm})\hat{j}$. Thus, Eq. 4-2 gives

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (-10\text{ cm})\hat{i} + (-10\text{ cm})\hat{j}.$$

The magnitude is given by $|\Delta\vec{r}| = \sqrt{(-10\text{ cm})^2 + (-10\text{ cm})^2} = 14\text{ cm}.$

(b) Using Eq. 3-6, the angle is

$$\theta = \tan^{-1}\left(\frac{-10\text{ cm}}{-10\text{ cm}}\right) = 45°\text{ or }-135°.$$

We choose $-135°$ since the desired angle is in the third quadrant. In terms of the magnitude-angle notation, one may write

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (-10\text{ cm})\hat{i} + (-10\text{ cm})\hat{j} \rightarrow (14\text{ cm}\angle -135°).$$

(c) In this case, we have $\vec{r}_1 = (-10\text{ cm})\hat{j}$ and $\vec{r}_2 = (10\text{ cm})\hat{j}$, and $\Delta\vec{r} = (20\text{ cm})\hat{j}$. Thus, $|\Delta\vec{r}| = 20\text{ cm}.$

(d) Using Eq. 3-6, the angle is given by

$$\theta = \tan^{-1}\left(\frac{20\text{ cm}}{0\text{ cm}}\right) = 90°.$$

(e) In a full-hour sweep, the hand returns to its starting position, and the displacement is zero.

(f) The corresponding angle for a full-hour sweep is also zero.

5. **THINK** This problem deals with the motion of a train in two dimensions. The entire trip consists of three parts, and we're interested in the overall average velocity.

EXPRESS The average velocity of the entire trip is given by Eq. 4-8, $\vec{v}_{\text{avg}} = \Delta\vec{r}/\Delta t$, where the total displacement $\Delta\vec{r} = \Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3$ is the sum of three displacements (each result of a constant velocity during a given time), and $\Delta t = \Delta t_1 + \Delta t_2 + \Delta t_3$ is the total amount of time for the trip. We use a coordinate system with +x for East and +y for North.

ANALYZE (a) In unit-vector notation, the first displacement is given by

$$\Delta\vec{r}_1 = \left(60.0\ \frac{\text{km}}{\text{h}}\right)\left(\frac{40.0\ \text{min}}{60\ \text{min/h}}\right)\hat{\text{i}} = (40.0\ \text{km})\hat{\text{i}}.$$

The second displacement has a magnitude of $(60.0\ \frac{\text{km}}{\text{h}})\cdot(\frac{20.0\ \text{min}}{60\ \text{min/h}}) = 20.0\ \text{km}$, and its direction is 40° north of east. Therefore,

$$\Delta\vec{r}_2 = (20.0\ \text{km})\cos(40.0°)\,\hat{\text{i}} + (20.0\ \text{km})\sin(40.0°)\,\hat{\text{j}} = (15.3\ \text{km})\,\hat{\text{i}} + (12.9\ \text{km})\,\hat{\text{j}}.$$

Similarly, the third displacement is

$$\Delta\vec{r}_3 = -\left(60.0\ \frac{\text{km}}{\text{h}}\right)\left(\frac{50.0\ \text{min}}{60\ \text{min/h}}\right)\hat{\text{i}} = (-50.0\ \text{km})\,\hat{\text{i}}.$$

Thus, the total displacement is

$$\Delta\vec{r} = \Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3 = (40.0\ \text{km})\hat{\text{i}} + (15.3\ \text{km})\,\hat{\text{i}} + (12.9\ \text{km})\,\hat{\text{j}} - (50.0\ \text{km})\,\hat{\text{i}}$$
$$= (5.30\ \text{km})\,\hat{\text{i}} + (12.9\ \text{km})\,\hat{\text{j}}.$$

The time for the trip is $\Delta t = (40.0 + 20.0 + 50.0)$ min = 110 min, which is equivalent to 1.83 h. Eq. 4-8 then yields

$$\vec{v}_{\text{avg}} = \frac{(5.30\ \text{km})\,\hat{\text{i}} + (12.9\ \text{km})\,\hat{\text{j}}}{1.83\ \text{h}} = (2.90\ \text{km/h})\,\hat{\text{i}} + (7.01\ \text{km/h})\,\hat{\text{j}}.$$

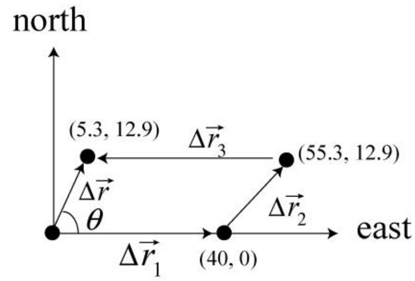
The magnitude of $\vec{v}_{\text{avg}}$ is $|\vec{v}_{\text{avg}}| = \sqrt{(2.90\ \text{km/h})^2 + (7.01\ \text{km/h})^2} = 7.59\ \text{km/h}$.

(b) The angle is given by

$$\theta = \tan^{-1}\left(\frac{v_{\text{avg},y}}{v_{\text{avg},x}}\right) = \tan^{-1}\left(\frac{7.01\ \text{km/h}}{2.90\ \text{km/h}}\right) = 67.5°\ \ \text{(north of east)},$$

or $22.5°$ east of due north.

LEARN The displacement of the train is depicted in the figure below:



Note that the net displacement $\Delta\vec{r}$ is found by adding $\Delta\vec{r}_1$, $\Delta\vec{r}_2$ and $\Delta\vec{r}_3$ vectorially.

6. To emphasize the fact that the velocity is a function of time, we adopt the notation $v(t)$ for dx/dt.

(a) Equation 4-10 leads to

$$v(t)=\frac{d}{dt}\,(3.00t\hat{\text{i}}\,-4.00t^2\hat{\text{j}}+2.00\hat{\text{k}})=(3.00\text{ m/s})\hat{\text{i}}\,-(8.00\text{ m/s}^2)t\,\hat{\text{j}}$$

(b) Evaluating this result at t = 2.00 s produces $\vec{v}$ = (3.00$\hat{\text{i}}$ − 16.0$\hat{\text{j}}$) m/s.

(c) The speed at t = 2.00 s is $v=|\vec{v}|=\sqrt{(3.00\text{ m/s})^2+(-16.0\text{ m/s})^2}=16.3$ m/s.

(d) The angle of $\vec{v}$ at that moment is

$$\tan^{-1}\left(\frac{-16.0\text{ m/s}}{3.00\text{ m/s}}\right)=-79.4°\text{ or }101°$$

where we choose the first possibility (79.4° measured *clockwise* from the +x direction, or 281° counterclockwise from +x) since the signs of the components imply the vector is in the fourth quadrant.

7. Using Eq. 4-3 and Eq. 4-8, we have

$$\vec{v}_{\text{avg}}=\frac{(-2.0\hat{\text{i}}+8.0\hat{\text{j}}-2.0\hat{\text{k}})\text{ m}-(5.0\hat{\text{i}}-6.0\hat{\text{j}}+2.0\hat{\text{k}})\text{ m}}{10\text{ s}}=(-0.70\hat{\text{i}}+1.40\hat{\text{j}}-0.40\hat{\text{k}})\text{ m/s}.$$

8. Our coordinate system has $\hat{\text{i}}$ pointed east and $\hat{\text{j}}$ pointed north. The first displacement is $\vec{r}_{AB}=(483\text{ km})\hat{\text{i}}$ and the second is $\vec{r}_{BC}=(-966\text{ km})\hat{\text{j}}$.

(a) The net displacement is

$$\vec{r}_{AC} = \vec{r}_{AB} + \vec{r}_{BC} = (483\ \text{km})\hat{\text{i}} - (966\ \text{km})\hat{\text{j}}$$

which yields $|\vec{r}_{AC}| = \sqrt{(483\ \text{km})^2 + (-966\ \text{km})^2} = 1.08\times10^3\ \text{km}$.

(b) The angle is given by

$$\theta = \tan^{-1}\left(\frac{-966\ \text{km}}{483\ \text{km}}\right) = -63.4°.$$

We observe that the angle can be alternatively expressed as 63.4° south of east, or 26.6° east of south.

(c) Dividing the magnitude of $\vec{r}_{AC}$ by the total time (2.25 h) gives

$$\vec{v}_{\text{avg}} = \frac{(483\ \text{km})\hat{\text{i}} - (966\ \text{km})\hat{\text{j}}}{2.25\ \text{h}} = (215\ \text{km/h})\hat{\text{i}} - (429\ \text{km/h})\hat{\text{j}}$$

with a magnitude $|\vec{v}_{\text{avg}}| = \sqrt{(215\ \text{km/h})^2 + (-429\ \text{km/h})^2} = 480\ \text{km/h}$.

(d) The direction of $\vec{v}_{\text{avg}}$ is 26.6° east of south, same as in part (b). In magnitude-angle notation, we would have $\vec{v}_{\text{avg}} = (480\ \text{km/h}\ \angle\ -63.4°)$.

(e) Assuming the *AB* trip was a straight one, and similarly for the *BC* trip, then $|\vec{r}_{AB}|$ is the distance traveled during the *AB* trip, and $|\vec{r}_{BC}|$ is the distance traveled during the *BC* trip. Since the average speed is the total distance divided by the total time, it equals

$$\frac{483\ \text{km} + 966\ \text{km}}{2.25\ \text{h}} = 644\ \text{km/h}.$$

9. The (x,y) coordinates (in meters) of the points are A = (15, –15), B = (30, –45), C = (20, –15), and D = (45, 45). The respective times are t_A = 0, t_B = 300 s, t_C = 600 s, and t_D = 900 s. Average velocity is defined by Eq. 4-8. Each displacement $\Delta\vec{r}$ is understood to originate at point A.

(a) The average velocity having the least magnitude (5.0 m/600 s) is for the displacement ending at point C: $|\vec{v}_{\text{avg}}| = 0.0083$ m/s.

(b) The direction of $\vec{v}_{\text{avg}}$ is 0° (measured counterclockwise from the +x axis).

(c) The average velocity having the greatest magnitude ($\sqrt{(15\text{ m})^2+(30\text{ m})^2}/300\text{ s}$) is for the displacement ending at point *B*: $|\vec{v}_{avg}|= 0.11$ m/s.

(d) The direction of $\vec{v}_{avg}$ is 297° (counterclockwise from +*x*) or −63° (which is equivalent to measuring 63° *clockwise* from the +*x* axis).

10. We differentiate $\vec{r} = 5.00t\,\hat{\text{i}} + (et + ft^2)\hat{\text{j}}$.

(a) The particle's motion is indicated by the derivative of $\vec{r}$: $\vec{v}$ = 5.00 $\hat{\text{i}}$ + (e + 2ft) $\hat{\text{j}}$. The angle of its direction of motion is consequently

$$\theta = \tan^{-1}(v_y/v_x) = \tan^{-1}[(e + 2ft)/5.00].$$

The graph indicates θ_o = 35.0°, which determines the parameter *e*:

$$e = (5.00\text{ m/s})\tan(35.0°) = 3.50\text{ m/s}.$$

(b) We note (from the graph) that θ = 0 when t = 14.0 s. Thus, $e + 2ft = 0$ at that time. This determines the parameter *f* :

$$f = \frac{-e}{2t} = \frac{-3.5\text{ m/s}}{2(14.0\text{ s})} = -0.125\text{ m/s}^2.$$

11. In parts (b) and (c), we use Eq. 4-10 and Eq. 4-16. For part (d), we find the direction of the velocity computed in part (b), since that represents the asked-for tangent line.

(a) Plugging into the given expression, we obtain

$$\vec{r}\Big|_{t=2.00} = [2.00(8)-5.00(2)]\hat{\text{i}} + [6.00-7.00(16)]\hat{\text{j}} = (6.00\hat{\text{i}} - 106\hat{\text{j}})\text{ m}$$

(b) Taking the derivative of the given expression produces

$$\vec{v}(t) = (6.00t^2 - 5.00)\,\hat{\text{i}} - 28.0t^3\,\hat{\text{j}}$$

where we have written $v(t)$ to emphasize its dependence on time. This becomes, at t = 2.00 s, $\vec{v}$ = (19.0$\hat{\text{i}}$ − 224$\hat{\text{j}}$) m/s.

(c) Differentiating the $\vec{v}(t)$ found above, with respect to t produces $12.0t\,\hat{\text{i}} - 84.0t^2\,\hat{\text{j}}$, which yields $\vec{a}$ =(24.0$\hat{\text{i}}$−336$\hat{\text{j}}$) m/s^2 at t = 2.00 s.

(d) The angle of $\vec{v}$, measured from +*x*, is either

$$\tan^{-1}\left(\frac{-224\text{ m/s}}{19.0\text{ m/s}}\right) = -85.2° \text{ or } 94.8°$$

where we settle on the first choice (–85.2°, which is equivalent to 275° measured counterclockwise from the +*x* axis) since the signs of its components imply that it is in the fourth quadrant.

12. We adopt a coordinate system with $\hat{\text{i}}$ pointed east and $\hat{\text{j}}$ pointed north; the coordinate origin is the flagpole. We "translate" the given information into unit-vector notation as follows:

$$\vec{r}_\text{o} = (40.0\text{ m})\hat{\text{i}} \quad \text{and} \quad \vec{v}_\text{o} = (-10.0\text{ m/s})\hat{\text{j}}$$
$$\vec{r} = (40.0\text{ m})\hat{\text{j}} \quad \text{and} \quad \vec{v} = (10.0\text{ m/s})\hat{\text{i}}.$$

(a) Using Eq. 4-2, the displacement $\Delta\vec{r}$ is

$$\Delta\vec{r} = \vec{r} - \vec{r}_\text{o} = (-40.0\text{ m})\hat{\text{i}} + (40.0\text{ m})\hat{\text{j}}$$

with a magnitude $|\Delta\vec{r}| = \sqrt{(-40.0\text{ m})^2 + (40.0\text{ m})^2} = 56.6\text{ m}.$

(b) The direction of $\Delta\vec{r}$ is

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{40.0\text{ m}}{-40.0\text{ m}}\right) = -45.0° \text{ or } 135°.$$

Since the desired angle is in the second quadrant, we pick $135°$ ($45°$ north of due west). Note that the displacement can be written as $\Delta\vec{r} = \vec{r} - \vec{r}_\text{o} = (56.6\angle 135°)$ in terms of the magnitude-angle notation.

(c) The magnitude of $\vec{v}_\text{avg}$ is simply the magnitude of the displacement divided by the time (Δt = 30.0 s). Thus, the average velocity has magnitude (56.6 m)/(30.0 s) = 1.89 m/s.

(d) Equation 4-8 shows that $\vec{v}_\text{avg}$ points in the same direction as $\Delta\vec{r}$, that is, $135°$ ($45°$ north of due west).

(e) Using Eq. 4-15, we have

$$\vec{a}_\text{avg} = \frac{\vec{v} - \vec{v}_\text{o}}{\Delta t} = (0.333\text{ m/s}^2)\hat{\text{i}} + (0.333\text{ m/s}^2)\hat{\text{j}}.$$

The magnitude of the average acceleration vector is therefore equal to $|\vec{a}_\text{avg}| = \sqrt{(0.333\text{ m/s}^2)^2 + (0.333\text{ m/s}^2)^2} = 0.471\text{ m/s}^2$.

(f) The direction of $\vec{a}_{\text{avg}}$ is

$$\theta = \tan^{-1}\left(\frac{0.333 \text{ m/s}^2}{0.333 \text{ m/s}^2}\right) = 45° \text{ or } -135°.$$

Since the desired angle is now in the first quadrant, we choose 45°, and $\vec{a}_{\text{avg}}$ points north of due east.

13. **THINK** Knowing the position of a particle as function of time allows us to calculate its corresponding velocity and acceleration by taking time derivatives.

EXPRESS From the position vector $\vec{r}(t)$, the velocity and acceleration of the particle can be found by differentiating $\vec{r}(t)$ with respect to time:

$$\vec{v} = \frac{d\vec{r}}{dt}, \quad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}.$$

ANALYZE (a) Taking the derivative of the position vector $\vec{r}(t) = \hat{\text{i}} + (4t^2)\hat{\text{j}} + t\hat{\text{k}}$ with respect to time, we have, in SI units (m/s),

$$\vec{v} = \frac{d}{dt}(\hat{\text{i}} + 4t^2\,\hat{\text{j}} + t\,\hat{\text{k}}) = 8t\,\hat{\text{j}} + \hat{\text{k}}.$$

(b) Taking another derivative with respect to time leads to, in SI units (m/s^2),

$$\vec{a} = \frac{d}{dt}(8t\,\hat{\text{j}} + \hat{\text{k}}) = 8\,\hat{\text{j}}.$$

LEARN The particle undergoes constant acceleration in the +y-direction. This can be seen by noting that the y component of $\vec{r}(t)$ is $4t^2$, which is quadratic in t.

14. We use Eq. 4-15 with $\vec{v}_1$ designating the initial velocity and $\vec{v}_2$ designating the later one.

(a) The average acceleration during the $\Delta t = 4$ s interval is

$$\vec{a}_{\text{avg}} = \frac{(-2.0\,\hat{\text{i}} - 2.0\,\hat{\text{j}} + 5.0\,\hat{\text{k}}) \text{ m/s} - (4.0\,\hat{\text{i}} - 22\,\hat{\text{j}} + 3.0\,\hat{\text{k}}) \text{ m/s}}{4 \text{ s}} = (-1.5 \text{ m/s}^2)\,\hat{\text{i}} + (0.5 \text{ m/s}^2)\,\hat{\text{k}}.$$

(b) The magnitude of $\vec{a}_{\text{avg}}$ is $\sqrt{(-1.5 \text{ m/s}^2)^2 + (0.5 \text{ m/s}^2)^2} = 1.6 \text{ m/s}^2$.

(c) Its angle in the xz plane (measured from the +x axis) is one of these possibilities:

$$\tan^{-1}\left(\frac{0.5\text{ m/s}^2}{-1.5\text{ m/s}^2}\right) = -18° \text{ or } 162°$$

where we settle on the second choice since the signs of its components imply that it is in the second quadrant.

15. **THINK** Given the initial velocity and acceleration of a particle, we're interested in finding its velocity and position at a later time.

EXPRESS Since the acceleration, $\vec{a} = a_x\hat{\text{i}} + a_y\hat{\text{j}} = (-1.0\text{ m/s}^2)\hat{\text{i}} + (-0.50\text{ m/s}^2)\hat{\text{j}}$, is constant in both x and y directions, we may use Table 2-1 for the motion along each direction. This can be handled individually (for x and y) or together with the unit-vector notation (for $\Delta\vec{r}$).

Since the particle started at the origin, the coordinates of the particle at any time t are given by $\vec{r} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$. The velocity of the particle at any time t is given by $\vec{v} = \vec{v}_0 + \vec{a}t$, where $\vec{v}_0$ is the initial velocity and $\vec{a}$ is the (constant) acceleration. Along the x-direction, we have

$$x(t) = v_{0x}t + \frac{1}{2}a_x t^2, \qquad v_x(t) = v_{0x} + a_x t$$

Similarly, along the y-direction, we get

$$y(t) = v_{0y}t + \frac{1}{2}a_y t^2, \qquad v_y(t) = v_{0y} + a_y t\ .$$

Known: $v_{0x} = 3.0\text{ m/s}, v_{0y} = 0,\ a_x = -1.0\text{ m/s}^2,\ a_y = -0.5\text{ m/s}^2$.

ANALYZE (a) Substituting the values given, the components of the velocity are

$$v_x(t) = v_{0x} + a_x t = (3.0\text{ m/s}) - (1.0\text{ m/s}^2)t$$
$$v_y(t) = v_{0y} + a_y t = -(0.50\text{ m/s}^2)t$$

When the particle reaches its maximum x coordinate at $t = t_m$, we must have $v_x = 0$. Therefore, $3.0 - 1.0t_m = 0$ or $t_m = 3.0$ s. The y component of the velocity at this time is

$$v_y(t = 3.0\text{ s}) = -(0.50\text{ m/s}^2)(3.0) = -1.5\text{ m/s}$$

Thus, $\vec{v}_m = (-1.5\text{ m/s})\hat{\text{j}}$.

(b) At $t = 3.0$ s , the components of the position are

$$x(t=3.0\,\text{s}) = v_{0x}t + \frac{1}{2}a_x t^2 = (3.0\,\text{m/s})(3.0\,\text{s}) + \frac{1}{2}(-1.0\,\text{m/s}^2)(3.0\,\text{s})^2 = 4.5\ \text{m}$$
$$y(t=3.0\,\text{s}) = v_{0y}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}(-0.5\,\text{m/s}^2)(3.0\,\text{s})^2 = -2.25\ \text{m}$$

Using unit-vector notation, the results can be written as $\vec{r}_m = (4.50\ \text{m})\,\hat{\text{i}} - (2.25\ \text{m})\,\hat{\text{j}}$.

LEARN The motion of the particle in this problem is two-dimensional, and the kinematics in the *x*- and *y*-directions can be analyzed separately.

16. We make use of Eq. 4-16.

(a) The acceleration as a function of time is

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\left(6.0t - 4.0t^2\right)\hat{\text{i}} + 8.0\,\hat{\text{j}}\right) = \left(6.0 - 8.0t\right)\hat{\text{i}}$$

in SI units. Specifically, we find the acceleration vector at $t = 3.0\,\text{s}$ to be $\left(6.0 - 8.0(3.0)\right)\hat{\text{i}} = (-18\ \text{m/s}^2)\hat{\text{i}}$.

(b) The equation is $\vec{a} = \left(6.0 - 8.0t\right)\hat{\text{i}} = 0$; we find t = 0.75 s.

(c) Since the *y* component of the velocity, v_y = 8.0 m/s, is never zero, the velocity cannot vanish.

(d) Since speed is the magnitude of the velocity, we have

$$v = |\vec{v}| = \sqrt{\left(6.0t - 4.0t^2\right)^2 + \left(8.0\right)^2} = 10$$

in SI units (m/s). To solve for t, we first square both sides of the above equation, followed by some rearrangement:

$$\left(6.0t - 4.0t^2\right)^2 + 64 = 100 \quad \Rightarrow \quad \left(6.0t - 4.0t^2\right)^2 = 36$$

Taking the square root of the new expression and making further simplification lead to

$$6.0t - 4.0t^2 = \pm 6.0 \quad \Rightarrow \quad 4.0t^2 - 6.0t \pm 6.0 = 0$$

Finally, using the quadratic formula, we obtain

$$t = \frac{6.0 \pm \sqrt{36 - 4(4.0)(\pm 6.0)}}{2(8.0)}$$

where the requirement of a real positive result leads to the unique answer: t = 2.2 s.

17. We find t by applying Eq. 2-11 to motion along the y axis (with v_y = 0 characterizing $y = y_{max}$):

$$0 = (12 \text{ m/s}) + (-2.0 \text{ m/s}^2)t \quad \Rightarrow \quad t = 6.0 \text{ s}.$$

Then, Eq. 2-11 applies to motion along the x axis to determine the answer:

$$v_x = (8.0 \text{ m/s}) + (4.0 \text{ m/s}^2)(6.0 \text{ s}) = 32 \text{ m/s}.$$

Therefore, the velocity of the cart, when it reaches $y = y_{max}$, is (32 m/s)$\hat{\text{i}}$.

18. We find t by solving $\Delta x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$:

$$12.0 \text{ m} = 0 + (4.00 \text{ m/s})t + \frac{1}{2}(5.00 \text{ m/s}^2)t^2$$

where we have used Δx = 12.0 m, v_x = 4.00 m/s, and a_x = 5.00 m/s^2 . We use the quadratic formula and find t = 1.53 s. Then, Eq. 2-11 (actually, its analog in two dimensions) applies with this value of t. Therefore, its velocity (when Δx = 12.00 m) is

$$\begin{aligned}\vec{v} &= \vec{v}_0 + \vec{a}t = (4.00 \text{ m/s})\hat{\text{i}} + (5.00 \text{ m/s}^2)(1.53 \text{ s})\hat{\text{i}} + (7.00 \text{ m/s}^2)(1.53 \text{ s})\hat{\text{j}} \\ &= (11.7 \text{ m/s})\hat{\text{i}} + (10.7 \text{ m/s})\hat{\text{j}}.\end{aligned}$$

Thus, the magnitude of $\vec{v}$ is $|\vec{v}| = \sqrt{(11.7 \text{ m/s})^2 + (10.7 \text{ m/s})^2} = 15.8$ m/s.

(b) The angle of $\vec{v}$, measured from +x, is

$$\tan^{-1}\left(\frac{10.7 \text{ m/s}}{11.7 \text{ m/s}}\right) = 42.6°.$$

19. We make use of Eq. 4-16 and Eq. 4-10.

Using $\vec{a} = 3t\hat{\text{i}} + 4t\hat{\text{j}}$, we have (in m/s)

$$\vec{v}(t) = \vec{v}_0 + \int_0^t \vec{a}\; dt = (5.00\hat{\text{i}} + 2.00\hat{\text{j}}) + \int_0^t (3t\hat{\text{i}} + 4t\hat{\text{j}})\; dt = \left(5.00 + 3t^2/2\right)\hat{\text{i}} + \left(2.00 + 2t^2\right)\hat{\text{j}}$$

Integrating using Eq. 4-10 then yields (in meters)

$$\vec{r}(t)=\vec{r}_0+\int_0^t \vec{v}dt=(20.0\hat{\mathrm{i}}+40.0\hat{\mathrm{j}})+\int_0^t[(5.00+3t^2/2)\hat{\mathrm{i}}+(2.00+2t^2)\hat{\mathrm{j}}]dt$$
$$=(20.0\hat{\mathrm{i}}+40.0\hat{\mathrm{j}})+(5.00t+t^3/2)\hat{\mathrm{i}}+(2.00t+2t^3/3)\hat{\mathrm{j}}$$
$$=(20.0+5.00t+t^3/2)\hat{\mathrm{i}}+(40.0+2.00t+2t^3/3)\hat{\mathrm{j}}$$

(a) At $t=4.00$ s, we have $\vec{r}(t=4.00\,\mathrm{s})=(72.0\text{ m})\hat{\mathrm{i}}+(90.7\text{ m})\hat{\mathrm{j}}$.

(b) $\vec{v}(t=4.00\,\mathrm{s})=(29.0\text{ m/s})\hat{\mathrm{i}}+(34.0\text{ m/s})\hat{\mathrm{j}}$. Thus, the angle between the direction of travel and +*x*, measured counterclockwise, is $\theta=\tan^{-1}[(34.0\,\mathrm{m/s})/(29.0\,\mathrm{m/s})]=49.5°$.

20. The acceleration is constant so that use of Table 2-1 (for both the *x* and *y* motions) is permitted. Where units are not shown, SI units are to be understood. Collision between particles *A* and *B* requires two things. First, the *y* motion of *B* must satisfy (using Eq. 2-15 and noting that θ is measured from the *y* axis)

$$y=\frac{1}{2}a_yt^2 \quad\Rightarrow\quad 30\text{ m}=\frac{1}{2}\left[(0.40\text{ m/s}^2)\cos\theta\right]t^2.$$

Second, the *x* motions of *A* and *B* must coincide:

$$vt=\frac{1}{2}a_xt^2 \quad\Rightarrow\quad (3.0\text{ m/s})t=\frac{1}{2}\left[(0.40\text{ m/s}^2)\sin\theta\right]t^2.$$

We eliminate a factor of *t* in the last relationship and formally solve for time:

$$t=\frac{2v}{a_x}=\frac{2(3.0\text{ m/s})}{(0.40\text{ m/s}^2)\sin\theta}.$$

This is then plugged into the previous equation to produce

$$30\text{ m}=\frac{1}{2}\left[(0.40\text{ m/s}^2)\cos\theta\right]\left(\frac{2(3.0\text{ m/s})}{(0.40\text{ m/s}^2)\sin\theta}\right)^2$$

which, with the use of $\sin^2\theta = 1-\cos^2\theta$, simplifies to

$$30=\frac{9.0}{0.20}\,\frac{\cos\theta}{1-\cos^2\theta} \quad\Rightarrow\quad 1-\cos^2\theta=\frac{9.0}{(0.20)(30)}\cos\theta.$$

We use the quadratic formula (choosing the positive root) to solve for $\cos\theta$:

$$\cos\theta=\frac{-1.5+\sqrt{1.5^2-4(1.0)(-1.0)}}{2}=\frac{1}{2}$$

which yields $\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60°$.

21. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The initial velocity is horizontal so that $v_{0y} = 0$ and $v_{0x} = v_0 = 10$ m/s.

(a) With the origin at the initial point (where the dart leaves the thrower's hand), the y coordinate of the dart is given by $y = -\frac{1}{2}gt^2$, so that with y = $-PQ$ we have $PQ = \frac{1}{2}\left(9.8 \text{ m/s}^2\right)\left(0.19 \text{ s}\right)^2 = 0.18 \text{ m}.$

(b) From $x = v_0t$ we obtain x = (10 m/s)(0.19 s) = 1.9 m.

22. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable.

(a) With the origin at the initial point (edge of table), the y coordinate of the ball is given by $y = -\frac{1}{2}gt^2$. If t is the time of flight and y = –1.20 m indicates the level at which the ball hits the floor, then

$$t = \sqrt{\frac{2\left(-1.20 \text{ m}\right)}{-9.80 \text{ m/s}^2}} = 0.495 \text{s}.$$

(b) The initial (horizontal) velocity of the ball is $\vec{v} = v_0\,\hat{\text{i}}$. Since x = 1.52 m is the horizontal position of its impact point with the floor, we have $x = v_0t$. Thus,

$$v_0 = \frac{x}{t} = \frac{1.52 \text{ m}}{0.495 \text{ s}} = 3.07 \text{ m/s}.$$

23. (a) From Eq. 4-22 (with θ_0 = 0), the time of flight is

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(45.0 \text{ m})}{9.80 \text{ m/s}^2}} = 3.03 \text{ s}.$$

(b) The horizontal distance traveled is given by Eq. 4-21:

$$\Delta x = v_0t = (250 \text{ m/s})(3.03 \text{ s}) = 758 \text{ m}.$$

(c) And from Eq. 4-23, we find

$$\left|v_y\right| = gt = (9.80 \text{ m/s}^2)(3.03 \text{ s}) = 29.7 \text{ m/s}.$$

24. We use Eq. 4-26

$$R_{\max} = \left(\frac{v_0^2}{g}\sin 2\theta_0\right)_{\max} = \frac{v_0^2}{g} = \frac{(9.50\text{m/s})^2}{9.80\text{m/s}^2} = 9.209\text{ m} \approx 9.21\text{m}$$

to compare with Powell's long jump; the difference from $R_{\max}$ is only ΔR =(9.21m – 8.95m) = 0.259 m.

25. Using Eq. (4-26), the take-off speed of the jumper is

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta_0}} = \sqrt{\frac{(9.80\text{ m/s}^2)(77.0\text{ m})}{\sin 2(12.0°)}} = 43.1\text{ m/s}$$

26. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is the throwing point (the stone's initial position). The x component of its initial velocity is given by $v_{0x} = v_0 \cos\theta_0$ and the y component is given by $v_{0y} = v_0 \sin\theta_0$, where v_0 = 20 m/s is the initial speed and θ_0 = 40.0° is the launch angle.

(a) At t = 1.10 s, its x coordinate is

$$x = v_0 t\cos\theta_0 = (20.0\text{ m/s})(1.10\text{ s})\cos 40.0° = 16.9\text{ m}$$

(b) Its y coordinate at that instant is

$$y = v_0 t\sin\theta_0 - \frac{1}{2}gt^2 = (20.0\text{m/s})(1.10\text{s})\sin 40.0° - \frac{1}{2}(9.80\text{m/s}^2)(1.10\text{s})^2 = 8.21\text{ m.}$$

(c) At t' = 1.80 s, its x coordinate is $x = (20.0\text{ m/s})(1.80\text{ s})\cos 40.0° = 27.6\text{ m.}$

(d) Its y coordinate at t' is

$$y = (20.0\text{m/s})(1.80\text{s})\sin 40.0° - \frac{1}{2}(9.80\text{m/s}^2)(1.80\text{s}^2) = 7.26\text{m.}$$

(e) The stone hits the ground earlier than t = 5.0 s. To find the time when it hits the ground solve $y = v_0 t\sin\theta_0 - \frac{1}{2}gt^2 = 0$ for t. We find

$$t = \frac{2v_0}{g}\sin\theta_0 = \frac{2(20.0\text{ m/s})}{9.8\text{ m/s}^2}\sin 40° = 2.62\text{ s.}$$

Its x coordinate on landing is

$$x = v_0 t \cos\theta_0 = (20.0 \text{ m/s})(2.62 \text{ s})\cos 40° = 40.2 \text{ m}.$$

(f) Assuming it stays where it lands, its vertical component at t = 5.00 s is y = 0.

27. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below the release point. We write θ_0 = –30.0° since the angle shown in the figure is measured clockwise from horizontal. We note that the initial speed of the decoy is the plane's speed at the moment of release: v_0 = 290 km/h, which we convert to SI units: (290)(1000/3600) = 80.6 m/s.

(a) We use Eq. 4-12 to solve for the time:

$$\Delta x = (v_0 \cos\theta_0)\, t \quad \Rightarrow \quad t = \frac{700 \text{ m}}{(80.6 \text{ m/s})\cos(-30.0°)} = 10.0 \text{ s}.$$

(b) And we use Eq. 4-22 to solve for the initial height y_0:

$$y - y_0 = (v_0 \sin\theta_0)\, t - \frac{1}{2} g t^2 \quad \Rightarrow \quad 0 - y_0 = (-40.3 \text{ m/s})(10.0 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(10.0 \text{ s})^2$$

which yields y_0 = 897 m.

28. (a) Using the same coordinate system assumed in Eq. 4-22, we solve for $y = h$:

$$h = y_0 + v_0 \sin\theta_0 t - \frac{1}{2} g t^2$$

which yields h = 51.8 m for y_0 = 0, v_0 = 42.0 m/s, θ_0 = 60.0°, and t = 5.50 s.

(b) The horizontal motion is steady, so $v_x = v_{0x} = v_0 \cos\theta_0$, but the vertical component of velocity varies according to Eq. 4-23. Thus, the speed at impact is

$$v = \sqrt{(v_0 \cos\theta_0)^2 + (v_0 \sin\theta_0 - g t)^2} = 27.4 \text{ m/s}.$$

(c) We use Eq. 4-24 with v_y = 0 and $y = H$:

$$H = \frac{(v_0 \sin\theta_0)^2}{2g} = 67.5 \text{ m}.$$

29. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at its initial position (where it is launched). At maximum height, we observe $v_y = 0$ and denote $v_x = v$ (which is also equal to v_{0x}). In this notation, we have $v_0 = 5v$. Next, we observe $v_0 \cos \theta_0 = v_{0x} = v$, so that we arrive at an equation (where $v \neq 0$ cancels) which can be solved for θ_0:

$$(5v)\cos\theta_0 = v \;\Rightarrow\; \theta_0 = \cos^{-1}\left(\frac{1}{5}\right) = 78.5°.$$

30. Although we could use Eq. 4-26 to find where it lands, we choose instead to work with Eq. 4-21 and Eq. 4-22 (for the soccer ball) since these will give information about where *and when* and these are also considered more fundamental than Eq. 4-26. With $\Delta y = 0$, we have

$$\Delta y = (v_0 \sin\theta_0)\, t - \frac{1}{2}gt^2 \;\Rightarrow\; t = \frac{(19.5 \text{ m/s})\sin 45.0°}{(9.80 \text{ m/s}^2)/2} = 2.81 \text{ s}.$$

Then Eq. 4-21 yields $\Delta x = (v_0 \cos \theta_0)t = 38.7$ m. Thus, using Eq. 4-8, the player must have an average velocity of

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{(38.7 \text{ m})\,\hat{\text{i}} - (55 \text{ m})\hat{\text{i}}}{2.81 \text{ s}} = (-5.8 \text{ m/s})\,\hat{\text{i}}$$

which means his average speed (assuming he ran in only one direction) is 5.8 m/s.

31. We first find the time it takes for the volleyball to hit the ground. Using Eq. 4-22, we have

$$y - y_0 = (v_0 \sin\theta_0)\, t - \frac{1}{2}gt^2 \;\Rightarrow\; 0 - 2.30 \text{ m} = (-20.0 \text{ m/s})\sin(18.0°)t - \frac{1}{2}(9.80 \text{ m/s}^2)t^2$$

which gives $t = 0.30$ s. Thus, the range of the volleyball is

$$R = (v_0 \cos\theta_0)t = (20.0 \text{ m/s})\cos 18.0°(0.30 \text{ s}) = 5.71 \text{ m}$$

On the other hand, when the angle is changed to $\theta_0' = 8.00°$, using the same procedure as shown above, we find

$$y - y_0 = (v_0 \sin\theta_0')\, t' - \frac{1}{2}gt'^2 \;\Rightarrow\; 0 - 2.30 \text{ m} = (-20.0 \text{ m/s})\sin(8.00°)t' - \frac{1}{2}(9.80 \text{ m/s}^2)t'^2$$

which yields $t' = 0.46$ s, and the range is

$$R' = (v_0 \cos\theta_0)t' = (20.0 \text{ m/s})\cos 18.0°(0.46 \text{ s}) = 9.06 \text{ m}$$

Thus, the ball travels an extra distance of

$$\Delta R = R' - R = 9.06\ \text{m} - 5.71\ \text{m} = 3.35\ \text{m}$$

32. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the release point (the initial position for the ball as it begins projectile motion in the sense of §4-5), and we let θ_0 be the angle of throw (shown in the figure). Since the horizontal component of the velocity of the ball is $v_x = v_0 \cos 40.0°$, the time it takes for the ball to hit the wall is

$$t = \frac{\Delta x}{v_x} = \frac{22.0\ \text{m}}{(25.0\ \text{m/s})\cos 40.0°} = 1.15\ \text{s}.$$

(a) The vertical distance is

$$\Delta y = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2 = (25.0\ \text{m/s})\sin 40.0°(1.15\ \text{s}) - \frac{1}{2}(9.80\ \text{m/s}^2)(1.15\ \text{s})^2 = 12.0\ \text{m}.$$

(b) The horizontal component of the velocity when it strikes the wall does not change from its initial value: $v_x = v_0 \cos 40.0° = 19.2$ m/s.

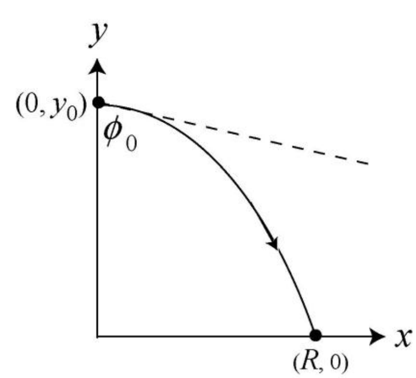
(c) The vertical component becomes (using Eq. 4-23)

$$v_y = v_0 \sin\theta_0 - gt = (25.0\ \text{m/s})\sin 40.0° - (9.80\ \text{m/s}^2)(1.15\ \text{s}) = 4.80\ \text{m/s}.$$

(d) Since $v_y > 0$ when the ball hits the wall, it has not reached the highest point yet.

33. **THINK** This problem deals with projectile motion. We're interested in the horizontal displacement and velocity of the projectile before it strikes the ground.

EXPRESS We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below the release point. We write $\theta_0 = -37.0°$ for the angle measured from $+x$, since the angle $\phi_0 = 53.0°$ given in the problem is measured from the $-y$ direction. The initial setup of the problem is shown in the figure to the right (not to scale).



ANALYZE (a) The initial speed of the projectile is the plane's speed at the moment of release. Given that $y_0 = 730$ m and $y = 0$ at $t = 5.00$ s, we use Eq. 4-22 to find v_0:

$$y - y_0 = (v_0 \sin\theta_0)\ t - \frac{1}{2}gt^2 \quad \Rightarrow \quad 0 - 730\ \text{m} = v_0 \sin(-37.0°)(5.00\ \text{s}) - \frac{1}{2}(9.80\ \text{m/s}^2)(5.00\ \text{s})^2$$

which yields v_0 = 202 m/s.

(b) The horizontal distance traveled is

$$R = v_x t = (v_0 \cos\theta_0)t = [(202\ \text{m/s})\cos(-37.0°)](5.00\ \text{s}) = 806\ \text{m}.$$

(c) The x component of the velocity (just before impact) is

$$v_x = v_0 \cos\theta_0 = (202\ \text{m/s})\cos(-37.0°) = 161\ \text{m/s}.$$

(d) The y component of the velocity (just before impact) is

$$v_y = v_0 \sin\theta_0 - gt = (202\ \text{m/s})\sin(-37.0°) - (9.80\ \text{m/s}^2)(5.00\ \text{s}) = -171\ \text{m/s}.$$

LEARN In this projectile problem we analyzed the kinematics in the vertical and horizontal directions separately since they do not affect each other. The x-component of the velocity, $v_x = v_0 \cos\theta_0$, remains unchanged throughout since there's no horizontal acceleration.

34. (a) Since the y-component of the velocity of the stone at the top of its path is zero, its speed is

$$v = \sqrt{v_x^2 + v_y^2} = v_x = v_0 \cos\theta_0 = (28.0\ \text{m/s})\cos 40.0° = 21.4\ \text{m/s}.$$

(b) Using the fact that $v_y = 0$ at the maximum height $y_{\max}$, the amount of time it takes for the stone to reach $y_{\max}$ is given by Eq. 4-23:

$$0 = v_y = v_0 \sin\theta_0 - gt \quad \Rightarrow \quad t = \frac{v_0 \sin\theta_0}{g}.$$

Substituting the above expression into Eq. 4-22, we find the maximum height to be

$$y_{\max} = (v_0 \sin\theta_0)\ t - \frac{1}{2}gt^2 = v_0 \sin\theta_0 \left(\frac{v_0 \sin\theta_0}{g}\right) - \frac{1}{2}g\left(\frac{v_0 \sin\theta_0}{g}\right)^2 = \frac{v_0^2 \sin^2\theta_0}{2g}.$$

To find the time the stone descends to $y = y_{\max}/2$, we solve the quadratic equation given in Eq. 4-22:

$$y = \frac{1}{2}y_{\max} = \frac{v_0^2 \sin^2\theta_0}{4g} = (v_0 \sin\theta_0)\ t - \frac{1}{2}gt^2 \quad \Rightarrow \quad t_{\pm} = \frac{(2 \pm \sqrt{2})v_0 \sin\theta_0}{2g}.$$

Choosing $t = t_+$ (for descending), we have

$$v_x = v_0 \cos\theta_0 = (28.0 \text{ m/s})\cos 40.0° = 21.4 \text{ m/s}$$
$$v_y = v_0 \sin\theta_0 - g\,\frac{(2+\sqrt{2})v_0 \sin\theta_0}{2g} = -\frac{\sqrt{2}}{2} v_0 \sin\theta_0 = -\frac{\sqrt{2}}{2}(28.0 \text{ m/s})\sin 40.0° = -12.7 \text{ m/s}$$

Thus, the speed of the stone when $y = y_{\max}/2$ is

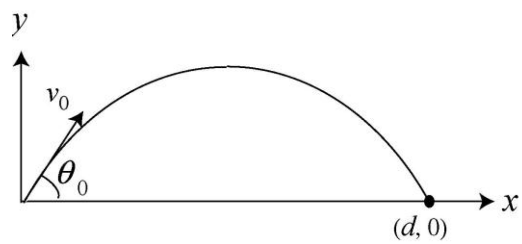
$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(21.4 \text{ m/s})^2 + (-12.7 \text{ m/s})^2} = 24.9 \text{ m/s}\,.$$

(c) The percentage difference is

$$\frac{24.9 \text{ m/s} - 21.4 \text{ m/s}}{21.4 \text{ m/s}} = 0.163 = 16.3\%\,.$$

35. **THINK** This problem deals with projectile motion of a bullet. We're interested in the firing angle that allows the bullet to strike a target at some distance away.

EXPRESS We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the end of the rifle (the initial point for the bullet as it begins projectile motion in the sense of § 4-5), and we let θ_0 be the firing angle. If the target is a distance d away, then its coordinates are $x = d$, $y = 0$.



The projectile motion equations lead to

$$d = (v_0 \cos\theta_0)t, \qquad 0 = v_0 t \sin\theta_0 - \frac{1}{2}gt^2$$

where θ_0 is the firing angle. The setup of the problem is shown in the figure above (scale exaggerated).

ANALYZE The time at which the bullet strikes the target is given by $t = d/(v_0 \cos\theta_0)$. Eliminating t leads to $2v_0^2 \sin\theta_0 \cos\theta_0 - gd = 0$. Using $\sin\theta_0 \cos\theta_0 = \frac{1}{2}\sin(2\theta_0)$, we obtain

$$v_0^2 \sin\ (2\theta_0) = gd \ \Rightarrow\ \sin(2\theta_0) = \frac{gd}{v_0^2} = \frac{(9.80\ \text{m/s}^2)(45.7\ \text{m})}{(460\ \text{m/s})^2}$$

which yields $\sin(2\theta_0) = 2.11\times10^{-3}$, or θ_0 = 0.0606°. If the gun is aimed at a point a distance ℓ above the target, then tan $\theta_0 = \ell/d$ so that

$$\ell = d\tan\theta_0 = (45.7\ \text{m})\tan(0.0606°) = 0.0484\ \text{m} = 4.84\ \text{cm}.$$

LEARN Due to the downward gravitational acceleration, in order for the bullet to strike the target, the gun must be aimed at a point slightly above the target.

36. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below the point where the ball was hit by the racquet.

(a) We want to know how high the ball is above the court when it is at x = 12.0 m. First, Eq. 4-21 tells us the time it is over the fence:

$$t = \frac{x}{v_0\cos\theta_0} = \frac{12.0\ \text{m}}{(23.6\ \text{m/s})\cos 0°} = 0.508\ \text{s}.$$

At this moment, the ball is at a height (above the court) of

$$y = y_0 + (v_0 \sin\theta_0)\,t - \frac{1}{2}gt^2 = 1.10\text{m}$$

which implies it does indeed clear the 0.90-m-high fence.

(b) At t = 0.508 s, the center of the ball is (1.10 m – 0.90 m) = 0.20 m above the net.

(c) Repeating the computation in part (a) with θ_0 = –5.0° results in t = 0.510 s and $y = 0.040$ m, which clearly indicates that it cannot clear the net.

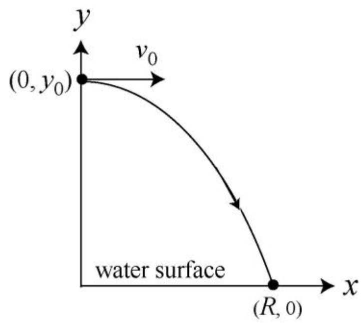
(d) In the situation discussed in part (c), the distance between the top of the net and the center of the ball at t = 0.510 s is 0.90 m – 0.040 m = 0.86 m.

37. **THINK** The trajectory of the diver is a projectile motion. We are interested in the displacement of the diver at a later time.

EXPRESS The initial velocity has no vertical component ($\theta_0 = 0$), but only an x component. Eqs. 4-21 and 4-22 can be simplified to

$$x - x_0 = v_{0x}t$$
$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2 = -\frac{1}{2}gt^2.$$

where $x_0 = 0$, $v_{0x} = v_0 = +2.0\text{ m/s}$ and y_0 = +10.0 m (taking the water surface to be at $y = 0$). The setup of the problem is shown in the figure below.



ANALYZE (a) At $t = 0.80\text{ s}$, the horizontal distance of the diver from the edge is

$$x = x_0 + v_{0x}t = 0 + (2.0\text{ m/s})(0.80\text{ s}) = 1.60\text{ m}.$$

(b) Similarly, using the second equation for the vertical motion, we obtain

$$y = y_0 - \frac{1}{2}gt^2 = 10.0\text{ m} - \frac{1}{2}(9.80\text{ m/s}^2)(0.80\text{ s})^2 = 6.86\text{ m}.$$

(c) At the instant the diver strikes the water surface, y = 0. Solving for t using the equation $y = y_0 - \frac{1}{2}gt^2 = 0$ leads to

$$t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2(10.0\text{ m})}{9.80\text{ m/s}^2}} = 1.43\text{ s}.$$

During this time, the x-displacement of the diver is $R = x$ = (2.00 m/s)(1.43 s) = 2.86 m.

LEARN Using Eq. 4-25 with $\theta_0 = 0$, the trajectory of the diver can also be written as

$$y = y_0 - \frac{gx^2}{2v_0^2}\ .$$

Part (c) can also be solved by using this equation:

$$y = y_0 - \frac{gx^2}{2v_0^2} = 0 \;\Rightarrow\; x = R = \sqrt{\frac{2v_0^2 y_0}{g}} = \sqrt{\frac{2(2.0\text{ m/s})^2(10.0\text{ m})}{9.8\text{ m/s}^2}} = 2.86\text{ m}.$$

38. In this projectile motion problem, we have $v_0 = v_x$ = constant, and what is plotted is $v = \sqrt{v_x^2 + v_y^2}$. We infer from the plot that at t = 2.5 s, the ball reaches its maximum height, where $v_y = 0$. Therefore, we infer from the graph that v_x = 19 m/s.

(a) During t = 5 s, the horizontal motion is $x - x_0 = v_x t$ = 95 m.

(b) Since $\sqrt{(19\text{ m/s})^2 + v_{0y}^2} = 31\text{ m/s}$ (the first point on the graph), we find $v_{0y} = 24.5\text{ m/s}$. Thus, with t = 2.5 s, we can use $y_{\max} - y_0 = v_{0y}t - \frac{1}{2}gt^2$ or $v_y^2 = 0 = v_{0y}^2 - 2g(y_{\max} - y_0)$, or $y_{\max} - y_0 = \frac{1}{2}\left(v_y + v_{0_y}\right)t$ to solve. Here we will use the latter:

$$y_{\max} - y_0 = \frac{1}{2}(v_y + v_{0y})\ t \Rightarrow y_{\max} = \frac{1}{2}(0 + 24.5\text{m/s})(2.5\text{ s}) = 31\text{ m}$$

where we have taken $y_0 = 0$ as the ground level.

39. Following the hint, we have the time-reversed problem with the ball thrown from the ground, toward the right, at 60° measured counterclockwise from a rightward axis. We see in this time-reversed situation that it is convenient to use the familiar coordinate system with $+x$ as *rightward* and with positive angles measured counterclockwise.

(a) The x-equation (with $x_0 = 0$ and x = 25.0 m) leads to

$$25.0\text{ m} = (v_0 \cos 60.0°)(1.50\text{ s}),$$

so that v_0 = 33.3 m/s. And with $y_0 = 0$, and $y = h > 0$ at t = 1.50 s, we have $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ where $v_{0y} = v_0 \sin 60.0°$. This leads to h = 32.3 m.

(b) We have

$$v_x = v_{0x} = (33.3\text{ m/s})\cos 60.0° = 16.7\text{ m/s}$$
$$v_y = v_{0y} - gt = (33.3\text{ m/s})\sin 60.0° - (9.80\text{ m/s}^2)(1.50\text{ s}) = 14.2\text{ m/s}.$$

The magnitude of $\vec{v}$ is given by

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(16.7\text{ m/s})^2 + (14.2\text{ m/s})^2} = 21.9\text{ m/s}.$$

(c) The angle is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{14.2\text{ m/s}}{16.7\text{ m/s}}\right) = 40.4°.$$

(d) We interpret this result ("undoing" the time reversal) as an initial velocity (from the edge of the building) of magnitude 21.9 m/s with angle (down from leftward) of 40.4°.

40. (a) Solving the quadratic equation Eq. 4-22:

$$y - y_0 = (v_0 \sin\theta_0)\, t - \frac{1}{2} g t^2 \;\; \Rightarrow \;\; 0 - 2.160 \text{ m} = (15.00 \text{ m/s}) \sin(45.00°) t - \frac{1}{2}(9.800 \text{ m/s}^2) t^2$$

the total travel time of the shot in the air is found to be $t = 2.352\,\text{s}$. Therefore, the horizontal distance traveled is

$$R = \left(v_0 \cos\theta_0\right) t = (15.00 \text{ m/s}) \cos 45.00° (2.352 \text{ s}) = 24.95 \text{ m}.$$

(b) Using the procedure outlined in (a) but for $\theta_0 = 42.00°$, we have

$$y - y_0 = (v_0 \sin\theta_0)\, t - \frac{1}{2} g t^2 \;\; \Rightarrow \;\; 0 - 2.160 \text{ m} = (15.00 \text{ m/s}) \sin(42.00°) t - \frac{1}{2}(9.800 \text{ m/s}^2) t^2$$

and the total travel time is $t = 2.245\,\text{s}$. This gives

$$R = \left(v_0 \cos\theta_0\right) t = (15.00 \text{ m/s}) \cos 42.00° (2.245 \text{ s}) = 25.02 \text{ m}.$$

41. With the Archer fish set to be at the origin, the position of the insect is given by (x, y) where $x = R/2 = v_0^2 \sin 2\theta_0 / 2g$, and y corresponds to the maximum height of the parabolic trajectory: $y = y_{\max} = v_0^2 \sin^2\theta_0 / 2g$. From the figure, we have

$$\tan\phi = \frac{y}{x} = \frac{v_0^2 \sin^2\theta_0 / 2g}{v_0^2 \sin 2\theta_0 / 2g} = \frac{1}{2} \tan\theta_0$$

Given that $\phi = 36.0°$, we find the launch angle to be

$$\theta_0 = \tan^{-1}\left(2\tan\phi\right) = \tan^{-1}\left(2\tan 36.0°\right) = \tan^{-1}\left(1.453\right) = 55.46° \approx 55.5°.$$

Note that θ_0 depends only on ϕ and is independent of d.

42. (a) Using the fact that the person (as the projectile) reaches the maximum height over the middle wheel located at $x = 23 \text{ m} + (23/2) \text{ m} = 34.5 \text{ m}$, we can deduce the initial launch speed from Eq. 4-26:

$$x = \frac{R}{2} = \frac{v_0^2 \sin 2\theta_0}{2g} \;\; \Rightarrow \;\; v_0 = \sqrt{\frac{2gx}{\sin 2\theta_0}} = \sqrt{\frac{2(9.8 \text{ m/s}^2)(34.5 \text{ m})}{\sin(2 \cdot 53°)}} = 26.5 \text{ m/s}.$$

Upon substituting the value to Eq. 4-25, we obtain

$$y = y_0 + x \tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0} = 3.0\text{ m} + (23\text{ m})\tan 53° - \frac{(9.8\text{ m/s}^2)(23\text{ m})^2}{2(26.5\text{ m/s})^2(\cos 53°)^2} = 23.3\text{ m}.$$

Since the height of the wheel is $h_w = 18$ m, the clearance over the first wheel is $\Delta y = y - h_w = 23.3\text{ m} - 18\text{ m} = 5.3\text{ m}$.

(b) The height of the person when he is directly above the second wheel can be found by solving Eq. 4-24. With the second wheel located at $x = 23\text{ m} + (23/2)\text{ m} = 34.5\text{ m}$, we have

$$y = y_0 + x \tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0} = 3.0\text{ m} + (34.5\text{ m})\tan 53° - \frac{(9.8\text{ m/s}^2)(34.5\text{ m})^2}{2(26.52\text{ m/s})^2(\cos 53°)^2}$$
$$= 25.9\text{ m}.$$

Therefore, the clearance over the second wheel is $\Delta y = y - h_w = 25.9\text{ m} - 18\text{ m} = 7.9\text{ m}$.

(c) The location of the center of the net is given by

$$0 = y - y_0 = x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0} \quad \Rightarrow \quad x = \frac{v_0^2 \sin 2\theta_0}{g} = \frac{(26.52\text{ m/s})^2 \sin(2\cdot 53°)}{9.8\text{ m/s}^2} = 69\text{ m}.$$

43. We designate the given velocity $\vec{v} = (7.6\text{ m/s})\hat{\text{i}} + (6.1\text{ m/s})\hat{\text{j}}$ as $\vec{v}_1$, as opposed to the velocity when it reaches the max height $\vec{v}_2$ or the velocity when it returns to the ground $\vec{v}_3$, and take $\vec{v}_0$ as the launch velocity, as usual. The origin is at its launch point on the ground.

(a) Different approaches are available, but since it will be useful (for the rest of the problem) to first find the initial y velocity, that is how we will proceed. Using Eq. 2-16, we have

$$v_{1y}^2 = v_{0y}^2 - 2g\Delta y \quad \Rightarrow \quad (6.1\text{ m/s})^2 = v_{0y}^2 - 2(9.8\text{ m/s}^2)(9.1\text{ m})$$

which yields v_{0y} = 14.7 m/s. Knowing that v_{2y} must equal 0, we use Eq. 2-16 again but now with $\Delta y = h$ for the maximum height:

$$v_{2y}^2 = v_{0y}^2 - 2gh \quad \Rightarrow \quad 0 = (14.7\text{ m/s})^2 - 2(9.8\text{ m/s}^2)h$$

which yields h = 11 m.

(b) Recalling the derivation of Eq. 4-26, but using v_{0y} for $v_0 \sin\theta_0$ and v_{0x} for $v_0\cos\theta_0$, we have

$$0 = v_{0y}t - \frac{1}{2}gt^2, \quad R = v_{0x}t$$

which leads to $R = 2v_{0x}v_{0y}/g$. Noting that $v_{0x} = v_{1x} = 7.6$ m/s, we plug in values and obtain

$$R = 2(7.6 \text{ m/s})(14.7 \text{ m/s})/(9.8 \text{ m/s}^2) = 23 \text{ m}.$$

(c) Since $v_{3x} = v_{1x} = 7.6$ m/s and $v_{3y} = -v_{0y} = -14.7$ m/s, we have

$$v_3 = \sqrt{v_{3x}^2 + v_{3y}^2} = \sqrt{(7.6 \text{ m/s})^2 + (-14.7 \text{ m/s})^2} = 17 \text{ m/s}.$$

(d) The angle (measured from horizontal) for $\vec{v}_3$ is one of these possibilities:

$$\tan^{-1}\left(\frac{-14.7 \text{ m}}{7.6 \text{ m}}\right) = -63° \text{ or } 117°$$

where we settle on the first choice (–63°, which is equivalent to 297°) since the signs of its components imply that it is in the fourth quadrant.

44. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The initial velocity is horizontal so that $v_{0y} = 0$ and $v_{0x} = v_0 = 161$ km/h. Converting to SI units, this is $v_0 = 44.7$ m/s.

(a) With the origin at the initial point (where the ball leaves the pitcher's hand), the y coordinate of the ball is given by $y = -\frac{1}{2}gt^2$, and the x coordinate is given by $x = v_0t$. From the latter equation, we have a simple proportionality between horizontal distance and time, which means the time to travel half the total distance is half the total time. Specifically, if $x = 18.3/2$ m, then $t = (18.3/2 \text{ m})/(44.7 \text{ m/s}) = 0.205$ s.

(b) And the time to travel the next 18.3/2 m must also be 0.205 s. It can be useful to write the horizontal equation as $\Delta x = v_0\Delta t$ in order that this result can be seen more clearly.

(c) Using the equation $y = -\frac{1}{2}gt^2$, we see that the ball has reached the height of $|-\frac{1}{2}(9.80 \text{ m/s}^2)(0.205 \text{ s})^2| = 0.205 \text{ m}$ at the moment the ball is halfway to the batter.

(d) The ball's height when it reaches the batter is $-\frac{1}{2}(9.80 \text{ m/s}^2)(0.409 \text{ s})^2 = -0.820 \text{m}$, which, when subtracted from the previous result, implies it has fallen another 0.615 m. Since the value of y is not simply proportional to t, we do not expect equal time-intervals to correspond to equal height-changes; in a physical sense, this is due to the fact that the initial y-velocity for the first half of the motion is not the same as the "initial" y-velocity for the second half of the motion.

45. (a) Let $m = \frac{d_2}{d_1} = 0.600$ be the slope of the ramp, so $y = mx$ there. We choose our coordinate origin at the point of launch and use Eq. 4-25. Thus,

$$y = \tan(50.0°)x - \frac{(9.80 \text{ m/s}^2)x^2}{2(10.0 \text{ m/s})^2(\cos 50.0°)^2} = 0.600x$$

which yields $x = 4.99$ m. This is less than d_1 so the ball *does* land on the ramp.

(b) Using the value of x found in part (a), we obtain $y = mx = 2.99$ m. Thus, the Pythagorean theorem yields a displacement magnitude of $\sqrt{x^2 + y^2} = 5.82$ m.

(c) The angle is, of course, the angle of the ramp: $\tan^{-1}(m) = 31.0°$.

46. Using the fact that $v_y = 0$ when the player is at the maximum height y_{max}, the amount of time it takes to reach y_{max} can be solved by using Eq. 4-23:

$$0 = v_y = v_0 \sin\theta_0 - gt \quad \Rightarrow \quad t_{max} = \frac{v_0 \sin\theta_0}{g}.$$

Substituting the above expression into Eq. 4-22, we find the maximum height to be

$$y_{max} = (v_0 \sin\theta_0)\, t_{max} - \frac{1}{2} g t_{max}^2 = v_0 \sin\theta_0 \left(\frac{v_0 \sin\theta_0}{g}\right) - \frac{1}{2} g \left(\frac{v_0 \sin\theta_0}{g}\right)^2 = \frac{v_0^2 \sin^2\theta_0}{2g}.$$

To find the time when the player is at $y = y_{max}/2$, we solve the quadratic equation given in Eq. 4-22:

$$y = \frac{1}{2} y_{max} = \frac{v_0^2 \sin^2\theta_0}{4g} = (v_0 \sin\theta_0)\, t - \frac{1}{2} g t^2 \quad \Rightarrow \quad t_\pm = \frac{(2 \pm \sqrt{2}) v_0 \sin\theta_0}{2g}.$$

With $t = t_-$ (for ascending), the amount of time the player spends at a height $y \geq y_{max}/2$ is

$$\Delta t = t_{max} - t_- = \frac{v_0 \sin\theta_0}{g} - \frac{(2 - \sqrt{2}) v_0 \sin\theta_0}{2g} = \frac{v_0 \sin\theta_0}{\sqrt{2} g} = \frac{t_{max}}{\sqrt{2}} \quad \Rightarrow \quad \frac{\Delta t}{t_{max}} = \frac{1}{\sqrt{2}} = 0.707.$$

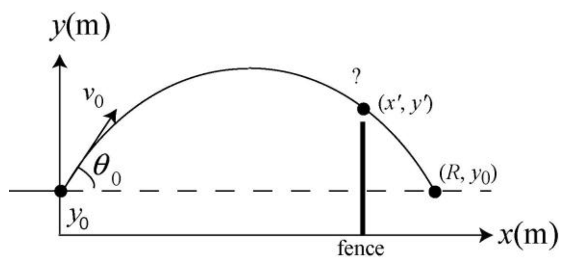
Therefore, the player spends about 70.7% of the time in the upper half of the jump. Note that the ratio $\Delta t / t_{max}$ is independent of v_0 and θ_0, even though Δt and t_{max} depend on these quantities.

47. **THINK** The baseball undergoes projectile motion after being hit by the batter. We'd like to know if the ball clears a high fence at some distance away.

EXPRESS We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below impact point between bat and ball. In the absence of a fence, with $\theta_0 = 45°$, the horizontal range (same launch level) is $R = 107$ m. We want to know how high the ball is from the ground when it is at $x' = 97.5$ m, which requires knowing the initial velocity. The trajectory of the baseball can be described by Eq. 4-25:

$$y - y_0 = (\tan\theta_0)x - \frac{gx^2}{2(v_0\cos\theta_0)^2}.$$

The setup of the problem is shown in the figure below (not to scale).



ANALYZE (a) We first solve for the initial speed v_0. Using the range information ($y = y_0$ when $x = R$) and θ_0 = 45°, Eq. 4-25 gives

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta_0}} = \sqrt{\frac{(9.8\text{ m/s}^2)(107\text{ m})}{\sin(2\cdot 45°)}} = 32.4\text{ m/s}.$$

Thus, the time at which the ball flies over the fence is:

$$x' = (v_0\cos\theta_0)t' \quad\Rightarrow\quad t' = \frac{x'}{v_0\cos\theta_0} = \frac{97.5\text{ m}}{(32.4\text{ m/s})\cos 45°} = 4.26\text{ s}.$$

At this moment, the ball is at a height (above the ground) of

$$\begin{aligned} y' &= y_0 + (v_0\sin\theta_0)t' - \frac{1}{2}gt'^2 \\ &= 1.22\text{ m} + [(32.4\text{ m/s})\sin 45°](4.26\text{ s}) - \frac{1}{2}(9.8\text{ m/s}^2)(4.26\text{ s})^2 \\ &= 9.88\text{ m} \end{aligned}$$

which implies it does indeed clear the 7.32 m high fence.

(b) At $t' = 4.26$ s, the center of the ball is 9.88 m – 7.32 m = 2.56 m above the fence.

LEARN Using the trajectory equation above, one can show that the minimum initial velocity required to clear the fence is given by

$$y' - y_0 = (\tan\theta_0)x' - \frac{gx'^2}{2(v_0\cos\theta_0)^2},$$

or about 31.9 m/s.

48. Following the hint, we have the time-reversed problem with the ball thrown from the roof, toward the left, at 60° measured clockwise from a leftward axis. We see in this time-reversed situation that it is convenient to take +x as *leftward* with positive angles measured clockwise. Lengths are in meters and time is in seconds.

(a) With y_0 = 20.0 m, and y = 0 at t = 4.00 s, we have $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ where $v_{0y} = v_0 \sin 60°$. This leads to v_0 = 16.9 m/s. This plugs into the x-equation $x - x_0 = v_{0x}t$ (with x_0 = 0 and $x = d$) to produce

$$d = (16.9\text{ m/s})\cos 60°(4.00\text{ s}) = 33.7\text{ m}.$$

(b) We have

$$v_x = v_{0x} = (16.9\text{ m/s})\cos 60.0° = 8.43\text{ m/s}$$
$$v_y = v_{0y} - gt = (16.9\text{ m/s})\sin 60.0° - (9.80\text{m/s}^2)(4.00\text{ s}) = -24.6\text{ m/s}.$$

The magnitude of $\vec{v}$ is $|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(8.43\text{ m/s})^2 + (-24.6\text{ m/s})^2} = 26.0\text{ m/s}.$

(c) The angle relative to horizontal is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-24.6\text{ m/s}}{8.43\text{ m/s}}\right) = -71.1°.$$

We may convert the result from rectangular components to magnitude-angle representation:

$$\vec{v} = (8.43, -24.6) \ \rightarrow\ (26.0 \angle -71.1°)$$

and we now interpret our result ("undoing" the time reversal) as an initial velocity of magnitude 26.0 m/s with angle (up from rightward) of 71.1°.

49. **THINK** In this problem a football is given an initial speed and it undergoes projectile motion. We'd like to know the smallest and greatest angles at which a field goal can be scored.

EXPRESS We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the point where the ball is kicked. We use x and y to denote the coordinates of the ball at the goalpost, and try to find the kicking angle(s) θ_0 so that y = 3.44 m when x = 50 m. Writing the kinematic equations for projectile motion:

$$x = v_0 \cos\theta_0, \quad y = v_0 t \sin\theta_0 - \tfrac{1}{2} g t^2,$$

we see the first equation gives $t = x/v_0 \cos\theta_0$, and when this is substituted into the second the result is

$$y = x \tan\theta_0 - \frac{g x^2}{2 v_0^2 \cos^2\theta_0}.$$

ANALYZE One may solve the above equation by trial and error: systematically trying values of θ_0 until you find the two that satisfy the equation. A little manipulation, however, will give an algebraic solution: Using the trigonometric identity

$$1 / \cos^2\theta_0 = 1 + \tan^2\theta_0,$$

we obtain

$$\frac{1}{2}\frac{g x^2}{v_0^2}\tan^2\theta_0 - x\tan\theta_0 + y + \frac{1}{2}\frac{g x^2}{v_0^2} = 0$$

which is a second-order equation for $\tan\theta_0$. To simplify writing the solution, we denote

$$c = \frac{1}{2} g x^2 / v_0^2 = \frac{1}{2}\left(9.80 \text{ m/s}^2\right)\left(50 \text{ m}\right)^2 / \left(25 \text{ m/s}\right)^2 = 19.6 \text{ m}.$$

Then the second-order equation becomes $c \tan^2\theta_0 - x \tan\theta_0 + y + c = 0$. Using the quadratic formula, we obtain its solution(s).

$$\tan\theta_0 = \frac{x \pm \sqrt{x^2 - 4(y+c)c}}{2c} = \frac{50 \text{ m} \pm \sqrt{(50 \text{ m})^2 - 4(3.44 \text{ m} + 19.6 \text{ m})(19.6 \text{ m})}}{2(19.6 \text{ m})}.$$

The two solutions are given by $\tan\theta_0 = 1.95$ and $\tan\theta_0 = 0.605$. The corresponding (first-quadrant) angles are $\theta_0 = 63°$ and $\theta_0 = 31°$. Thus,

(a) The smallest elevation angle is $\theta_0 = 31°$, and

(b) The greatest elevation angle is $\theta_0 = 63°$.

LEARN If kicked at any angle between 31° and 63°, the ball will travel above the cross bar on the goalposts.

50. We apply Eq. 4-21, Eq. 4-22, and Eq. 4-23.

(a) From $\Delta x = v_{0x} t$, we find $v_{0x} = 40 \text{ m} / 2 \text{ s} = 20$ m/s.

(b) From $\Delta y = v_{0y} t - \tfrac{1}{2} g t^2$, we find $v_{0y} = \left(53 \text{ m} + \tfrac{1}{2}(9.8 \text{ m/s}^2)(2 \text{ s})^2\right) / 2 = 36 \text{ m/s}.$

(c) From $v_y = v_{0y} - gt'$ with v_y = 0 as the condition for maximum height, we obtain $t' = (36 \text{ m/s})/(9.8 \text{ m/s}^2) = 3.7 \text{ s}$. During that time the x-motion is constant, so $x' - x_0 = (20 \text{ m/s})(3.7 \text{ s}) = 74 \text{ m}$.

51. (a) The skier jumps up at an angle of $\theta_0 = 11.3°$ up from the horizontal and thus returns to the launch level with his velocity vector 11.3° below the horizontal. With the snow surface making an angle of $\alpha = 9.0°$ (downward) with the horizontal, the angle between the slope and the velocity vector is $\phi = \theta_0 - \alpha = 11.3° - 9.0° = 2.3°$.

(b) Suppose the skier lands at a distance d down the slope. Using Eq. 4-25 with $x = d\cos\alpha$ and $y = -d\sin\alpha$ (the edge of the track being the origin), we have

$$-d\sin\alpha = d\cos\alpha\tan\theta_0 - \frac{g(d\cos\alpha)^2}{2v_0^2\cos^2\theta_0}.$$

Solving for d, we obtain

$$d = \frac{2v_0^2\cos^2\theta_0}{g\cos^2\alpha}\left(\cos\alpha\tan\theta_0 + \sin\alpha\right) = \frac{2v_0^2\cos\theta_0}{g\cos^2\alpha}\left(\cos\alpha\sin\theta_0 + \cos\theta_0\sin\alpha\right)$$
$$= \frac{2v_0^2\cos\theta_0}{g\cos^2\alpha}\sin(\theta_0 + \alpha).$$

Substituting the values given, we find

$$d = \frac{2(10 \text{ m/s})^2\cos(11.3°)}{(9.8 \text{ m/s}^2)\cos^2(9.0°)}\sin(11.3° + 9.0°) = 7.117 \text{ m}.$$

which gives

$$y = -d\sin\alpha = -(7.117 \text{ m})\sin(9.0°) = -1.11 \text{ m}.$$

Therefore, at landing the skier is approximately 1.1 m below the launch level.

(c) The time it takes for the skier to land is

$$t = \frac{x}{v_x} = \frac{d\cos\alpha}{v_0\cos\theta_0} = \frac{(7.117 \text{ m})\cos(9.0°)}{(10 \text{ m/s})\cos(11.3°)} = 0.72 \text{ s}.$$

Using Eq. 4-23, the x-and y-components of the velocity at landing are

$$v_x = v_0\cos\theta_0 = (10 \text{ m/s})\cos(11.3°) = 9.81 \text{ m/s}$$
$$v_y = v_0\sin\theta_0 - gt = (10 \text{ m/s})\sin(11.3°) - (9.8 \text{ m/s}^2)(0.72 \text{ s}) = -5.07 \text{ m/s}$$

Thus, the direction of travel at landing is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-5.07\ \text{m/s}}{9.81\ \text{m/s}}\right) = -27.3°.$$

or 27.3° below the horizontal. The result implies that the angle between the skier's path and the slope is $\phi = 27.3° - 9.0° = 18.3°$, or approximately $18°$ to two significant figures.

52. From Eq. 4-21, we find $t = x / v_{0x}$. Then Eq. 4-23 leads to

$$v_y = v_{0y} - gt = v_{0y} - \frac{gx}{v_{0x}}.$$

Since the slope of the graph is –0.500, we conclude

$$\frac{g}{v_{0x}} = \frac{1}{2} \;\Rightarrow v_{ox} = 19.6\ \text{m/s}.$$

And from the "*y* intercept" of the graph, we find v_{oy} = 5.00 m/s. Consequently,

$$\theta_o = \tan^{-1}(v_{oy} / v_{ox}) = 14.3° \approx 14°.$$

53. Let $y_0 = h_0$ = 1.00 m at x_0 = 0 when the ball is hit. Let $y_1 = h$ (the height of the wall) and x_1 describe the point where it first rises above the wall one second after being hit; similarly, $y_2 = h$ and x_2 describe the point where it passes back down behind the wall four seconds later. And y_f = 1.00 m at $x_f = R$ is where it is caught. Lengths are in meters and time is in seconds.

(a) Keeping in mind that v_x is constant, we have $x_2 - x_1$ = 50.0 m = v_{1x} (4.00 s), which leads to v_{1x} = 12.5 m/s. Thus, applied to the full six seconds of motion:

$$x_f - x_0 = R = v_x(6.00\ \text{s}) = 75.0\ \text{m}.$$

(b) We apply $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ to the motion above the wall,

$$y_2 - y_1 = 0 = v_{1y}\left(4.00\ \text{s}\right) - \frac{1}{2}g\left(4.00\ \text{s}\right)^2$$

and obtain v_{1y} = 19.6 m/s. One second earlier, using $v_{1y} = v_{0y} - g$(1.00 s), we find $v_{0y} = 29.4$ m/s. Therefore, the velocity of the ball just after being hit is

$$\vec{v} = v_{0x}\hat{\text{i}} + v_{0y}\hat{\text{j}} = (12.5\ \text{m/s})\ \hat{\text{i}} + (29.4\ \text{m/s})\ \hat{\text{j}}$$

Its magnitude is $|\vec{v}| = \sqrt{(12.5\ \text{m/s})^2 + (29.4\ \text{m/s})^2} = 31.9\ \text{m/s}$.

(c) The angle is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{29.4\ \text{m/s}}{12.5\ \text{m/s}}\right) = 67.0°.$$

We interpret this result as a velocity of magnitude 31.9 m/s, with angle (up from rightward) of 67.0°.

(d) During the first 1.00 s of motion, $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$ yields

$$h = 1.0\ \text{m} + (29.4\ \text{m/s})(1.00\ \text{s}) - \tfrac{1}{2}(9.8\ \text{m/s}^2)(1.00\ \text{s})^2 = 25.5\ \text{m}.$$

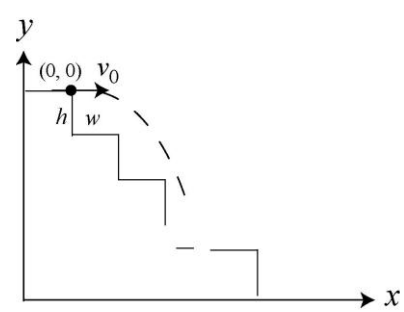
54. For $\Delta y = 0$, Eq. 4-22 leads to $t = 2v_o\sin\theta_o/g$, which immediately implies $t_{max} = 2v_o/g$ (which occurs for the "straight up" case: $\theta_o = 90°$). Thus,

$$\tfrac{1}{2}t_{max} = v_o/g \ \Rightarrow\ \tfrac{1}{2} = \sin\theta_o.$$

Therefore, the half-maximum-time flight is at angle θ_o = 30.0°. Since the least speed occurs at the top of the trajectory, which is where the velocity is simply the x-component of the initial velocity ($v_o\cos\theta_o = v_o\cos 30°$ for the half-maximum-time flight), then we need to refer to the graph in order to find v_o – in order that we may complete the solution. In the graph, we note that the range is 240 m when θ_o = 45.0°. Equation 4-26 then leads to v_o = 48.5 m/s. The answer is thus (48.5 m/s)cos30.0° = 42.0 m/s.

55. **THINK** In this problem a ball rolls off the top of a stairway with an initial speed, and we'd like to know on which step it lands first.

EXPRESS We denote h as the height of a step and w as the width. To hit step n, the ball must fall a distance nh and travel horizontally a distance between $(n-1)w$ and nw. We take the origin of a coordinate system to be at the point where the ball leaves the top of the stairway, and we choose the y axis to be positive in the upward direction, as shown in the figure.



The coordinates of the ball at time t are given by $x = v_{0x}t$ and $y = -\frac{1}{2}gt^2$ (since v_{0y} = 0).

ANALYZE We equate y to $-nh$ and solve for the time to reach the level of step n:

$$t = \sqrt{\frac{2nh}{g}}.$$

The x coordinate then is

$$x = v_{0x}\sqrt{\frac{2nh}{g}} = (1.52 \text{ m/s})\sqrt{\frac{2n(0.203 \text{ m})}{9.8 \text{ m/s}^2}} = (0.309 \text{ m})\sqrt{n}.$$

The method is to try values of n until we find one for which x/w is less than n but greater than $n - 1$. For $n = 1$, $x = 0.309$ m and $x/w = 1.52$, which is greater than n. For $n = 2$, $x = 0.437$ m and $x/w = 2.15$, which is also greater than n. For $n = 3$, $x = 0.535$ m and $x/w = 2.64$. Now, this is less than n and greater than $n - 1$, so the ball hits the third step.

LEARN To check the consistency of our calculation, we can substitute $n = 3$ into the above equations. The results are $t = 0.353$ s, $y = 0.609$ m and $x = 0.535$ m. This indeed corresponds to the third step.

56. We apply Eq. 4-35 to solve for speed v and Eq. 4-34 to find acceleration a.

(a) Since the radius of Earth is 6.37×10^6 m, the radius of the satellite orbit is

$$r = (6.37 \times 10^6 + 640 \times 10^3) \text{ m} = 7.01 \times 10^6 \text{ m}.$$

Therefore, the speed of the satellite is

$$v = \frac{2\pi r}{T} = \frac{2\pi\left(7.01 \times 10^6 \text{ m}\right)}{(98.0 \text{ min})(60 \text{ s/min})} = 7.49 \times 10^3 \text{ m/s}.$$

(b) The magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{\left(7.49 \times 10^3 \text{ m/s}\right)^2}{7.01 \times 10^6 \text{ m}} = 8.00 \text{ m/s}^2.$$

57. The magnitude of centripetal acceleration ($a = v^2/r$) and its direction (toward the center of the circle) form the basis of this problem.

(a) If a passenger at this location experiences $\vec{a} = 1.83 \text{ m/s}^2$ east, then the center of the circle is *east* of this location. The distance is $r = v^2/a = (3.66 \text{ m/s})^2/(1.83 \text{ m/s}^2) = 7.32$ m.

(b) Thus, relative to the center, the passenger at that moment is located 7.32 m toward the west.

(c) If the direction of $\vec{a}$ experienced by the passenger is now *south*—indicating that the center of the merry-go-round is south of him, then relative to the center, the passenger at that moment is located 7.32 m toward the north.

58. (a) The circumference is $c = 2\pi r = 2\pi(0.15 \text{ m}) = 0.94$ m.

(b) With $T = (60 \text{ s})/1200 = 0.050$ s, the speed is $v = c/T = (0.94 \text{ m})/(0.050 \text{ s}) = 19$ m/s. This is equivalent to using Eq. 4-35.

(c) The magnitude of the acceleration is $a = v^2/r = (19 \text{ m/s})^2/(0.15 \text{ m}) = 2.4 \times 10^3 \text{ m/s}^2$.

(d) The period of revolution is $(1200 \text{ rev/min})^{-1} = 8.3 \times 10^{-4}$ min, which becomes, in SI units, $T = 0.050$ s = 50 ms.

59. (a) Since the wheel completes 5 turns each minute, its period is one-fifth of a minute, or 12 s.

(b) The magnitude of the centripetal acceleration is given by $a = v^2/R$, where R is the radius of the wheel, and v is the speed of the passenger. Since the passenger goes a distance $2\pi R$ for each revolution, his speed is

$$v = \frac{2\pi(15 \text{ m})}{12 \text{ s}} = 7.85 \text{ m/s}$$

and his centripetal acceleration is $a = \dfrac{(7.85 \text{ m/s})^2}{15 \text{ m}} = 4.1 \text{ m/s}^2$.

(c) When the passenger is at the highest point, his centripetal acceleration is downward, toward the center of the orbit.

(d) At the lowest point, the centripetal acceleration is $a = 4.1 \text{ m/s}^2$, same as part (b).

(e) The direction is up, toward the center of the orbit.

60. (a) During constant-speed circular motion, the velocity vector is perpendicular to the acceleration vector at every instant. Thus, $\vec{v} \cdot \vec{a} = 0$.

(b) The acceleration in this vector, at every instant, points toward the center of the circle, whereas the position vector points from the center of the circle to the object in motion. Thus, the angle between $\vec{r}$ and $\vec{a}$ is 180° at every instant, so $\vec{r} \times \vec{a} = 0$.

61. We apply Eq. 4-35 to solve for speed v and Eq. 4-34 to find centripetal acceleration a.

(a) $v = 2\pi r/T = 2\pi(20 \text{ km})/1.0 \text{ s} = 126 \text{ km/s} = 1.3 \times 10^5$ m/s.

(b) The magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{(126 \text{ km/s})^2}{20 \text{ km}} = 7.9 \times 10^5 \text{ m/s}^2.$$

(c) Clearly, both *v* and *a* will increase if *T* is reduced.

62. The magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{(10 \text{ m/s})^2}{25 \text{ m}} = 4.0 \text{ m/s}^2.$$

63. We first note that $\vec{a}_1$ (the acceleration at t_1 = 2.00 s) is perpendicular to $\vec{a}_2$ (the acceleration at t_2=5.00 s), by taking their scalar (dot) product:

$$\vec{a}_1 \cdot \vec{a}_2 = [(6.00 \text{ m/s}^2)\hat{\mathrm{i}} + (4.00 \text{ m/s}^2)\hat{\mathrm{j}}] \cdot [(4.00 \text{ m/s}^2)\hat{\mathrm{i}} + (-6.00 \text{ m/s}^2)\hat{\mathrm{j}}] = 0.$$

Since the acceleration vectors are in the (negative) radial directions, then the two positions (at t_1 and t_2) are a quarter-circle apart (or three-quarters of a circle, depending on whether one measures clockwise or counterclockwise). A quick sketch leads to the conclusion that if the particle is moving counterclockwise (as the problem states) then it travels three-quarters of a circumference in moving from the position at time t_1 to the position at time t_2 . Letting *T* stand for the period, then $t_2 - t_1$ = 3.00 s = 3*T*/4. This gives *T* = 4.00 s. The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(6.00 \text{ m/s}^2)^2 + (4.00 \text{ m/s})^2} = 7.21 \text{ m/s}^2.$$

Using Eqs. 4-34 and 4-35, we have $a = 4\pi^2 r / T^2$, which yields

$$r = \frac{aT^2}{4\pi^2} = \frac{(7.21 \text{ m/s}^2)(4.00 \text{ s})^2}{4\pi^2} = 2.92 \text{ m}.$$

64. When traveling in circular motion with constant speed, the instantaneous acceleration vector necessarily points toward the center. Thus, the center is “straight up” from the cited point.

(a) Since the center is “straight up” from (4.00 m, 4.00 m), the *x* coordinate of the center is 4.00 m.

(b) To find out “how far up” we need to know the radius. Using Eq. 4-34 we find

$$r=\frac{v^2}{a}=\frac{(5.00\text{ m/s})^2}{12.5\text{ m/s}^2}=2.00\text{ m}.$$

Thus, the y coordinate of the center is 2.00 m + 4.00 m = 6.00 m. Thus, the center may be written as (x, y) = (4.00 m, 6.00 m).

65. Since the period of a uniform circular motion is $T=2\pi r/v$, where r is the radius and v is the speed, the centripetal acceleration can be written as

$$a=\frac{v^2}{r}=\frac{1}{r}\left(\frac{2\pi r}{T}\right)^2=\frac{4\pi^2 r}{T^2}.$$

Based on this expression, we compare the (magnitudes) of the wallet and purse accelerations, and find their ratio is the ratio of r values. Therefore, a_{wallet} = 1.50 a_{purse}. Thus, the wallet acceleration vector is

$$a=1.50[(2.00\text{ m/s}^2)\hat{\text{i}}+(4.00\text{ m/s}^2)\hat{\text{j}}]=(3.00\text{ m/s}^2)\hat{\text{i}}+(6.00\text{ m/s}^2)\hat{\text{j}}.$$

66. The fact that the velocity is in the +y direction and the acceleration is in the +x direction at t_1 = 4.00 s implies that the motion is clockwise. The position corresponds to the "9:00 position." On the other hand, the position at t_2 = 10.0 s is in the "6:00 position" since the velocity points in the –x direction and the acceleration is in the +y direction. The time interval $\Delta t=10.0\text{ s}-4.00\text{ s}=6.00\text{ s}$ is equal to 3/4 of a period:

$$6.00\text{ s}=\frac{3}{4}T \quad\Rightarrow\quad T=8.00\text{ s}.$$

Equation 4-35 then yields

$$r=\frac{vT}{2\pi}=\frac{(3.00\text{ m/s})(8.00\text{ s})}{2\pi}=3.82\text{ m}.$$

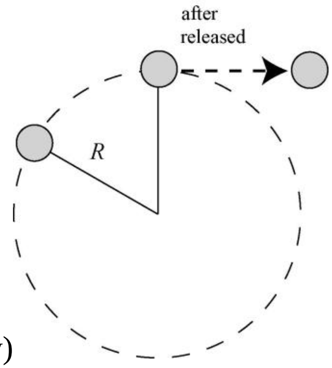
(a) The x coordinate of the center of the circular path is $x=5.00\text{ m}+3.82\text{ m}=8.82\text{ m}.$

(b) The y coordinate of the center of the circular path is $y=6.00\text{ m}.$

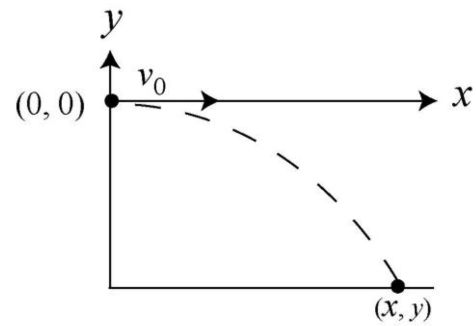
In other words, the center of the circle is at (x,y) = (8.82 m, 6.00 m).

67. **THINK** In this problem we have a stone whirled in a horizontal circle. After the string breaks, the stone undergoes projectile motion.

EXPRESS The stone moves in a circular path (top view shown below left) initially, but undergoes projectile motion after the string breaks (side view shown below right). Since $a=v^2/R$, to calculate the centripetal acceleration of the stone, we need to know its speed during its circular motion (this is also its initial speed when it flies off). We use the kinematic equations of projectile motion (discussed in §4-6) to find that speed.



(top view)



(side view)

Taking the $+y$ direction to be upward and placing the origin at the point where the stone leaves its circular orbit, then the coordinates of the stone during its motion as a projectile are given by $x = v_0t$ and $y = -\frac{1}{2}gt^2$ (since $v_{0y} = 0$). It hits the ground at $x = 10$ m and $y = -2.0\text{ m}$.

ANALYZE Formally solving the y-component equation for the time, we obtain $t = \sqrt{-2y/g}$, which we substitute into the first equation:

$$v_0 = x\sqrt{-\frac{g}{2y}} = (10\text{ m})\sqrt{-\frac{9.8\text{ m/s}^2}{2(-2.0\text{ m})}} = 15.7\text{ m/s}.$$

Therefore, the magnitude of the centripetal acceleration is

$$a = \frac{v_0^2}{R} = \frac{(15.7\text{ m/s})^2}{1.5\text{ m}} = 160\text{ m/s}^2.$$

LEARN The above equations can be combined to give $a = \dfrac{gx^2}{-2yR}$. The equation implies that the greater the centripetal acceleration, the greater the initial speed of the projectile, and the greater the distance traveled by the stone. This is precisely what we expect.

68. We note that after three seconds have elapsed ($t_2 - t_1 = 3.00$ s) the velocity (for this object in circular motion of period T) is reversed; we infer that it takes three seconds to reach the opposite side of the circle. Thus, $T = 2(3.00\text{ s}) = 6.00$ s.

(a) Using Eq. 4-35, $r = vT/2\pi$, where $v = \sqrt{(3.00\text{ m/s})^2 + (4.00\text{ m/s})^2} = 5.00\text{ m/s}$, we obtain $r = 4.77\text{ m}$. The magnitude of the object's centripetal acceleration is therefore $a = v^2/r = 5.24\text{ m/s}^2$.

(b) The average acceleration is given by Eq. 4-15:

$$\vec{a}_{\text{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{(-3.00\hat{\text{i}} - 4.00\hat{\text{j}})\text{ m/s} - (3.00\hat{\text{i}} + 4.00\hat{\text{j}})\text{ m/s}}{5.00\text{ s} - 2.00\text{ s}} = (-2.00\text{ m/s}^2)\hat{\text{i}} + (-2.67\text{ m/s}^2)\hat{\text{j}}$$

which implies $|\vec{a}_{\text{avg}}| = \sqrt{(-2.00\text{ m/s}^2)^2 + (-2.67\text{ m/s}^2)^2} = 3.33\text{ m/s}^2$.

69. We use Eq. 4-15 first using velocities relative to the truck (subscript t) and then using velocities relative to the ground (subscript g). We work with SI units, so $20\text{ km/h} \to 5.6\text{ m/s}$, $30\text{ km/h} \to 8.3\text{ m/s}$, and $45\text{ km/h} \to 12.5\text{ m/s}$. We choose east as the $+\hat{\text{i}}$ direction.

(a) The velocity of the cheetah (subscript c) at the end of the 2.0 s interval is (from Eq. 4-44)

$$\vec{v}_{\text{c t}} = \vec{v}_{\text{c g}} - \vec{v}_{\text{t g}} = (12.5\text{ m/s})\ \hat{\text{i}} - (-5.6\text{ m/s})\ \hat{\text{i}} = (18.1\text{ m/s})\ \hat{\text{i}}$$

relative to the truck. Since the velocity of the cheetah relative to the truck at the beginning of the 2.0 s interval is $(-8.3\text{ m/s})\hat{\text{i}}$, the (average) acceleration vector relative to the cameraman (in the truck) is

$$\vec{a}_{\text{avg}} = \frac{(18.1\text{ m/s})\hat{\text{i}} - (-8.3\text{ m/s})\hat{\text{i}}}{2.0\text{ s}} = (13\text{ m/s}^2)\hat{\text{i}},$$

or $|\vec{a}_{\text{avg}}| = 13\text{ m/s}^2$.

(b) The direction of $\vec{a}_{\text{avg}}$ is $+\hat{\text{i}}$, or eastward.

(c) The velocity of the cheetah at the start of the 2.0 s interval is (from Eq. 4-44)

$$\vec{v}_{0\text{cg}} = \vec{v}_{0\text{ct}} + \vec{v}_{0\text{tg}} = (-8.3\text{ m/s})\hat{\text{i}} + (-5.6\text{ m/s})\hat{\text{i}} = (-13.9\text{ m/s})\hat{\text{i}}$$

relative to the ground. The (average) acceleration vector relative to the crew member (on the ground) is

$$\vec{a}_{\text{avg}} = \frac{(12.5\text{ m/s})\hat{\text{i}} - (-13.9\text{ m/s})\hat{\text{i}}}{2.0\text{ s}} = (13\text{ m/s}^2)\hat{\text{i}}, \quad |\vec{a}_{\text{avg}}| = 13\text{ m/s}^2$$

identical to the result of part (a).

(d) The direction of $\vec{a}_{\text{avg}}$ is $+\hat{\text{i}}$, or eastward.

70. We use Eq. 4-44, noting that the upstream corresponds to the $+\hat{\text{i}}$ direction.

(a) The subscript b is for the boat, w is for the water, and g is for the ground.

$$\vec{v}_{bg} = \vec{v}_{bw} + \vec{v}_{wg} = (14 \text{ km/h})\,\hat{i} + (-9 \text{ km/h})\,\hat{i} = (5 \text{ km/h})\,\hat{i}.$$

Thus, the magnitude is $|\vec{v}_{bg}| = 5$ km/h.

(b) The direction of $\vec{v}_{bg}$ is $+x$, or upstream.

(c) We use the subscript c for the child, and obtain

$$\vec{v}_{cg} = \vec{v}_{cb} + \vec{v}_{bg} = (-6 \text{ km/h})\,\hat{i} + (5 \text{ km/h})\,\hat{i} = (-1 \text{ km/h})\,\hat{i}\,.$$

The magnitude is $|\vec{v}_{cg}| = 1$ km/h.

(d) The direction of $\vec{v}_{cg}$ is $-x$, or downstream.

71. While moving in the same direction as the sidewalk's motion (covering a distance d relative to the ground in time t_1 = 2.50 s), Eq. 4-44 leads to

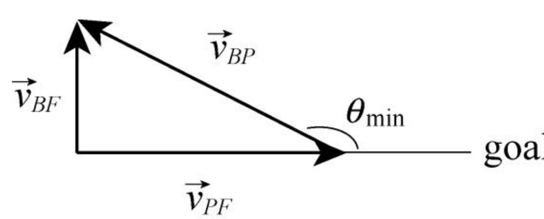
$$v_{\text{sidewalk}} + v_{\text{man running}} = \frac{d}{t_1}\ .$$

While he runs back (taking time t_2 = 10.0 s) we have

$$v_{\text{sidewalk}} - v_{\text{man running}} = -\frac{d}{t_2}\ .$$

Dividing these equations and solving for the desired ratio, we get $\frac{12.5}{7.5} = \frac{5}{3} = 1.67$.

72. We denote the velocity of the player with $\vec{v}_{PF}$ and the relative velocity between the player and the ball be $\vec{v}_{BP}$. Then the velocity $\vec{v}_{BF}$ of the ball relative to the field is given by $\vec{v}_{BF} = \vec{v}_{PF} + \vec{v}_{BP}$. The smallest angle $\theta_{\min}$ corresponds to the case when $\vec{v}_{BF} \perp \vec{v}_{PF}$. Hence,



$$\theta_{\min} = 180° - \cos^{-1}\left(\frac{|\vec{v}_{PF}|}{|\vec{v}_{BP}|}\right) = 180° - \cos^{-1}\left(\frac{4.0 \text{ m/s}}{6.0 \text{ m/s}}\right) = 130°.$$

73. We denote the police and the motorist with subscripts p and m, respectively. The coordinate system is indicated in Fig. 4-46.

(a) The velocity of the motorist with respect to the police car is

$$\vec{v}_{mp} = \vec{v}_m - \vec{v}_p = (-60 \text{ km/h})\hat{j} - (-80 \text{ km/h})\hat{i} = (80 \text{ km/h})\hat{i} - (60 \text{ km/h})\hat{j}.$$

(b) $\vec{v}_{mp}$ does happen to be along the line of sight. Referring to Fig. 4-46, we find the vector pointing from one car to another is $\vec{r} = (800 \text{ m})\hat{\text{i}} - (600 \text{ m})\,\hat{\text{j}}$ (from M to P). Since the ratio of components in $\vec{r}$ is the same as in $\vec{v}_{mp}$, they must point the same direction.

(c) No, they remain unchanged.

74. Velocities are taken to be constant; thus, the velocity of the plane relative to the ground is $\vec{v}_{PG} = (55 \text{ km})/(1/4 \text{ hour})\,\hat{\text{j}} = (220 \text{ km/h})\hat{\text{j}}$. In addition,

$$\vec{v}_{AG} = (42 \text{ km/h})(\cos 20°\hat{\text{i}} - \sin 20°\hat{\text{j}}) = (39 \text{ km/h})\hat{\text{i}} - (14 \text{ km/h})\hat{\text{j}}.$$

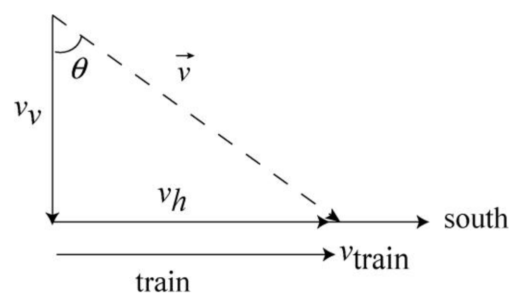
Using $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$, we have

$$\vec{v}_{PA} = \vec{v}_{PG} - \vec{v}_{AG} = -(39 \text{ km/h})\hat{\text{i}} + (234 \text{ km/h})\hat{\text{j}}.$$

which implies $|\vec{v}_{PA}| = 237 \text{ km/h}$, or 240 km/h (to two significant figures.)

75. **THINK** This problem deals with relative motion in two dimensions. Raindrops appear to fall vertically by an observer on a moving train.

EXPRESS Since the raindrops fall vertically relative to the train, the horizontal component of the velocity of a raindrop, v_h = 30 m/s, must be the same as the speed of the train, i.e., $v_h = v_{\text{train}}$ (see figure).



On the other hand, if v_v is the vertical component of the velocity and θ is the angle between the direction of motion and the vertical, then tan $\theta = v_h/v_v$. Knowing v_v and v_h allows us to determine the speed of the raindrops.

ANALYZE With $\theta = 70°$, we find the vertical component of the velocity to be

$$v_v = v_h/\tan\theta = (30 \text{ m/s})/\tan 70° = 10.9 \text{ m/s}.$$

Therefore, the speed of a raindrop is

$$v = \sqrt{v_h^2 + v_v^2} = \sqrt{(30 \text{ m/s})^2 + (10.9 \text{ m/s})^2} = 32 \text{ m/s}.$$

LEARN As long as the horizontal component of the velocity of the raindrops coincides with the speed of the train, the passenger on board will see the rain falling perfectly vertically.

76. The destination is $\vec{D}$ = 800 km $\hat{\text{j}}$ where we orient axes so that +*y* points north and +*x* points east. This takes two hours, so the (constant) velocity of the plane (relative to the ground) is $\vec{v}_{\text{pg}}$ = (400 km/h) $\hat{\text{j}}$. This must be the vector sum of the plane's velocity with respect to the air which has (*x*,*y*) components (500cos70º, 500sin70º), and the velocity of the air (*wind*) relative to the ground $\vec{v}_{\text{ag}}$. Thus,

$$(400\text{ km/h})\,\hat{\text{j}} = (500\text{ km/h})\cos 70^\circ\,\hat{\text{i}} + (500\text{ km/h})\sin 70^\circ\,\hat{\text{j}} + \vec{v}_{\text{ag}}$$

which yields

$$\vec{v}_{\text{ag}} = (-171\text{ km/h})\hat{\text{i}} - (70.0\text{ km/h})\hat{\text{j}}\,.$$

(a) The magnitude of $\vec{v}_{\text{ag}}$ is $|\vec{v}_{\text{ag}}| = \sqrt{(-171\text{ km/h})^2 + (-70.0\text{ km/h})^2} = 185\text{ km/h}.$

(b) The direction of $\vec{v}_{\text{ag}}$ is

$$\theta = \tan^{-1}\left(\frac{-70.0\text{ km/h}}{-171\text{ km/h}}\right) = 22.3^\circ \text{ (south of west).}$$

77. **THINK** This problem deals with relative motion in two dimensions. Snowflakes falling vertically downward are seen to fall at an angle by a moving observer.

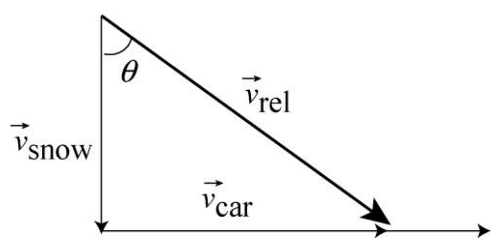
EXPRESS Relative to the car the velocity of the snowflakes has a vertical component of $v_v = 8.0\text{ m/s}$ and a horizontal component of $v_h = 50\text{ km/h} = 13.9\text{ m/s}$.

ANALYZE The angle θ from the vertical is found from

$$\tan\theta = \frac{v_h}{v_v} = \frac{13.9\text{ m/s}}{8.0\text{ m/s}} = 1.74$$

which yields $\theta = 60°$.

LEARN The problem can also be solved by expressing the velocity relation in vector notation: $\vec{v}_{\text{rel}} = \vec{v}_{\text{car}} + \vec{v}_{\text{snow}}$, as shown in the figure.



78. We make use of Eq. 4-44 and Eq. 4-45.

The velocity of Jeep *P* relative to *A* at the instant is

$$\vec{v}_{PA} = (40.0 \text{ m/s})(\cos 60°\hat{\text{i}} + \sin 60°\hat{\text{j}}) = (20.0 \text{ m/s})\hat{\text{i}} + (34.6 \text{ m/s})\hat{\text{j}}.$$

Similarly, the velocity of Jeep *B* relative to *A* at the instant is

$$\vec{v}_{BA} = (20.0 \text{ m/s})(\cos 30°\hat{\text{i}} + \sin 30°\hat{\text{j}}) = (17.3 \text{ m/s})\hat{\text{i}} + (10.0 \text{ m/s})\hat{\text{j}}.$$

Thus, the velocity of *P* relative to *B* is

$$\vec{v}_{PB} = \vec{v}_{PA} - \vec{v}_{BA} = (20.0\hat{\text{i}} + 34.6\hat{\text{j}}) \text{ m/s} - (17.3\hat{\text{i}} + 10.0\hat{\text{j}}) \text{ m/s} = (2.68 \text{ m/s})\hat{\text{i}} + (24.6 \text{ m/s})\hat{\text{j}}.$$

(a) The magnitude of $\vec{v}_{PB}$ is $|\vec{v}_{PB}| = \sqrt{(2.68 \text{ m/s})^2 + (24.6 \text{ m/s})^2} = 24.8 \text{ m/s}$.

(b) The direction of $\vec{v}_{PB}$ is $\theta = \tan^{-1}[(24.6 \text{ m/s})/(2.68 \text{ m/s})] = 83.8°$ north of east (or 6.2° east of north).

(c) The acceleration of *P* is

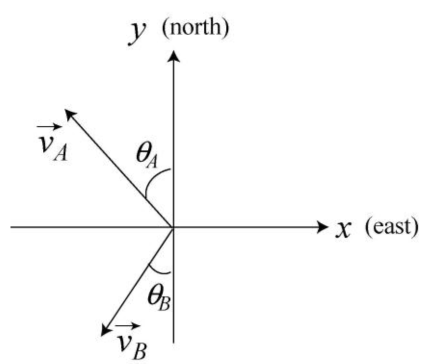
$$\vec{a}_{PA} = (0.400 \text{ m/s}^2)(\cos 60.0°\hat{\text{i}} + \sin 60.0°\hat{\text{j}}) = (0.200 \text{ m/s}^2)\hat{\text{i}} + (0.346 \text{ m/s}^2)\hat{\text{j}},$$

and $\vec{a}_{PA} = \vec{a}_{PB}$. Thus, we have $|\vec{a}_{PB}| = 0.400 \text{ m/s}^2$.

(d) The direction is 60.0° north of east (or 30.0° east of north).

79. **THINK** This problem involves analyzing the relative motion of two ships sailing in different directions.

EXPRESS Given that $\theta_A = 45°$, and $\theta_B = 40°$, as defined in the figure, the velocity vectors (relative to the shore) for ships *A* and *B* are given by



$$\vec{v}_A = -(v_A \cos 45°)\,\hat{\text{i}} + (v_A \sin 45°)\,\hat{\text{j}}$$
$$\vec{v}_B = -(v_B \sin 40°)\,\hat{\text{i}} - (v_B \cos 40°)\,\hat{\text{j}},$$

with v_A = 24 knots and v_B = 28 knots. We take east as $+\hat{\text{i}}$ and north as $\hat{\text{j}}$.

The velocity of ship *A* relative to ship *B* is simply given by $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$.

ANALYZE (a) The relative velocity is

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = (v_B \sin 40° - v_A \cos 45°)\hat{\text{i}} + (v_B \cos 40° + v_A \sin 45°)\,\hat{\text{j}}$$
$$= (1.03 \text{ knots})\hat{\text{i}} + (38.4 \text{ knots})\hat{\text{j}}$$

the magnitude of which is $|\vec{v}_{AB}| = \sqrt{(1.03 \text{ knots})^2 + (38.4 \text{ knots})^2} \approx 38.4 \text{ knots}.$

(b) The angle θ_{AB} which $\vec{v}_{AB}$ makes with north is given by

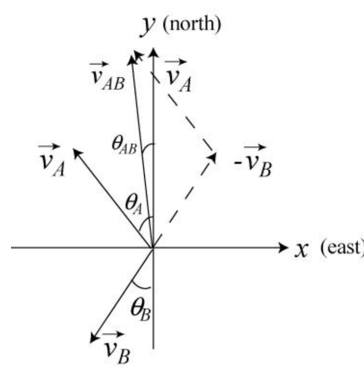
$$\theta_{AB} = \tan^{-1}\left(\frac{v_{AB,x}}{v_{AB,y}}\right) = \tan^{-1}\left(\frac{1.03 \text{ knots}}{38.4 \text{ knots}}\right) = 1.5°$$

which is to say that $\vec{v}_{AB}$ points 1.5° east of north.

(c) Since the two ships started at the same time, their relative velocity describes at what rate the distance between them is increasing. Because the rate is steady, we have

$$t = \frac{|\Delta r_{AB}|}{|\vec{v}_{AB}|} = \frac{160 \text{ nautical miles}}{38.4 \text{ knots}} = 4.2 \text{ h.}$$

(d) The velocity $\vec{v}_{AB}$ does not change with time in this problem, and $\vec{r}_{AB}$ is in the same direction as $\vec{v}_{AB}$ since they started at the same time. Reversing the points of view, we have $\vec{v}_{AB} = -\vec{v}_{BA}$ so that $\vec{r}_{AB} = -\vec{r}_{BA}$ (i.e., they are 180° opposite to each other). Hence, we conclude that *B* stays at a bearing of 1.5° west of south relative to *A* during the journey (neglecting the curvature of Earth).



LEARN The relative velocity is depicted in the figure on the right. When analyzing relative motion in two dimensions, a vector diagram such as the one shown can be very helpful.

80. This is a classic problem involving two-dimensional relative motion. We align our coordinates so that *east* corresponds to +*x* and *north* corresponds to +*y*. We write the vector addition equation as $\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG}$. We have $\vec{v}_{WG} = (2.0\angle 0°)$ in the magnitude-angle notation (with the unit m/s understood), or $\vec{v}_{WG} = 2.0\hat{\text{i}}$ in unit-vector notation. We also have $\vec{v}_{BW} = (8.0\angle 120°)$ where we have been careful to phrase the angle in the 'standard' way (measured counterclockwise from the +*x* axis), or $\vec{v}_{BW} = (-4.0\hat{\text{i}}+6.9\hat{\text{j}})$ m/s.

(a) We can solve the vector addition equation for $\vec{v}_{BG}$:

$$\vec{v}_{BG} = v_{BW} + \vec{v}_{WG} = (2.0 \text{ m/s})\hat{\text{i}} + (-4.0\hat{\text{i}} + 6.9\hat{\text{j}}) \text{ m/s} = (-2.0 \text{ m/s})\hat{\text{i}} + (6.9 \text{ m/s})\hat{\text{j}}.$$

Thus, we find $|\vec{v}_{BG}| = 7.2$ m/s.

(b) The direction of $\vec{v}_{BG}$ is $\theta = \tan^{-1}[(6.9 \text{ m/s})/(-2.0 \text{ m/s})] = 106°$ (measured counterclockwise from the +*x* axis), or 16° west of north.

(c) The velocity is constant, and we apply $y - y_0 = v_y t$ in a reference frame. Thus, in the *ground* reference frame, we have $(200 \text{ m}) = (7.2 \text{ m/s})\sin(106°)t \rightarrow t = 29$ s. Note: If a student obtains "28 s," then the student has probably neglected to take the *y* component properly (a common mistake).

81. Here, the subscript *W* refers to the water. Our coordinates are chosen with +*x* being *east* and +*y* being *north*. In these terms, the angle specifying *east* would be 0° and the angle specifying *south* would be –90° or 270°. Where the length unit is not displayed, km is to be understood.

(a) We have $\vec{v}_{AW} = \vec{v}_{AB} + \vec{v}_{BW}$, so that

$$\vec{v}_{AB} = (22 \angle -90°) - (40 \angle 37°) = (56 \angle -125°)$$

in the magnitude-angle notation (conveniently done with a vector-capable calculator in polar mode). Converting to rectangular components, we obtain

$$\vec{v}_{AB} = (-32 \text{km/h})\,\hat{\text{i}} - (46 \text{ km/h})\,\hat{\text{j}} .$$

Of course, this could have been done in unit-vector notation from the outset.

(b) Since the velocity-components are constant, integrating them to obtain the position is straightforward $(\vec{r} - \vec{r}_0 = \int \vec{v}\, dt)$

$$\vec{r} = (2.5 - 32t)\,\hat{\text{i}} + (4.0 - 46t)\,\hat{\text{j}}$$

with lengths in kilometers and time in hours.

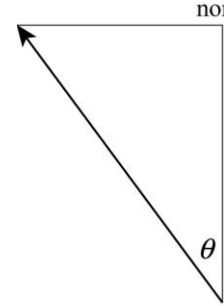
(c) The magnitude of this $\vec{r}$ is $r = \sqrt{(2.5 - 32t)^2 + (4.0 - 46t)^2}$. We minimize this by taking a derivative and requiring it to equal zero — which leaves us with an equation for *t*

$$\frac{dr}{dt} = \frac{1}{2}\frac{6286t - 528}{\sqrt{(2.5 - 32t)^2 + (4.0 - 46t)^2}} = 0$$

which yields $t = 0.084$ h.

(d) Plugging this value of t back into the expression for the distance between the ships (r), we obtain r = 0.2 km. Of course, the calculator offers more digits (r = 0.225…), but they are not significant; in fact, the uncertainties implicit in the given data, here, should make the ship captains worry.

82. We construct a right triangle starting from the clearing on the south bank, drawing a line (200 m long) due north (*upward* in our sketch) across the river, and then a line due west (upstream, leftward in our sketch) along the north bank for a distance $(82\text{ m})+(1.1\text{ m/s})t$, where the t-dependent contribution is the distance that the river will carry the boat downstream during time t.



The hypotenuse of this right triangle (the arrow in our sketch) also depends on t and on the boat's speed (relative to the water), and we set it equal to the Pythagorean "sum" of the triangle's sides:

$$(4.0)t = \sqrt{200^2 + (82+1.1t)^2}$$

which leads to a quadratic equation for t

$$46724 + 180.4t - 14.8t^2 = 0.$$

(b) We solve for t first and find a positive value: t = 62.6 s.

(a) The angle between the northward (200 m) leg of the triangle and the hypotenuse (which is measured "west of north") is then given by

$$\theta = \tan^{-1}\left(\frac{82+1.1t}{200}\right) = \tan^{-1}\left(\frac{151}{200}\right) = 37°.$$

83. We establish coordinates with $\hat{\text{i}}$ pointing to the far side of the river (perpendicular to the current) and $\hat{\text{j}}$ pointing in the direction of the current. We are told that the magnitude (presumed constant) of the velocity of the boat relative to the water is $|\vec{v}_{bw}|$ = 6.4 km/h. Its angle, relative to the x axis is θ. With km and h as the understood units, the velocity of the water (relative to the ground) is $\vec{v}_{wg} = (3.2\text{ km/h})\hat{\text{j}}$.

(a) To reach a point "directly opposite" means that the velocity of her boat relative to ground must be $\vec{v}_{bg} = v_{bg}\hat{\text{i}}$ where $v_{bg} > 0$ is unknown. Thus, all $\hat{\text{j}}$ components must cancel in the vector sum $\vec{v}_{bw} + \vec{v}_{wg} = \vec{v}_{bg}$, which means the $\vec{v}_{bw}$ sin θ = (–3.2 km/h) $\hat{\text{j}}$, so

$$\theta = \sin^{-1}[(-3.2\text{ km/h})/(6.4\text{ km/h})] = -30°.$$

(b) Using the result from part (a), we find $v_{bg} = v_{bw}\cos\theta = 5.5$ km/h. Thus, traveling a distance of $\ell = 6.4$ km requires a time of (6.4 km)/(5.5 km/h) = 1.15 h or 69 min.

(c) If her motion is completely along the y axis (as the problem implies) then with v_{wg} = 3.2 km/h (the water speed) we have

$$t_{\text{total}} = \frac{D}{v_{bw} + v_{wg}} + \frac{D}{v_{bw} - v_{wg}} = 1.33 \text{ h}$$

where D = 3.2 km. This is equivalent to 80 min.

(d) Since

$$\frac{D}{v_{bw}+v_{wg}} + \frac{D}{v_{bw}-v_{wg}} = \frac{D}{v_{bw}-v_{wg}} + \frac{D}{v_{bw}+v_{wg}}$$

the answer is the same as in the previous part, that is, $t_{\text{total}} = 80 \text{ min}$.

(e) The shortest-time path should have $\theta = 0°$. This can also be shown by noting that the case of general θ leads to

$$\vec{v}_{bg} = \vec{v}_{bw} + \vec{v}_{wg} = v_{bw}\cos\theta\,\hat{\text{i}} + (v_{bw}\sin\theta + v_{wg})\,\hat{\text{j}}$$

where the x component of $\vec{v}_{bg}$ must equal l/t. Thus,

$$t = \frac{l}{v_{bw}\cos\theta}$$

which can be minimized using $dt/d\theta = 0$.

(f) The above expression leads to t = (6.4 km)/(6.4 km/h) = 1.0 h, or 60 min.

84. Relative to the sled, the launch velocity is $\vec{v}_{0\text{rel}} = v_{ox}\,\hat{\text{i}} + v_{oy}\,\hat{\text{j}}$. Since the sled's motion is in the negative direction with speed v_s (note that we are treating v_s as a positive number, so the sled's velocity is actually $-v_s\,\hat{\text{i}}$), then the launch velocity relative to the ground is $\vec{v}_0 = (v_{ox} - v_s)\,\hat{\text{i}} + v_{oy}\,\hat{\text{j}}$. The horizontal and vertical displacement (relative to the ground) are therefore

$$x_{\text{land}} - x_{\text{launch}} = \Delta x_{\text{bg}} = (v_{ox} - v_s)\, t_{\text{flight}}$$

$$y_{\text{land}} - y_{\text{launch}} = 0 = v_{oy}\, t_{\text{flight}} + \tfrac{1}{2}(-g)(t_{\text{flight}})^2 .$$

Combining these equations leads to

$$\Delta x_{\text{bg}} = \frac{2v_{0x}v_{0y}}{g} - \left(\frac{2v_{0y}}{g}\right)v_s .$$

The first term corresponds to the "*y* intercept" on the graph, and the second term (in parentheses) corresponds to the magnitude of the "slope." From the figure, we have

$$\Delta x_{bg} = 40 - 4v_s .$$

This implies v_{oy} = (4.0 s)(9.8 m/s^2)/2 = 19.6 m/s, and that furnishes enough information to determine v_{ox}.

(a) $v_{ox} = 40g/2v_{oy}$ = (40 m)(9.8 m/s^2)/(39.2 m/s) = 10 m/s.

(b) As noted above, v_{oy} = 19.6 m/s.

(c) Relative to the sled, the displacement Δx_{bs} does not depend on the sled's speed, so $\Delta x_{bs} = v_{ox}\, t_{\text{flight}}$ = 40 m.

(d) As in (c), relative to the sled, the displacement Δx_{bs} does not depend on the sled's speed, and $\Delta x_{bs} = v_{ox}\, t_{\text{flight}}$ = 40 m.

85. Using displacement = velocity × time (for each constant-velocity part of the trip), along with the fact that 1 hour = 60 minutes, we have the following vector addition exercise (using notation appropriate to many vector-capable calculators):

(1667 m ∠ 0º) + (1333 m ∠ –90º) + (333 m ∠ 180º) + (833 m ∠ –90º) + (667 m ∠ 180º) + (417 m ∠ –90º) = (2668 m ∠ –76º).

(a) Thus, the magnitude of the net displacement is 2.7 km.

(b) Its direction is 76° clockwise (relative to the initial direction of motion).

86. We use a coordinate system with +*x* eastward and +*y* upward.

(a) We note that 123° is the angle between the initial position and later position vectors, so that the angle from +*x* to the later position vector is 40° + 123° = 163°. In unit-vector notation, the position vectors are

$$\vec{r}_1 = (360\text{ m})\cos(40°)\,\hat{\text{i}} + (360\text{ m})\sin(40°)\,\hat{\text{j}} = (276\text{ m})\hat{\text{i}} + (231\text{ m})\,\hat{\text{j}}$$
$$\vec{r}_2 = (790\text{ m})\cos(163°)\,\hat{\text{i}} + (790\text{ m})\sin(163°)\,\hat{\text{j}} = (-755\text{ m})\hat{\text{i}} + (231\text{ m})\hat{\text{j}}$$

respectively. Consequently, we plug into Eq. 4-3

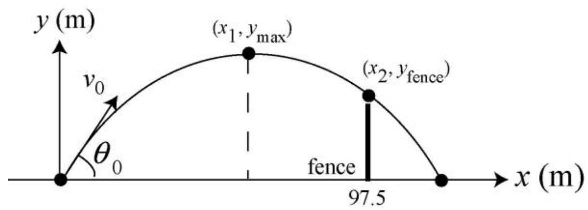
$$\Delta\vec{r} = [(-755\text{ m}) - (276\text{ m})]\hat{\text{i}} + (231\text{ m} - 231\text{ m})\hat{\text{j}} = -(1031\text{ m})\,\hat{\text{i}}.$$

The magnitude of the displacement $\Delta\vec{r}$ is $|\Delta\vec{r}| = 1031\text{ m}$.

(b) The direction of $\Delta\vec{r}$ is $-\hat{\text{i}}$, or westward.

87. **THINK** This problem deals with the projectile motion of a baseball. Given the information on the position of the ball at two instants, we are asked to analyze its trajectory.

EXPRESS The trajectory of the baseball is shown in the figure on the right. According to the problem statement, at $t_1 = 3.0\text{ s}$, the ball reaches it maximum height $y_{\max}$, and at $t_2 = t_1 + 2.5\text{ s} = 5.5\text{ s}$, it barely clears a fence at $x_2 = 97.5\text{ m}$.



Eq. 2-15 can be applied to the vertical (y axis) motion related to reaching the maximum height (when t_1 = 3.0 s and v_y = 0):

$$y_{\max} - y_0 = v_y t - \frac{1}{2}gt^2 .$$

ANALYZE (a) With ground level chosen so y_0 = 0, this equation gives the result

$$y_{\max} = \frac{1}{2}gt_1^2 = \frac{1}{2}(9.8\text{ m/s}^2)(3.0\text{ s})^2 = 44.1\text{ m}$$

(b) After the moment it reached maximum height, it is falling; at $t_2 = t_1 + 2.5\text{ s} = 5.5\text{ s}$, it will have fallen an amount given by Eq. 2-18:

$$y_{\text{fence}} - y_{\max} = 0 - \frac{1}{2}g(t_2 - t_1)^2 .$$

Thus, the height of the fence is

$$y_{\text{fence}} = y_{\max} - \frac{1}{2}g(t_2 - t_1)^2 = 44.1\text{ m} - \frac{1}{2}(9.8\text{ m/s}^2)(2.5\text{ s})^2 = 13.48\text{ m} .$$

(c) Since the horizontal component of velocity in a projectile-motion problem is constant (neglecting air friction), we find from 97.5 m = v_{0x}(5.5 s) that v_{0x} = 17.7 m/s. The total flight time of the ball is $T = 2t_1 = 2(3.0\text{ s}) = 6.0\text{ s}$. Thus, the range of the baseball is

$$R = v_{0x}T = (17.7\text{ m/s})(6.0\text{ s}) = 106.4\text{ m}$$

which means that the ball travels an additional distance

$$\Delta x = R - x_2 = 106.4\text{ m} - 97.5\text{ m} = 8.86\text{ m}$$

beyond the fence before striking the ground.

LEARN Part (c) can also be solved by noting that after passing the fence, the ball will strike the ground in 0.5 s (so that the total "fall-time" equals the "rise-time"). With v_{0x} = 17.7 m/s, we have Δx = (17.7 m/s)(0.5 s) = 8.86 m.

88. When moving in the same direction as the jet stream (of speed v_s), the time is

$$t_1 = \frac{d}{v_{ja} + v_s},$$

where d = 4000 km is the distance and v_{ja} is the speed of the jet relative to the air (1000 km/h). When moving against the jet stream, the time is

$$t_2 = \frac{d}{v_{ja} - v_s},$$

where $t_2 - t_1 = \frac{70}{60}\text{ h}$. Combining these equations and using the quadratic formula to solve gives v_s = 143 km/h.

89. **THINK** We have a particle moving in a two-dimensional plane with a constant acceleration. Since the x and y components of the acceleration are constants, we can use Table 2-1 for the motion along both axes.

EXPRESS Using vector notation with $\vec{r}_0 = 0$, the position and velocity of the particle as a function of time are given by $\vec{r}(t) = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$ and $\vec{v}(t) = \vec{v}_0 + \vec{a}t$, respectively. Where units are not shown, SI units are to be understood.

ANALYZE (a) Given the initial velocity $\vec{v}_0 = (8.0\text{ m/s})\hat{\text{j}}$ and the acceleration $\vec{a} = (4.0\text{ m/s}^2)\hat{\text{i}} + (2.0\text{ m/s}^2)\hat{\text{j}}$, the position vector of the particle is

$$\vec{r} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 = \left(8.0\,\hat{\text{j}}\right)t + \frac{1}{2}\left(4.0\,\hat{\text{i}} + 2.0\,\hat{\text{j}}\right)t^2 = \left(2.0t^2\right)\hat{\text{i}} + \left(8.0t + 1.0t^2\right)\hat{\text{j}}.$$

Therefore, the time that corresponds to x = 29 m can be found by solving the equation $2.0t^2 = 29$, which leads to t = 3.8 s. The y coordinate at that time is

$$y = (8.0\text{ m/s})(3.8\text{ s}) + (1.0\text{ m/s}^2)(3.8\text{ s})^2 = 45\text{ m}.$$

(b) The velocity of the particle is given by $\vec{v} = \vec{v}_0 + \vec{a}t$. Thus, at t = 3.8 s, the velocity is

$$\vec{v} = (8.0\text{ m/s})\,\hat{\text{j}} + \left((4.0\text{ m/s}^2)\,\hat{\text{i}} + (2.0\text{ m/s}^2)\,\hat{\text{j}}\right)(3.8\text{ s}) = (15.2\text{ m/s})\,\hat{\text{i}} + (15.6\text{ m/s})\,\hat{\text{j}}$$

which has a magnitude of $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15.2\text{ m/s})^2 + (15.6\text{ m/s})^2} = 22\text{ m/s}.$

LEARN Instead of using the vector notation, we can also deal with the x- and the y-components individually.

90. Using the same coordinate system assumed in Eq. 4-25, we rearrange that equation to solve for the initial speed:

$$v_0 = \frac{x}{\cos\theta_0}\sqrt{\frac{g}{2\,(x\tan\theta_0 - y)}}$$

which yields v_0 = 23 ft/s for g = 32 ft/s^2, x = 13 ft, y = 3 ft and θ_0 = 55°.

91. We make use of Eq. 4-25.

(a) By rearranging Eq. 4-25, we obtain the initial speed:

$$v_0 = \frac{x}{\cos\theta_0}\sqrt{\frac{g}{2(x\tan\theta_0 - y)}}$$

which yields $v_0 = 255.5 \approx 2.6 \times 10^2$ m/s for x = 9400 m, y = –3300 m, and θ_0 = 35°.

(b) From Eq. 4-21, we obtain the time of flight:

$$t = \frac{x}{v_0\cos\theta_0} = \frac{9400\text{ m}}{(255.5\text{ m/s})\cos 35^\circ} = 45\text{ s}.$$

(c) We expect the air to provide resistance but no appreciable lift to the rock, so we would need a greater launching speed to reach the same target.

92. We apply Eq. 4-34 to solve for speed v and Eq. 4-35 to find the period T.

(a) We obtain

$$v = \sqrt{ra} = \sqrt{(5.0\text{ m})(7.0)(9.8\text{ m/s}^2)} = 19\text{ m/s}.$$

(b) The time to go around once (the period) is $T = 2\pi r/v$ = 1.7 s. Therefore, in one minute (t = 60 s), the astronaut executes

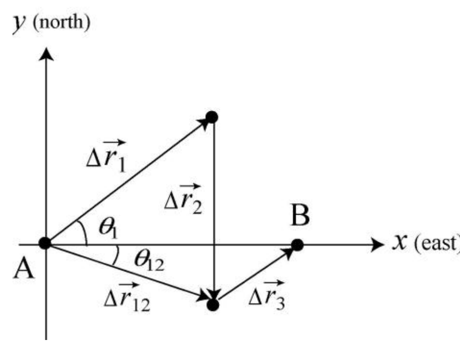
$$\frac{t}{T}=\frac{60\text{ s}}{1.7\text{ s}}=35$$

revolutions. Thus, 35 rev/min is needed to produce a centripetal acceleration of 7*g* when the radius is 5.0 m.

(c) As noted above, T = 1.7 s.

93. **THINK** This problem deals with the two-dimensional kinematics of a desert camel moving from oasis A to oasis B.

EXPRESS The journey of the camel is illustrated in the figure on the right. We use a 'standard' coordinate system with +*x* East and +*y* North. Lengths are in kilometers and times are in hours. Using vector notation, we write the displacements for the first two segments of the trip as:



$$\Delta\vec{r}_1=(75\text{ km})\cos(37°)\,\hat{\text{i}}+(75\text{ km})\sin(37°)\,\hat{\text{j}}$$

$$\Delta\vec{r}_2=(-65\text{ km})\,\hat{\text{j}}$$

The net displacement is $\Delta\vec{r}_{12}=\Delta\vec{r}_1+\Delta\vec{r}_2$. As can be seen from the figure, to reach oasis B requires an additional displacement $\Delta\vec{r}_3$.

ANALYZE (a) We perform the vector addition of individual displacements to find the net displacement of the camel: $\Delta\vec{r}_{12}=\Delta\vec{r}_1+\Delta\vec{r}_2=(60\text{ km})\,\hat{\text{i}}-(20\text{ km})\,\hat{\text{j}}$. Its corresponding magnitude is

$$|\Delta\vec{r}_{12}|=\sqrt{(60\text{ km})^2+(-20\text{ km})^2}=63\text{ km}.$$

(b) The direction of $\Delta\vec{r}_{12}$ is $\theta_{12}=\tan^{-1}[(-20\text{ km})/(60\text{ km})]=-18°$, or $18°$ south of east.

(c) To calculate the average velocity for the first two segments of the journey (including rest), we use the result from part (a) in Eq. 4-8 along with the fact that

$$\Delta t_{12}=\Delta t_1+\Delta t_2+\Delta t_{\text{rest}}=50\text{ h}+35\text{ h}+5.0\text{ h}=90\text{ h}.$$

In unit vector notation, we have $\vec{v}_{12,\text{avg}}=\dfrac{(60\,\hat{\text{i}}-20\,\hat{\text{j}})\text{ km}}{90\text{ h}}=(0.67\,\hat{\text{i}}-0.22\,\hat{\text{j}})\text{ km/h}.$

This leads to $|\vec{v}_{12,\text{avg}}|=0.70\text{ km/h}.$

(d) The direction of $\vec{v}_{12,\text{avg}}$ is $\theta_{12}=\tan^{-1}[(-0.22\text{ km/h})/(0.67\text{ km/h})]=-18°$, or $18°$ south of east.

(e) The average speed is distinguished from the magnitude of average velocity in that it depends on the total distance as opposed to the net displacement. Since the camel travels 140 km, we obtain (140 km)/(90 h) = 1.56 km/h $\approx$1.6 km/h.

(f) The net displacement is required to be the 90 km East from *A* to *B*. The displacement from the resting place to *B* is denoted $\Delta\vec{r}_3$. Thus, we must have

$$\Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3 = (90\text{ km})\hat{\text{i}}$$

which produces $\Delta\vec{r}_3 = (30\text{ km})\hat{\text{i}} + (20\text{ km})\hat{\text{j}}$ in unit-vector notation, or $(36 \angle 33°)$ in magnitude-angle notation. Therefore, using Eq. 4-8 we obtain

$$|\vec{v}_{3,\text{avg}}| = \frac{36\text{ km}}{(120-90)\text{ h}} = 1.2\text{ km/h}.$$

(g) The direction of $\vec{v}_{3,\text{avg}}$ is the same as $\Delta\vec{r}_3$ (that is, 33° north of east).

LEARN With a vector-capable calculator in polar mode, we could perform the vector addition of the displacements as $(75 \angle 37°) + (65 \angle -90°) = (63 \angle -18°)$. Note the distinction between average velocity and average speed.

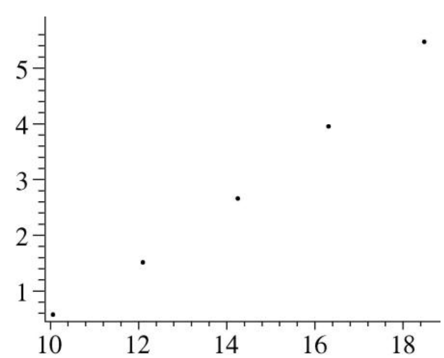
94. We compute the coordinate pairs (*x*, *y*) from $x = (v_0 \cos\theta)t$ and $y = v_0 \sin\theta t - \frac{1}{2}gt^2$ for *t* = 20 s and the speeds and angles given in the problem.

(a) We obtain

$$(x_A, y_A) = (10.1\text{ km}, 0.556\text{ km}) \qquad (x_B, y_B) = (12.1\text{ km}, 1.51\text{ km})$$
$$(x_C, y_C) = (14.3\text{ km}, 2.68\text{ km}) \qquad (x_D, y_D) = (16.4\text{ km}, 3.99\text{ km})$$

and (x_E, y_E) = (18.5 km, 5.53 km) which we plot in the next part.

(b) The vertical (*y*) and horizontal (*x*) axes are in kilometers. The graph does not start at the origin. The curve to "fit" the data is not shown, but is easily imagined (forming the "curtain of death").



95. (a) With Δx = 8.0 m, $t = \Delta t_1$, $a = a_x$, and v_{ox} = 0, Eq. 2-15 gives

$$8.0\text{ m} = \tfrac{1}{2}a_x(\Delta t_1)^2 ,$$

and the corresponding expression for motion along the y axis leads to

$$\Delta y = 12\text{ m} = \tfrac{1}{2}a_y(\Delta t_1)^2 .$$

Dividing the second expression by the first leads to $a_y / a_x = 3/2 = 1.5$.

(b) Letting $t = 2\Delta t_1$, then Eq. 2-15 leads to $\Delta x = (8.0\text{ m})(2)^2 = 32$ m, which implies that its x coordinate is now (4.0 + 32) m = 36 m. Similarly, $\Delta y = (12\text{ m})(2)^2 = 48$ m, which means its y coordinate has become (6.0 + 48) m = 54 m.

96. We assume the ball's initial velocity is perpendicular to the plane of the net. We choose coordinates so that $(x_0, y_0) = (0, 3.0)$ m, and $v_x > 0$ (note that $v_{0y} = 0$).

(a) To (barely) clear the net, we have

$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2 \quad \Rightarrow \; 2.24\text{ m} - 3.0\text{ m} = 0 - \frac{1}{2}\left(9.8\text{ m/s}^2\right)t^2$$

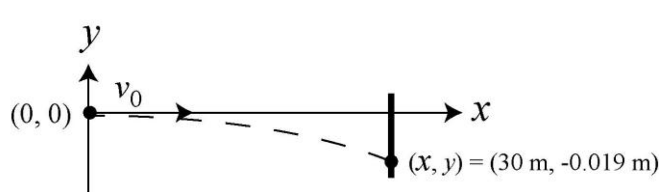
which gives $t = 0.39$ s for the time it is passing over the net. This is plugged into the x-equation to yield the (minimum) initial velocity $v_x = (8.0\text{ m})/(0.39\text{ s}) = 20.3$ m/s.

(b) We require $y = 0$ and find time t from the equation $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$. This value $\left(t = \sqrt{2(3.0\text{ m})/(9.8\text{ m/s}^2)} = 0.78\text{ s}\right)$ is plugged into the x-equation to yield the (maximum) initial velocity

$$v_x = (17.0\text{ m})/(0.78\text{ s}) = 21.7\text{ m/s}.$$

97. **THINK** A bullet fired horizontally from a rifle strikes the target at some distance below its aiming point. We're asked to find its total flight time and speed.

EXPRESS The trajectory of the bullet is shown in the figure on the right (not to scale). Note that the origin is chosen to be at the firing point. With this convention, the y coordinate of the bullet is given by $y = -\frac{1}{2}gt^2$. Knowing the coordinates (x, y) at the target allows us to calculate the total flight time and speed of the bullet.



ANALYZE (a) If t is the time of flight and $y = -0.019$ m indicates where the bullet hits the target, then

$$t=\sqrt{\frac{-2y}{g}}=\sqrt{\frac{-2(-0.019\text{ m})}{9.8\text{ m/s}^2}}=6.2\times10^{-2}\text{ s.}$$

(b) The muzzle velocity is the initial (horizontal) velocity of the bullet. Since x = 30 m is the horizontal position of the target, we have $x = v_0t$. Thus,

$$v_0=\frac{x}{t}=\frac{30\text{ m}}{6.3\times10^{-2}\text{ s}}=4.8\times10^{2}\text{ m/s.}$$

LEARN Alternatively, we may use Eq. 4-25 to solve for the initial velocity. With $\theta_0=0$ and $y_0=0$, the equation simplifies to $y=-\dfrac{gx^2}{2v_0^2}$, from which we find

$$v_0=\sqrt{-\frac{gx^2}{2y}}=\sqrt{-\frac{(9.8\text{ m/s}^2)(30\text{ m})^2}{2(-0.019\text{ m})}}=4.8\times10^2\text{ m/s},$$

in agreement with what we calculated in part (b).

98. For circular motion, we must have $\vec{v}$ with direction perpendicular to $\vec{r}$ and (since the speed is constant) magnitude $v=2\pi r/T$ where $r=\sqrt{(2.00\text{ m})^2+(-3.00\text{ m})^2}$ and $T=7.00\text{ s}$. The $\vec{r}$ (given in the problem statement) specifies a point in the fourth quadrant, and since the motion is clockwise then the velocity must have both components negative. Our result, satisfying these three conditions, (using unit-vector notation which makes it easy to double-check that $\vec{r}\cdot\vec{v}=0$) for $\vec{v}$ = $(-2.69\text{ m/s})\hat{\text{i}} + (-1.80\text{ m/s})\hat{\text{j}}$.

99. Let v_o = 2π(0.200 m)/(0.00500 s) ≈ 251 m/s (using Eq. 4-35) be the speed it had in circular motion and θ_o = (1 hr)(360º/12 hr [for full rotation]) = 30.0º. Then Eq. 4-25 leads to

$$y=(2.50\text{ m})\tan30.0°-\frac{(9.8\text{ m/s}^2)(2.50\text{ m})^2}{2(251\text{ m/s})^2(\cos30.0°)^2}\approx1.44\text{ m}$$

which means its height above the floor is 1.44 m + 1.20 m = 2.64 m.

100. Noting that $\vec{v}_2=0$, then, using Eq. 4-15, the average acceleration is

$$\vec{a}_{\text{avg}}=\frac{\Delta\vec{v}}{\Delta t}=\frac{0-\left(6.30\,\hat{\text{i}}-8.42\,\hat{\text{j}}\right)\text{m/s}}{3\text{ s}}=\left(-2.1\hat{\text{i}}+2.8\,\hat{\text{j}}\right)\text{m/s}^2$$

101. Using Eq. 2-16, we obtain $v^2=v_0^2-2gh$, or $h=(v_0^2-v^2)/2g$.

(a) Since $v = 0$ at the maximum height of an upward motion, with $v_0 = 7.00$ m/s, we have

$$h = (7.00 \text{ m/s})^2 / 2(9.80 \text{ m/s}^2) = 2.50 \text{ m}.$$

(b) The relative speed is $v_r = v_0 - v_c = 7.00 \text{ m/s} - 3.00 \text{ m/s} = 4.00 \text{ m/s}$ with respect to the floor. Using the above equation we obtain $h = (4.00 \text{ m/s})^2 / 2(9.80 \text{ m/s}^2) = 0.82$ m.

(c) The acceleration, or the rate of change of speed of the ball with respect to the ground is 9.80 m/s^2 (downward).

(d) Since the elevator cab moves at constant velocity, the rate of change of speed of the ball with respect to the cab floor is also 9.80 m/s^2 (downward).

102. (a) With r = 0.15 m and $a = 3.0 \times 10^{14}$ m/s^2, Eq. 4-34 gives

$$v = \sqrt{ra} = 6.7 \times 10^6 \text{ m/s}.$$

(b) The period is given by Eq. 4-35:

$$T = \frac{2\pi r}{v} = 1.4 \times 10^{-7} \text{ s}.$$

103. (a) The magnitude of the displacement vector $\Delta\vec{r}$ is given by

$$|\Delta\vec{r}| = \sqrt{(21.5 \text{ km})^2 + (9.7 \text{ km})^2 + (2.88 \text{ km})^2} = 23.8 \text{ km}.$$

Thus,

$$|\vec{v}_{\text{avg}}| = \frac{|\Delta\vec{r}|}{\Delta t} = \frac{23.8 \text{ km}}{3.50 \text{ h}} = 6.79 \text{ km/h}.$$

(b) The angle θ in question is given by

$$\theta = \tan^{-1}\left(\frac{2.88 \text{ km}}{\sqrt{(21.5 \text{ km})^2 + (9.7 \text{ km})^2}}\right) = 6.96°.$$

104. The initial velocity has magnitude v_0 and because it is horizontal, it is equal to v_x the horizontal component of velocity at impact. Thus, the speed at impact is

$$\sqrt{v_0^2 + v_y^2} = 3v_0$$

where $v_y = \sqrt{2gh}$ and we have used Eq. 2-16 with Δx replaced with h = 20 m. Squaring both sides of the first equality and substituting from the second, we find

$$v_0^2 + 2gh = (3v_0)^2$$

which leads to $gh = 4v_0^2$ and therefore to $v_0 = \sqrt{(9.8\text{ m/s}^2)(20\text{ m})}/2 = 7.0$ m/s.

105. We choose horizontal x and vertical y axes such that both components of $\vec{v}_0$ are positive. Positive angles are counterclockwise from +x and negative angles are clockwise from it. In unit-vector notation, the velocity at each instant during the projectile motion is

$$\vec{v} = v_0 \cos\theta_0\,\hat{\text{i}} + (v_0 \sin\theta_0 - gt)\,\hat{\text{j}}.$$

(a) With v_0 = 30 m/s and θ_0 = 60°, we obtain $\vec{v} = (15\hat{\text{i}} + 6.4\hat{\text{j}})$ m/s, for t = 2.0 s. The magnitude of $\vec{v}$ is $|\vec{v}| = \sqrt{(15\text{ m/s})^2 + (6.4\text{ m/s})^2} = 16$ m/s.

(b) The direction of $\vec{v}$ is

$$\theta = \tan^{-1}[(6.4\text{ m/s})/(15\text{ m/s})] = 23°,$$

measured counterclockwise from +x.

(c) Since the angle is positive, it is above the horizontal.

(d) With t = 5.0 s, we find $\vec{v} = (15\hat{\text{i}} - 23\hat{\text{j}})$ m/s, which yields

$$|\vec{v}| = \sqrt{(15\text{ m/s})^2 + (-23\text{ m/s})^2} = 27\text{ m/s}.$$

(e) The direction of $\vec{v}$ is $\theta = \tan^{-1}[(-23\text{ m/s})/(15\text{ m/s})] = -57°$, or 57° measured *clockwise* from +x.

(f) Since the angle is negative, it is below the horizontal.

106. We use Eq. 4-2 and Eq. 4-3.

(a) With the initial position vector as $\vec{r}_1$ and the later vector as $\vec{r}_2$, Eq. 4-3 yields

$$\Delta r = [(-2.0\text{ m}) - 5.0\text{ m}]\hat{\text{i}} + [(6.0\text{ m}) - (-6.0\text{ m})]\hat{\text{j}} + (2.0\text{ m} - 2.0\text{ m})\hat{\text{k}} = (-7.0\text{ m})\hat{\text{i}} + (12\text{ m})\hat{\text{j}}$$

for the displacement vector in unit-vector notation.

(b) Since there is no z component (that is, the coefficient of $\hat{\text{k}}$ is zero), the displacement vector is in the xy plane.

107. We write our magnitude-angle results in the form $(R \angle \theta)$ with SI units for the magnitude understood (m for distances, m/s for speeds, m/s^2 for accelerations). All angles θ are measured counterclockwise from +x, but we will occasionally refer to angles ϕ, which are measured counterclockwise from the vertical line between the circle-center and the coordinate origin and the line drawn from the circle-center to the particle location (see r in the figure). We note that the speed of the particle is $v = 2\pi r/T$ where r = 3.00 m and T = 20.0 s; thus, v = 0.942 m/s. The particle is moving counterclockwise in Fig. 4-56.

(a) At t = 5.0 s, the particle has traveled a fraction of

$$\frac{t}{T} = \frac{5.00\text{ s}}{20.0\text{ s}} = \frac{1}{4}$$

of a full revolution around the circle (starting at the origin). Thus, relative to the circle-center, the particle is at

$$\phi = \frac{1}{4}(360°) = 90°$$

measured from vertical (as explained above). Referring to Fig. 4-56, we see that this position (which is the "3 o'clock" position on the circle) corresponds to x = 3.0 m and y = 3.0 m relative to the coordinate origin. In our magnitude-angle notation, this is expressed as $(R \angle \theta) = (4.2 \angle 45°)$. Although this position is easy to analyze without resorting to trigonometric relations, it is useful (for the computations below) to note that these values of x and y relative to coordinate origin can be gotten from the angle ϕ from the relations

$$x = r\sin\phi, \quad y = r - r\cos\phi.$$

Of course, $R = \sqrt{x^2 + y^2}$ and θ comes from choosing the appropriate possibility from $\tan^{-1}(y/x)$ (or by using particular functions of vector-capable calculators).

(b) At t = 7.5 s, the particle has traveled a fraction of 7.5/20 = 3/8 of a revolution around the circle (starting at the origin). Relative to the circle-center, the particle is therefore at ϕ = 3/8 (360°) = 135° measured from vertical in the manner discussed above. Referring to Fig. 4-56, we compute that this position corresponds to

$$x = (3.00\text{ m})\sin 135° = 2.1\text{ m}$$
$$y = (3.0\text{ m}) - (3.0\text{ m})\cos 135° = 5.1\text{ m}$$

relative to the coordinate origin. In our magnitude-angle notation, this is expressed as $(R \angle \theta) = (5.5 \angle 68°)$.

(c) At t = 10.0 s, the particle has traveled a fraction of 10/20 = 1/2 of a revolution around the circle. Relative to the circle-center, the particle is at ϕ = 180° measured from vertical (see explanation above). Referring to Fig. 4-56, we see that this position corresponds to x

= 0 and y = 6.0 m relative to the coordinate origin. In our magnitude-angle notation, this is expressed as $\left(R\angle\theta\right)=\left(6.0\angle 90°\right)$.

(d) We subtract the position vector in part (a) from the position vector in part (c):

$$\left(6.0\angle 90°\right)-\left(4.2\angle 45°\right)=\left(4.2\angle 135°\right)$$

using magnitude-angle notation (convenient when using vector-capable calculators). If we wish instead to use unit-vector notation, we write

$$\Delta\vec{R}=(0-3.0\text{ m})\,\hat{\text{i}}+(6.0\text{ m}-3.0\text{ m})\,\hat{\text{j}}=(-3.0\text{ m})\hat{\text{i}}+(3.0\text{ m})\hat{\text{j}}$$

which leads to $|\Delta\vec{R}|=4.2\text{ m}$ and θ = 135°.

(e) From Eq. 4-8, we have $\vec{v}_{\text{avg}}=\Delta\vec{R}/\Delta t$. With $\Delta t = 5.0\text{ s}$, we have

$$\vec{v}_{\text{avg}}=(-0.60\text{ m/s})\,\hat{\text{i}}+(0.60\text{ m/s})\,\hat{\text{j}}$$

in unit-vector notation or (0.85 $\angle$ 135°) in magnitude-angle notation.

(f) The speed has already been noted (v = 0.94 m/s), but its direction is best seen by referring again to Fig. 4-56. The velocity vector is tangent to the circle at its “3 o’clock position” (see part (a)), which means $\vec{v}$ is vertical. Thus, our result is $\left(0.94\angle 90°\right)$.

(g) Again, the speed has been noted above (v = 0.94 m/s), but its direction is best seen by referring to Fig. 4-56. The velocity vector is tangent to the circle at its “12 o’clock position” (see part (c)), which means $\vec{v}$ is horizontal. Thus, our result is $\left(0.94\angle 180°\right)$.

(h) The acceleration has magnitude $a = v^2/r$ = 0.30 m/s^2, and at this instant (see part (a)) it is horizontal (toward the center of the circle). Thus, our result is $\left(0.30\angle 180°\right)$.

(i) Again, $a = v^2/r$ = 0.30 m/s^2, but at this instant (see part (c)) it is vertical (toward the center of the circle). Thus, our result is $\left(0.30\angle 270°\right)$.

108. Equation 4-34 describes an inverse proportionality between r and a, so that a large acceleration results from a small radius. Thus, an upper limit for a corresponds to a lower limit for r.

(a) The minimum turning radius of the train is given by

$$r_{min} = \frac{v^2}{a_{max}} = \frac{(216\ \text{km/h})^2}{(0.050)(9.8\ \text{m/s}^2)} = 7.3 \times 10^3\ \text{m}.$$

(b) The speed of the train must be reduced to no more than

$$v = \sqrt{a_{max} r} = \sqrt{0.050(9.8\ \text{m/s}^2)(1.00 \times 10^3\ \text{m})} = 22\ \text{m/s}$$

which is roughly 80 km/h.

109. (a) Using the same coordinate system assumed in Eq. 4-25, we find

$$y = x \tan\theta_0 - \frac{gx^2}{2(v_0 \cos\theta_0)^2} = -\frac{gx^2}{2v_0^2} \quad \text{if } \theta_0 = 0.$$

Thus, with $v_0 = 3.0 \times 10^6$ m/s and $x = 1.0$ m, we obtain $y = -5.4 \times 10^{-13}$ m, which is not practical to measure (and suggests why gravitational processes play such a small role in the fields of atomic and subatomic physics).

(b) It is clear from the above expression that $|y|$ decreases as v_0 is increased.

110. When the escalator is stalled the speed of the person is $v_p = \ell/t$, where ℓ is the length of the escalator and t is the time the person takes to walk up it. This is v_p = (15 m)/(90 s) = 0.167 m/s. The escalator moves at v_e = (15 m)/(60 s) = 0.250 m/s. The speed of the person walking up the moving escalator is

$$v = v_p + v_e = 0.167\ \text{m/s} + 0.250\ \text{m/s} = 0.417\ \text{m/s}$$

and the time taken to move the length of the escalator is

$$t = \ell / v = (15\ \text{m}) / (0.417\ \text{m/s}) = 36\ \text{s}.$$

If the various times given are independent of the escalator length, then the answer does not depend on that length either. In terms of ℓ (in meters) the speed (in meters per second) of the person walking on the stalled escalator is $\ell/90$, the speed of the moving escalator is $\ell/60$, and the speed of the person walking on the moving escalator is $v = (\ell/90) + (\ell/60) = 0.0278\ell$. The time taken is $t = \ell/v = \ell/0.0278\ell = 36\text{s}$ and is independent of ℓ.

111. The radius of Earth may be found in Appendix C.

(a) The speed of an object at Earth's equator is $v = 2\pi R/T$, where R is the radius of Earth (6.37×10^6 m) and T is the length of a day (8.64×10^4 s):

$$v = 2\pi(6.37 \times 10^6 \text{ m})/(8.64 \times 10^4 \text{ s}) = 463 \text{ m/s}.$$

The magnitude of the acceleration is given by

$$a = \frac{v^2}{R} = \frac{(463 \text{ m/s})^2}{6.37 \times 10^6 \text{ m}} = 0.034 \text{ m/s}^2.$$

(b) If T is the period, then $v = 2\pi R/T$ is the speed and the magnitude of the acceleration is

$$a = \frac{v^2}{R} = \frac{(2\pi R/T)^2}{R} = \frac{4\pi^2 R}{T^2}.$$

Thus,

$$T = 2\pi\sqrt{\frac{R}{a}} = 2\pi\sqrt{\frac{6.37 \times 10^6 \text{ m}}{9.8 \text{ m/s}^2}} = 5.1 \times 10^3 \text{ s} = 84 \text{ min.}$$

112. With $g_B = 9.8128$ m/s^2 and $g_M = 9.7999$ m/s^2, we apply Eq. 4-26:

$$R_M - R_B = \frac{v_0^2 \sin 2\theta_0}{g_M} - \frac{v_0^2 \sin 2\theta_0}{g_B} = \frac{v_0^2 \sin 2\theta_0}{g_B}\left(\frac{g_B}{g_M} - 1\right)$$

which becomes

$$R_M - R_B = R_B\left(\frac{9.8128 \text{ m/s}^2}{9.7999 \text{ m/s}^2} - 1\right)$$

and yields (upon substituting $R_B = 8.09$ m) $R_M - R_B = 0.01$ m = 1 cm.

113. From the figure, the three displacements can be written as

$$\vec{d}_1 = d_1(\cos\theta_1\hat{\text{i}} + \sin\theta_1\hat{\text{j}}) = (5.00 \text{ m})(\cos 30°\hat{\text{i}} + \sin 30°\hat{\text{j}}) = (4.33 \text{ m})\hat{\text{i}} + (2.50 \text{ m})\hat{\text{j}}$$

$$\vec{d}_2 = d_2[\cos(180° + \theta_1 - \theta_2)\hat{\text{i}} + \sin(180° + \theta_1 - \theta_2)\hat{\text{j}}] = (8.00 \text{ m})(\cos 160°\hat{\text{i}} + \sin 160°\hat{\text{j}})$$
$$= (-7.52 \text{ m})\hat{\text{i}} + (2.74 \text{ m})\hat{\text{j}}$$

$$\vec{d}_3 = d_3[\cos(360° - \theta_3 - \theta_2 + \theta_1)\hat{\text{i}} + \sin(360° - \theta_3 - \theta_2 + \theta_1)\hat{\text{j}}] = (12.0 \text{ m})(\cos 260°\hat{\text{i}} + \sin 260°\hat{\text{j}})$$
$$= (-2.08 \text{ m})\hat{\text{i}} - (11.8 \text{ m})\hat{\text{j}}$$

where the angles are measured from the $+x$ axis. The net displacement is

$$\vec{d} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 = (-5.27 \text{ m})\hat{\text{i}} - (6.58 \text{ m})\hat{\text{j}}.$$

(a) The magnitude of the net displacement is

$$|\vec{d}| = \sqrt{(-5.27\text{ m})^2 + (-6.58\text{ m})^2} = 8.43\text{ m}.$$

(b) The direction of $\vec{d}$ is $\theta = \tan^{-1}\left(\frac{d_y}{d_x}\right) = \tan^{-1}\left(\frac{-6.58\text{ m}}{-5.27\text{ m}}\right) = 51.3° \text{ or } 231°.$

We choose $231°$ (measured counterclockwise from +x) since the desired angle is in the third quadrant. An equivalent answer is $-129°$ (measured clockwise from +x).

114. Taking derivatives of $\vec{r} = 2t\hat{\text{i}} + 2\sin(\pi t/4)\hat{\text{j}}$ (with lengths in meters, time in seconds, and angles in radians) provides expressions for velocity and acceleration:

$$\vec{v} = \frac{d\vec{r}}{dt} = 2\hat{\text{i}} + \frac{\pi}{2}\cos\left(\frac{\pi t}{4}\right)\hat{\text{j}}$$
$$\vec{a} = \frac{d\vec{v}}{dt} = -\frac{\pi^2}{8}\sin\left(\frac{\pi t}{4}\right)\hat{\text{j}}.$$

Thus, we obtain:

time t (s)			0.0	1.0	2.0	3.0	4.0
(a)	$\vec{r}$ position	x (m)	0.0	2.0	4.0	6.0	8.0
		y (m)	0.0	1.4	2.0	1.4	0.0
(b)	$\vec{v}$ velocity	v_x(m/s)		2.0	2.0	2.0	
		v_y (m/s)		1.1	0.0	–1.1	
(c)	$\vec{a}$ acceleration	a_x (m/s^2)		0.0	0.0	0.0	
		a_y (m/s^2)		–0.87	–1.2	–0.87	

115. Since this problem involves constant downward acceleration of magnitude a, similar to the projectile motion situation, we use the equations of §4-6 as long as we substitute a for g. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The initial velocity is horizontal so that $v_{0y} = 0$ and

$$v_{0x} = v_0 = 1.00 \times 10^9\text{ cm/s}.$$

(a) If ℓ is the length of a plate and t is the time an electron is between the plates, then $\ell = v_0 t$, where v_0 is the initial speed. Thus

$$t = \frac{\ell}{v_0} = \frac{2.00\text{ cm}}{1.00 \times 10^9\text{ cm/s}} = 2.00 \times 10^{-9}\text{ s}.$$

(b) The vertical displacement of the electron is

$$y = -\frac{1}{2}at^2 = -\frac{1}{2}\left(1.00\times10^{17}\ \text{cm/s}^2\right)\left(2.00\times10^{-9}\ \text{s}\right)^2 = -0.20\,\text{cm} = -2.00\ \text{mm},$$

or $|y| = 2.00$ mm.

(c) The *x* component of velocity does not change:

$$v_x = v_0 = 1.00 \times 10^9\ \text{cm/s} = 1.00 \times 10^7\ \text{m/s}.$$

(d) The *y* component of the velocity is

$$v_y = a_y t = \left(1.00\times10^{17}\ \text{cm/s}^2\right)\left(2.00\times10^{-9}\ \text{s}\right) = 2.00\times10^8\,\text{cm/s} \\ = 2.00\times10^6\,\text{m/s}.$$

116. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion of the shot ball. We are allowed to use Table 2-1 (with Δy replacing Δx) because the ball has constant acceleration motion. We use primed variables (except *t*) with the constant-velocity elevator (so $v' = 10$ m/s), and unprimed variables with the ball (with initial velocity $v_0 = v' + 20 = 30$ m/s, relative to the ground). SI units are used throughout.

(a) Taking the time to be zero at the instant the ball is shot, we compute its maximum height *y* (relative to the ground) with $v^2 = v_0^2 - 2g(y - y_0)$, where the highest point is characterized by $v = 0$. Thus,

$$y = y_o + \frac{v_0^2}{2g} = 76\ \text{m}$$

where $y_o = y'_o + 2 = 30$ m (where $y'_o = 28$ m is given in the problem) and $v_0 = 30$ m/s relative to the ground as noted above.

(b) There are a variety of approaches to this question. One is to continue working in the frame of reference adopted in part (a) (which treats the ground as motionless and "fixes" the coordinate origin to it); in this case, one describes the elevator motion with $y' = y'_o + v't$ and the ball motion with Eq. 2-15, and solves them for the case where they reach the same point at the same time. Another is to work in the frame of reference of the elevator (the boy in the elevator might be oblivious to the fact the elevator is moving since it isn't accelerating), which is what we show here in detail:

$$\Delta y_e = v_{0_e}t - \frac{1}{2}gt^2 \quad\Rightarrow\quad t = \frac{v_{0_e} + \sqrt{v_{0e}^2 - 2g\Delta y_e}}{g}$$

where v_{0e} = 20 m/s is the initial velocity of the ball relative to the elevator and Δy_e = –2.0 m is the ball's displacement relative to the floor of the elevator. The positive root is chosen to yield a positive value for t; the result is t = 4.2 s.

117. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the initial position for the football as it begins projectile motion in the sense of §4-5), and we let θ_0 be the angle of its initial velocity measured from the +x axis.

(a) x = 46 m and y = –1.5 m are the coordinates for the landing point; it lands at time t = 4.5 s. Since $x = v_{0x}t$,

$$v_{0x} = \frac{x}{t} = \frac{46\ \text{m}}{4.5\ \text{s}} = 10.2\ \text{m/s}.$$

Since $y = v_{0y}t - \frac{1}{2}gt^2$,

$$v_{0y} = \frac{y + \frac{1}{2}gt^2}{t} = \frac{(-1.5\ \text{m}) + \frac{1}{2}(9.8\ \text{m/s}^2)(4.5\ \text{s})^2}{4.5\ \text{s}} = 21.7\ \text{m/s}.$$

The magnitude of the initial velocity is

$$v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{(10.2\ \text{m/s})^2 + (21.7\ \text{m/s})^2} = 24\ \text{m/s}.$$

(b) The initial angle satisfies tan $\theta_0 = v_{0y}/v_{0x}$. Thus,

$$\theta_0 = \tan^{-1}\left[(21.7\ \text{m/s})/(10.2\ \text{m/s})\right] = 65°.$$

118. The velocity of Larry is v_1 and that of Curly is v_2. Also, we denote the length of the corridor by L. Now, Larry's time of passage is t_1 = 150 s (which must equal L/v_1), and Curly's time of passage is t_2 = 70 s (which must equal L/v_2). The time Moe takes is therefore

$$t = \frac{L}{v_1 + v_2} = \frac{1}{v_1/L + v_2/L} = \frac{1}{\frac{1}{150\,\text{s}} + \frac{1}{70\,\text{s}}} = 48\text{s}.$$

119. The boxcar has velocity $\vec{v}_{cg} = v_1\,\hat{\text{i}}$ relative to the ground, and the bullet has velocity

$$\vec{v}_{0bg} = v_2\cos\theta\,\hat{\text{i}} + v_2\sin\theta\,\hat{\text{j}}$$

relative to the ground before entering the car (we are neglecting the effects of gravity on the bullet). While in the car, its velocity relative to the outside ground is

$$\vec{v}_{bg} = 0.8v_2\cos\theta\,\hat{\text{i}} + 0.8v_2\sin\theta\,\hat{\text{j}}$$

(due to the 20% reduction mentioned in the problem). The problem indicates that the velocity of the bullet in the car *relative to the car* is (with v_3 unspecified) $\vec{v}_{b\,c} = v_3\,\hat{\mathrm{j}}$. Now, Eq. 4-44 provides the condition

$$\vec{v}_{b\,g} = \vec{v}_{b\,c} + \vec{v}_{c\,g}$$

$$0.8v_2 \cos\theta\ \hat{\mathrm{i}} + 0.8v_2 \sin\theta\ \hat{\mathrm{j}}\ = v_3\ \hat{\mathrm{j}} + v_1\ \hat{\mathrm{i}}$$

so that equating x components allows us to find θ. If one wished to find v_3 one could also equate the y components, and from this, if the car width were given, one could find the time spent by the bullet in the car, but this information is not asked for (which is why the width is irrelevant). Therefore, examining the x components in SI units leads to

$$\theta = \cos^{-1}\left(\frac{v_1}{0.8v_2}\right) = \cos^{-1}\ \left(\frac{85\text{ km/h}\left(\frac{1000\text{ m/km}}{3600\text{ s/h}}\right)}{0.8\ (650\text{ m/s})}\right)$$

which yields 87° for the direction of $\vec{v}_{b\,g}$ (measured from $\hat{\mathrm{i}}$, which is the direction of motion of the car). The problem asks, "from what direction was it fired?" — which means the answer is not 87° but rather its supplement 93° (measured from the direction of motion). Stating this more carefully, in the coordinate system we have adopted in our solution, the bullet velocity vector is in the first quadrant, at 87° measured counterclockwise from the +x direction (the direction of train motion), which means that the direction from which the bullet came (where the sniper is) is in the third quadrant, at –93° (that is, 93° measured clockwise from +x).

120. (a) Using $a = v^2 / R,$ the radius of the track is

$$R = \frac{v^2}{a} = \frac{(9.20\text{ m/s})^2}{3.80\text{ m/s}^2} = 22.3\text{ m}\,.$$

(b) Using $T = 2\pi R / v,$ the period of the circular motion is

$$T = \frac{2\pi R}{v} = \frac{2\pi(22.3\text{ m})}{9.20\text{ m/s}} = 15.2\text{ s}$$

121. (a) With $v = c/10 = 3\times10^7$ m/s and $a = 20g = 196\text{ m/s}^2$, Eq. 4-34 gives

$$r = v^2 / a = 4.6\times10^{12}\text{ m.}$$

(b) The period is given by Eq. 4-35: $T = 2\pi r / v = 9.6\times10^5$ s. Thus, the time to make a quarter-turn is $T/4 = 2.4 \times 10^5$ s or about 2.8 days.

122. Since $v_y^2 = v_{0y}^2 - 2g\Delta y$, and v_y=0 at the target, we obtain

$$v_{0y} = \sqrt{2\left(9.80 \text{ m/s}^2\right)\left(5.00 \text{ m}\right)} = 9.90 \text{ m/s}$$

(a) Since $v_0 \sin \theta_0 = v_{0y}$, with v_0 = 12.0 m/s, we find θ_0 = 55.6°.

(b) Now, $v_y = v_{0y} - gt$ gives t = (9.90 m/s)/(9.80 m/s^2) = 1.01 s. Thus,

$$\Delta x = (v_0 \cos \theta_0)t = 6.85 \text{ m}.$$

(c) The velocity at the target has only the v_x component, which is equal to $v_{0x} = v_0 \cos \theta_0$ = 6.78 m/s.

123. With v_0 = 30.0 m/s and R = 20.0 m, Eq. 4-26 gives

$$\sin 2\theta_0 = \frac{gR}{v_0^2} = 0.218.$$

Because $\sin \phi = \sin(180° - \phi)$, there are two roots of the above equation:

$$2\theta_0 = \sin^{-1}(0.218) = 12.58° \text{ and } 167.4°.$$

which correspond to the two possible launch angles that will hit the target (in the absence of air friction and related effects).

(a) The smallest angle is θ_0 = 6.29°.

(b) The greatest angle is and θ_0 = 83.7°.

An alternative approach to this problem in terms of Eq. 4-25 (with y = 0 and 1/cos^2 = 1 + tan^2) is possible — and leads to a quadratic equation for tanθ_0 with the roots providing these two possible θ_0 values.

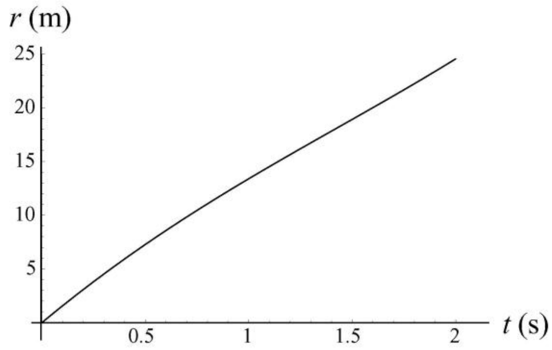
124. We make use of Eq. 4-21 and Eq.4-22.

(a) With v_o = 16 m/s, we square Eq. 4-21 and Eq. 4-22 and add them, then (using Pythagoras' theorem) take the square root to obtain r:

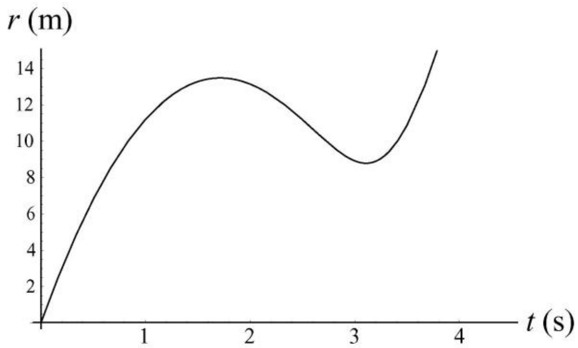
$$r = \sqrt{(x - x_0)^2 + (y - y_0)^2} = \sqrt{(v_0 \cos \theta_0 t)^2 + (v_0 \sin \theta_0 t - gt^2/2)^2}$$

$$= t\sqrt{v_0^2 - v_0 g \sin \theta_0 t + g^2 t^2/4}$$

Below we plot r as a function of time for θ_o = 40.0°:



(b) For this next graph for r versus t we set $\theta_0 = 80.0°$.



(c) Differentiating r with respect to t, we obtain

$$\frac{dr}{dt} = \frac{v_0^2 - 3v_0 gt\sin\theta_0/2 + g^2t^2/2}{\sqrt{v_0^2 - v_0 g\sin\theta_0 t + g^2t^2/4}}$$

Setting $dr/dt = 0$, with $v_0 = 16.0$ m/s and $\theta_0 = 40.0°$, we have $256 - 151t + 48t^2 = 0$. The equation has no real solution. This means that the maximum is reached at the end of the flight, with

$$t_{total} = 2v_0 \sin\theta_0 / g = 2(16.0\text{ m/s})\sin(40.0°)/(9.80\text{ m/s}^2) = 2.10\text{ s}.$$

(d) The value of r is given by

$$r = (2.10)\sqrt{(16.0)^2 - (16.0)(9.80)\sin 40.0°(2.10) + (9.80)^2(2.10)^2/4} = 25.7\text{ m}.$$

(e) The horizontal distance is $r_x = v_0 \cos\theta_0 t = (16.0\text{ m/s})\cos 40.0°(2.10\text{ s}) = 25.7\text{ m}.$

(f) The vertical distance is $r_y = 0$.

(g) For the θ_0 = 80° launch, the condition for maximum r is $256 - 232t + 48t^2 = 0$, or $t = 1.71$ s (the other solution, t = 3.13 s, corresponds to a minimum.)

(h) The distance traveled is

$$r = (1.71)\sqrt{(16.0)^2 - (16.0)(9.80)\sin 80.0°(1.71) + (9.80)^2(1.71)^2/4} = 13.5 \text{ m}.$$

(i) The horizontal distance is

$$r_x = v_0 \cos\theta_0 t = (16.0 \text{ m/s})\cos 80.0°(1.71 \text{ s}) = 4.75 \text{ m}.$$

(j) The vertical distance is

$$r_y = v_0 \sin\theta_0 t - \frac{gt^2}{2} = (16.0 \text{ m/s})\sin 80°(1.71 \text{ s}) - \frac{(9.80 \text{ m/s}^2)(1.71 \text{ s})^2}{2} = 12.6 \text{ m}.$$

125. Using the same coordinate system assumed in Eq. 4-25, we find x for the elevated cannon from

$$y = x\tan\theta_0 - \frac{gx^2}{2(v_0\cos\theta_0)^2} \quad \text{where } y = -30 \text{ m}.$$

Using the quadratic formula (choosing the positive root), we find

$$x = v_0\cos\theta_0\left(\frac{v_0\sin\theta_0 + \sqrt{(v_0\sin\theta_0)^2 - 2gy}}{g}\right)$$

which yields x = 715 m for v_0 = 82 m/s and θ_0 = 45°. This is 29 m longer than the distance of 686 m.

126. At maximum height, the y-component of a projectile's velocity vanishes, so the given 10 m/s is the (constant) x-component of velocity.

(a) Using v_{0y} to denote the y-velocity 1.0 s before reaching the maximum height, then (with v_y = 0) the equation $v_y = v_{0y} - gt$ leads to v_{0y} = 9.8 m/s. The magnitude of the velocity vector (or *speed*) at that moment is therefore

$$\sqrt{v_x^2 + v_{0y}{}^2} = \sqrt{(10 \text{ m/s})^2 + (9.8 \text{ m/s})^2} = 14 \text{ m/s}.$$

(b) It is clear from the symmetry of the problem that the speed is the same 1.0 s after reaching the top, as it was 1.0 s before (14 m/s again). This may be verified by using $v_y = v_{0y} - gt$ again but now "starting the clock" at the highest point so that v_{0y} = 0 (and $t = 1.0$ s). This leads to v_y = –9.8 m/s and $\sqrt{(10 \text{ m/s})^2 + (-9.8 \text{ m/s})^2} = 14$ m/s.

(c) The x_0 value may be obtained from $x = 0 = x_0 + (10 \text{ m/s})(1.0\text{s})$, which yields $x_0 = -10\,\text{m}$.

(d) With $v_{0y} = 9.8$ m/s denoting the y-component of velocity one second before the top of the trajectory, then we have $y = 0 = y_0 + v_{0y}t - \frac{1}{2}gt^2$ where $t = 1.0$ s. This yields $y_0 = -4.9$ m.

(e) By using $x - x_0 = (10 \text{ m/s})(1.0 \text{ s})$ where $x_0 = 0$, we obtain $x = 10$ m.

(f) Let $t = 0$ at the top with $y_0 = v_{0y} = 0$. From $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$, we have, for $t = 1.0$ s,

$$y = -(9.8 \text{ m/s}^2)(1.0 \text{ s})^2/2 = -4.9 \text{ m}.$$

127. With no acceleration in the x direction yet a constant acceleration of 1.40 m/s^2 in the y direction, the position (in meters) as a function of time (in seconds) must be

$$\vec{r} = (6.00t)\hat{\text{i}} + \left(\frac{1}{2}(1.40)t^2\right)\hat{\text{j}}$$

and $\vec{v}$ is its derivative with respect to t.

(a) At $t = 3.00$ s, therefore, $\vec{v} = (6.00\hat{\text{i}} + 4.20\hat{\text{j}})$ m/s.

(b) At $t = 3.00$ s, the position is $\vec{r} = (18.0\hat{\text{i}} + 6.30\hat{\text{j}})$ m.

128. We note that

$$\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$$

describes a right triangle, with one leg being $\vec{v}_{PG}$ (east), another leg being $\vec{v}_{AG}$ (magnitude = 20, direction = south), and the hypotenuse being $\vec{v}_{PA}$ (magnitude = 70). Lengths are in kilometers and time is in hours. Using the Pythagorean theorem, we have

$$|\vec{v}_{PA}| = \sqrt{|\vec{v}_{PG}|^2 + |\vec{v}_{AG}|^2} \;\Rightarrow\; 70 \text{ km/h} = \sqrt{|\vec{v}_{PG}|^2 + (20 \text{ km/h})^2}$$

which can be solved to give the ground speed: $|\vec{v}_{PG}| = 67$ km/h.

129. The figure offers many interesting points to analyze, and others are easily inferred (such as the point of maximum height). The focus here, to begin with, will be the final point shown (1.25 s after the ball is released) which is when the ball returns to its original height. In English units, $g = 32$ ft/s^2.

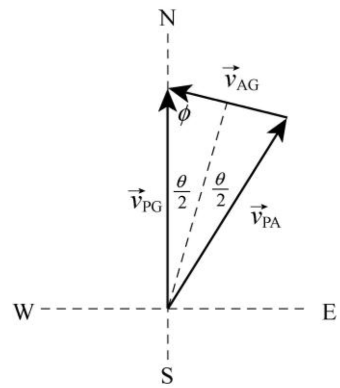
(a) Using $x - x_0 = v_x t$ we obtain v_x = (40 ft)/(1.25 s) = 32 ft/s. And $y - y_0 = 0 = v_{0y}t - \frac{1}{2}gt^2$ yields $v_{0y} = \frac{1}{2}\left(32 \text{ ft/s}^2\right)\left(1.25\text{ s}\right) = 20 \text{ ft/s}$. Thus, the initial speed is

$$v_0 = |\vec{v}_0| = \sqrt{(32 \text{ ft/s})^2 + (20 \text{ ft/s})^2} = 38 \text{ ft/s}.$$

(b) Since v_y = 0 at the maximum height and the horizontal velocity stays constant, then the speed at the top is the same as v_x = 32 ft/s.

(c) We can infer from the figure (or compute from $v_y = 0 = v_{0y} - gt$) that the time to reach the top is 0.625 s. With this, we can use $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ to obtain 9.3 ft (where y_0 = 3 ft has been used). An alternative approach is to use $v_y^2 = v_{0y}^2 - 2g\left(y - y_0\right)$.

130. We denote $\vec{v}_{\text{PG}}$ as the velocity of the plane relative to the ground, $\vec{v}_{\text{AG}}$ as the velocity of the air relative to the ground, and $\vec{v}_{\text{PA}}$ as the velocity of the plane relative to the air.



(a) The vector diagram is shown on the right: $\vec{v}_{\text{PG}} = \vec{v}_{\text{PA}} + \vec{v}_{\text{AG}}$. Since the magnitudes v_{PG} and v_{PA} are equal the triangle is isosceles, with two sides of equal length.

Consider either of the right triangles formed when the bisector of θ is drawn (the dashed line). It bisects $\vec{v}_{\text{AG}}$, so

$$\sin\left(\theta/2\right) = \frac{v_{\text{AG}}}{2v_{\text{PG}}} = \frac{70.0\text{ mi/h}}{2\left(135\text{ mi/h}\right)}$$

which leads to θ = 30.1°. Now $\vec{v}_{\text{AG}}$ makes the same angle with the E-W line as the dashed line does with the N-S line. The wind is blowing in the direction 15.0° north of west. Thus, it is blowing *from* 75.0° east of south.

(b) The plane is headed along $\vec{v}_{\text{PA}}$, in the direction 30.0° east of north. There is another solution, with the plane headed 30.0° west of north and the wind blowing 15° north of east (that is, from 75° west of south).

131. We make use of Eq. 4-24 and Eq. 4-25.

(a) With x = 180 m, θ_o = 30°, and v_o = 43 m/s, we obtain

$$y = \tan(30°)(180 \text{ m}) - \frac{(9.8 \text{ m/s}^2)(180 \text{ m})^2}{2(43 \text{ m/s})^2(\cos 30°)^2} = -11 \text{ m}$$

or $|y| = 11$ m. This implies the rise is roughly eleven meters above the fairway.

(b) The horizontal component (in the absence of air friction) is unchanged, but the vertical component increases (see Eq. 4-24). The Pythagorean theorem then gives the magnitude of final velocity (right before striking the ground): 45 m/s.

132. We let g_p denote the magnitude of the gravitational acceleration on the planet. A number of the points on the graph (including some "inferred" points — such as the max height point at $x = 12.5$ m and $t = 1.25$ s) can be analyzed profitably; for future reference, we label (with subscripts) the first $((x_0, y_0) = (0, 2)$ at $t_0 = 0)$ and last ("final") points $((x_f, y_f) = (25, 2)$ at $t_f = 2.5)$, with lengths in meters and time in seconds.

(a) The x-component of the initial velocity is found from $x_f - x_0 = v_{0x} t_f$. Therefore, $v_{0x} = 25/2.5 = 10$ m/s. We try to obtain the y-component from

$$y_f - y_0 = 0 = v_{0y}t_f - \tfrac{1}{2}g_p t_f^2.$$

This gives us $v_{0y} = 1.25g_p$, and we see we need another equation (by analyzing another point, say, the next-to-last one) $y - y_0 = v_{0y}t - \frac{1}{2}g_p t^2$ with $y = 6$ and $t = 2$; this produces our second equation $v_{0y} = 2 + g_p$. Simultaneous solution of these two equations produces results for v_{0y} and g_p (relevant to part (b)). Thus, our complete answer for the initial velocity is $\vec{v} = (10\text{ m/s})\hat{\text{i}} + (10\text{ m/s})\hat{\text{j}}$.

(b) As a by-product of the part (a) computations, we have $g_p = 8.0$ m/s^2.

(c) Solving for t_g (the time to reach the ground) in $y_g = 0 = y_0 + v_{0y}t_g - \frac{1}{2}g_p t_g^2$ leads to a positive answer: $t_g = 2.7$ s.

(d) With $g = 9.8$ m/s^2, the method employed in part (c) would produce the quadratic equation $-4.9t_g^2 + 10t_g + 2 = 0$ and then the positive result $t_g = 2.2$ s.

133. (a) The helicopter's speed is $v' = 6.2$ m/s, which implies that the speed of the package is $v_0 = 12 - v' = 5.8$ m/s, relative to the ground.

(b) Letting $+x$ be in the direction of $\vec{v}_0$ for the package and $+y$ be downward, we have (for the motion of the package)

$$\Delta x = v_0 t \quad \text{and} \quad \Delta y = \frac{1}{2}gt^2$$

where $\Delta y = 9.5$ m. From these, we find $t = 1.39$ s and $\Delta x = 8.08$ m for the package, while $\Delta x'$ (for the helicopter, which is moving in the opposite direction) is $-v' t = -8.63$ m. Thus, the horizontal separation between them is $8.08 - (-8.63) = 16.7$ m ≈ 17 m.

(c) The components of $\vec{v}$ at the moment of impact are (v_x, v_y) = (5.8, 13.6) in SI units. The vertical component has been computed using Eq. 2-11. The angle (which is below horizontal) for this vector is $\tan^{-1}(13.6/5.8) = 67°$.

134. The type of acceleration involved in steady-speed circular motion is the centripetal acceleration $a = v^2/r$ which is at each moment directed towards the center of the circle. The radius of the circle is $r = (12)^2/3 = 48$ m.

(a) Thus, if at the instant the car is traveling *clockwise* around the circle, it is 48 m west of the center of its circular path.

(b) The same result holds here if at the instant the car is traveling *counterclockwise*. That is, it is 48 m west of the center of its circular path.

135. (a) Using the same coordinate system assumed in Eq. 4-21 and Eq. 4-22 (so that θ_0 = –20.0°), we use v_0 = 15.0 m/s and find the horizontal displacement of the ball at t = 2.30 s:

$$\Delta x = (v_0 \cos\theta_0)t = 32.4 \text{ m}.$$

(b) The vertical displacement is $\Delta y = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2 = -37.7$ m.

136. We take the initial (x, y) specification to be (0.000, 0.762) m, and the positive x direction to be towards the "green monster." The components of the initial velocity are $(33.53 \angle 55°) \rightarrow (19.23,\ 27.47)$ m/s.

(a) With t = 5.00 s, we have $x = x_0 + v_x t$ = 96.2 m.

(b) At that time, $y = y_0 + v_{0y}t - \frac{1}{2}gt^2 = 15.59$ m, which is 4.31 m above the wall.

(c) The moment in question is specified by t = 4.50 s. At that time, $x - x_0$ = (19.23)(4.50) = 86.5 m.

(d) The vertical displacement is $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$ = 25.1 m.

137. When moving in the same direction as the jet stream (of speed v_s), the time is $t = d/(v_{ja} + v_s)$, where d = 4350 km is the distance and $v_{ja} = 966$ km/h is the speed of the jet relative to the air. When moving against the jet stream, the time is $t' = d/(v_{ja} - v_s)$, with $t' - t = 50 \text{ min} = (5/6)\text{h}$. Combining the expressions gives

$$t' - t = \frac{d}{v_{ja} - v_s} - \frac{d}{v_{ja} + v_s} = \frac{2dv_s}{v_{ja}^2 - v_s^2} = \frac{5}{6}\text{ h}$$

Upon rearranging and using the quadratic formula to solve for v_s, we get v_s = 88.63 km/h.

138. We establish coordinates with $\hat{\text{i}}$ pointing to the far side of the river (perpendicular to the current) and $\hat{\text{j}}$ pointing in the direction of the current. We are told that the magnitude (presumed constant) of the velocity of the boat relative to the water is $|\vec{v}_{bw}|$ = 6.4 km/h. Its angle, relative to the x axis is θ. With km and h as the understood units, the velocity of the water (relative to the ground) is $\vec{v}_{wg} = 3.2\hat{\text{j}}$.

(a) To reach a point "directly opposite" means that the velocity of her boat relative to ground must be $\vec{v}_{bg} = v_{bg}\hat{\text{i}}$ where $v > 0$ is unknown. Thus, all $\hat{\text{j}}$ components must cancel in the vector sum

$$\vec{v}_{bw} + \vec{v}_{wg} = \vec{v}_{bg}$$

which means the $u \sin\theta = -3.2$, so $\theta = \sin^{-1}(-3.2/6.4) = -30°$.

(b) Using the result from part (a), we find $v_{bg} = v_{bw}\cos\theta$ = 5.5 km/h. Thus, traveling a distance of ℓ = 6.4 km requires a time of 6.4/5.5 = 1.15 h or 69 min.

(c) If her motion is completely along the y axis (as the problem implies) then with v_{wg} = 3.2 km/h (the water speed) we have

$$t_{\text{total}} = \frac{D}{v_{bw} + v_{wg}} + \frac{D}{v_{bw} - v_{wg}} = 1.33 \text{ h}$$

where D = 3.2 km. This is equivalent to 80 min.

(d) Since

$$\frac{D}{v_{bw} + v_{wg}} + \frac{D}{v_{bw} - v_{wg}} = \frac{D}{v_{bw} - v_{wg}} + \frac{D}{v_{bw} + v_{wg}}$$

the answer is the same as in the previous part, i.e., $t_{\text{total}} = 80 \text{ min}$.

(e) The shortest-time path should have $\theta = 0$. This can also be shown by noting that the case of general θ leads to

$$\vec{v}_{bg} = \vec{v}_{bw} + \vec{v}_{wg} = v_{bw}\cos\theta\,\hat{\text{i}} + (v_{bw}\sin\theta + v_{wg})\,\hat{\text{j}}$$

where the x component of $\vec{v}_{bg}$ must equal l/t. Thus, $t = \dfrac{l}{v_{bw}\cos\theta}$, which can be minimized using the condition $dt/d\theta = 0$. The above expression leads to t = 6.4/6.4 = 1.0 h, or 60 min.