Chapter 13

1. The gravitational force between the two parts is

$$F = \frac{Gm(M-m)}{r^2} = \frac{G}{r^2}\left(mM - m^2\right)$$

which we differentiate with respect to *m* and set equal to zero:

$$\frac{dF}{dm} = 0 = \frac{G}{r^2}(M - 2m) \Rightarrow M = 2m\,.$$

This leads to the result $m/M = 1/2$.

2. The gravitational force between you and the moon at its initial position (directly opposite of Earth from you) is

$$F_0 = \frac{GM_m m}{(R_{ME} + R_E)^2}$$

where M_m is the mass of the moon, R_{ME} is the distance between the moon and the Earth, and R_E is the radius of the Earth. At its final position (directly above you), the gravitational force between you and the moon is

$$F_1 = \frac{GM_m m}{(R_{ME} - R_E)^2}\,.$$

(a) The ratio of the moon's gravitational pulls at the two different positions is

$$\frac{F_1}{F_0} = \frac{GM_m m/(R_{ME} - R_E)^2}{GM_m m/(R_{ME} + R_E)^2} = \left(\frac{R_{ME} + R_E}{R_{ME} - R_E}\right)^2 = \left(\frac{3.82\times10^8\ \text{m} + 6.37\times10^6\ \text{m}}{3.82\times10^8\ \text{m} - 6.37\times10^6\ \text{m}}\right)^2 = 1.06898.$$

Therefore, the increase is 0.06898, or approximately 6.9%.

(b) The change of the gravitational pull may be approximated as

$$\begin{aligned} F_1 - F_0 &= \frac{GM_m m}{(R_{ME} - R_E)^2} - \frac{GM_m m}{(R_{ME} + R_E)^2} \approx \frac{GM_m m}{R_{ME}^2}\left(1 + 2\frac{R_E}{R_{ME}}\right) - \frac{GM_m m}{R_{ME}^2}\left(1 - 2\frac{R_E}{R_{ME}}\right) \\ &= \frac{4GM_m m R_E}{R_{ME}^3}. \end{aligned}$$

On the other hand, your weight, as measured on a scale on Earth, is

$$F_g = mg_E = \frac{GM_E m}{R_E^2}.$$

Since the moon pulls you "up," the percentage decrease of weight is

$$\frac{F_1 - F_0}{F_g} = 4\left(\frac{M_m}{M_E}\right)\left(\frac{R_E}{R_{ME}}\right)^3 = 4\left(\frac{7.36\times10^{22}\ \text{kg}}{5.98\times10^{24}\ \text{kg}}\right)\left(\frac{6.37\times10^{6}\ \text{m}}{3.82\times10^{8}\ \text{m}}\right)^3 = 2.27\times10^{-7} \approx (2.3\times10^{-5})\%.$$

3. **THINK** The magnitude of gravitational force between two objects depends on their distance of separation.

EXPRESS The magnitude of the gravitational force of one particle on the other is given by $F = Gm_1m_2/r^2$, where m_1 and m_2 are the masses, r is their separation, and G is the universal gravitational constant.

ANALYZE Solve for r using the values given, we obtain

$$r = \sqrt{\frac{Gm_1m_2}{F}} = \sqrt{\frac{\left(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)(5.2\text{kg})(2.4\text{kg})}{2.3\times10^{-12}\,\text{N}}} = 19\,\text{m}.$$

LEARN The force of gravitation is inversely proportional to r^2.

4. We use subscripts *s*, *e*, and *m* for the Sun, Earth and Moon, respectively. Plugging in the numerical values (say, from Appendix C) we find

$$\frac{F_{sm}}{F_{em}} = \frac{Gm_sm_m/r_{sm}^2}{Gm_em_m/r_{em}^2} = \frac{m_s}{m_e}\left(\frac{r_{em}}{r_{sm}}\right)^2 = \frac{1.99\times10^{30}\ \text{kg}}{5.98\times10^{24}\ \text{kg}}\left(\frac{3.82\times10^{8}\ \text{m}}{1.50\times10^{11}\ \text{m}}\right)^2 = 2.16.$$

5. The gravitational force from Earth on you (with mass m) is

$$F_g = \frac{GM_E m}{R_E^2} = mg$$

where $g = GM_E/R_E^2 = 9.8\ \text{m/s}^2$. If r is the distance between you and a tiny black hole of mass $M_b = 1\times10^{11}\ \text{kg}$ that has the same gravitational pull on you as the Earth, then

$$F_g = \frac{GM_b m}{r^2} = mg.$$

Combining the two equations, we obtain

$$mg = \frac{GM_E m}{R_E^2} = \frac{GM_b m}{r^2} \Rightarrow r = \sqrt{\frac{GM_b}{g}} = \sqrt{\frac{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(1\times10^{11}\ \text{kg})}{9.8\ \text{m/s}^2}} \approx 0.8\ \text{m}.$$

6. The gravitational forces on m_5 from the two 5.00 g masses m_1 and m_4 cancel each other. Contributions to the net force on m_5 come from the remaining two masses:

$$F_{\text{net}} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(2.50\times10^{-3}\ \text{kg}\right)\left(3.00\times10^{-3}\ \text{kg} - 1.00\times10^{-3}\ \text{kg}\right)}{\left(\sqrt{2}\times10^{-1}\ \text{m}\right)^2}$$

$$= 1.67\times10^{-14}\ \text{N}.$$

The force is directed along the diagonal between m_2 and m_3, toward m_2. In unit-vector notation, we have

$$\vec{F}_{\text{net}} = F_{\text{net}}(\cos 45°\,\hat{\text{i}} + \sin 45°\,\hat{\text{j}}) = (1.18\times10^{-14}\,\text{N})\,\hat{\text{i}} + (1.18\times10^{-14}\,\text{N})\,\hat{\text{j}}.$$

7. We require the magnitude of force (given by Eq. 13-1) exerted by particle *C* on *A* be equal to that exerted by *B* on *A*. Thus,

$$\frac{Gm_A m_C}{r^2} = \frac{Gm_A m_B}{d^2}\ .$$

We substitute in $m_B = 3m_A$ and $m_B = 3m_A$, and (after canceling "m_A") solve for *r*. We find $r = 5d$. Thus, particle *C* is placed on the *x* axis, to the left of particle *A* (so it is at a negative value of *x*), at $x = -5.00d$.

8. Using $F = GmM/r^2$, we find that the topmost mass pulls upward on the one at the origin with 1.9×10^{-8} N, and the rightmost mass pulls rightward on the one at the origin with 1.0×10^{-8} N. Thus, the (*x*, *y*) components of the net force, which can be converted to polar components (here we use magnitude-angle notation), are

$$\vec{F}_{\text{net}} = \left(1.04\times10^{-8}, 1.85\times10^{-8}\right) \Rightarrow \left(2.13\times10^{-8} \angle 60.6°\right).$$

(a) The magnitude of the force is 2.13×10^{-8} N.

(b) The direction of the force relative to the +*x* axis is $60.6°$.

9. **THINK** Both the Sun and the Earth exert a gravitational pull on the space probe. The net force can be calculated by using superposition principle.

EXPRESS At the point where the two forces balance, we have $GM_E m/r_1^2 = GM_S m/r_2^2$, where M_E is the mass of Earth, M_S is the mass of the Sun, *m* is the mass of the space

probe, r_1 is the distance from the center of Earth to the probe, and r_2 is the distance from the center of the Sun to the probe. We substitute $r_2 = d - r_1$, where d is the distance from the center of Earth to the center of the Sun, to find

$$\frac{M_E}{r_1^2} = \frac{M_S}{(d - r_1)^2}.$$

ANALYZE Using the values for M_E, M_S, and d given in Appendix C, we take the positive square root of both sides to solve for r_1. A little algebra yields

$$r_1 = \frac{d}{1 + \sqrt{M_S / M_E}} = \frac{1.50 \times 10^{11}\ \text{m}}{1 + \sqrt{(1.99 \times 10^{30}\ \text{kg})/(5.98 \times 10^{24}\ \text{kg})}} = 2.60 \times 10^{8}\ \text{m}.$$

LEARN The fact that $r_1 \ll d$ indicates that the probe is much closer to the Earth than the Sun.

10. Using Eq. 13-1, we find

$$\vec{F}_{\text{AB}} = \frac{2Gm_{\text{A}}^2}{d^2}\,\hat{\text{j}}, \quad \vec{F}_{\text{AC}} = -\frac{4Gm_{\text{A}}^2}{3d^2}\,\hat{\text{i}}.$$

Since the vector sum of all three forces must be zero, we find the third force (using magnitude-angle notation) is

$$\vec{F}_{\text{AD}} = \frac{Gm_{\text{A}}^2}{d^2}(2.404 \angle -56.3^\circ).$$

This tells us immediately the direction of the vector $\vec{r}$ (pointing from the origin to particle D), but to find its magnitude we must solve (with $m_D = 4m_{\text{A}}$) the following equation:

$$2.404\left(\frac{Gm_{\text{A}}^2}{d^2}\right) = \frac{Gm_{\text{A}}m_D}{r^2}.$$

This yields $r = 1.29d$. In magnitude-angle notation, then, $\vec{r} = (1.29 \angle -56.3^\circ)$, with SI units understood. The "exact" answer without regard to significant figure considerations is

$$\vec{r} = \left(2\sqrt{\frac{6}{13\sqrt{13}}},\ -3\sqrt{\frac{6}{13\sqrt{13}}}\right).$$

(a) In (x, y) notation, the x coordinate is $x = 0.716d$.

(b) Similarly, the y coordinate is $y = -1.07d$.

11. (a) The distance between any of the spheres at the corners and the sphere at the center is

$$r = \ell / 2\cos 30^\circ = \ell / \sqrt{3}$$

where ℓ is the length of one side of the equilateral triangle. The net (downward) contribution caused by the two bottom-most spheres (each of mass m) to the total force on m_4 has magnitude

$$2F_y = 2\left(\frac{Gm_4m}{r^2}\right)\sin 30° = 3\frac{Gm_4m}{\ell^2}.$$

This must equal the magnitude of the pull from M, so

$$3\frac{Gm_4m}{\ell^2} = \frac{Gm_4m}{(\ell/\sqrt{3})^2}$$

which readily yields $m = M$.

(b) Since m_4 cancels in that last step, then the amount of mass in the center sphere is not relevant to the problem. The net force is still zero.

12. (a) We are told the value of the force when particle C is removed (that is, as its position x goes to infinity), which is a situation in which any force caused by C vanishes (because Eq. 13-1 has r^2 in the denominator). Thus, this situation only involves the force exerted by A on B:

$$F_{\text{net},x} = F_{AB} = \frac{Gm_Am_B}{r_{AB}^2} = 4.17 \times 10^{-10}\ \text{N}.$$

Since $m_B = 1.0$ kg and $r_{AB} = 0.20$ m, then this yields

$$m_A = \frac{r_{AB}^2F_{AB}}{Gm_B} = \frac{(0.20\ \text{m})^2(4.17\times10^{-10}\ \text{N})}{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(1.0\ \text{kg})} = 0.25\ \text{kg}.$$

(b) We note (from the graph) that the net force on B is zero when $x = 0.40$ m. Thus, at that point, the force exerted by C must have the same magnitude (but opposite direction) as the force exerted by A (which is the one discussed in part (a)). Therefore

$$\frac{Gm_Cm_B}{(0.40\ \text{m})^2} = 4.17 \times 10^{-10}\ \text{N} \quad \Rightarrow m_C = 1.00\ \text{kg}.$$

13. If the lead sphere were not hollowed the magnitude of the force it exerts on m would be $F_1 = GMm/d^2$. Part of this force is due to material that is removed. We calculate the force exerted on m by a sphere that just fills the cavity, at the position of the cavity, and subtract it from the force of the solid sphere.

The cavity has a radius $r = R/2$. The material that fills it has the same density (mass to volume ratio) as the solid sphere, that is, $M_c/r^3 = M/R^3$, where M_c is the mass that fills the cavity. The common factor $4\pi/3$ has been canceled. Thus,

$$M_c = \left(\frac{r^3}{R^3}\right)M = \left(\frac{R^3}{8R^3}\right)M = \frac{M}{8}.$$

The center of the cavity is $d - r = d - R/2$ from m, so the force it exerts on m is

$$F_2 = \frac{G(M/8)m}{(d - R/2)^2}.$$

The force of the hollowed sphere on m is

$$F = F_1 - F_2 = GMm\left(\frac{1}{d^2} - \frac{1}{8(d - R/2)^2}\right) = \frac{GMm}{d^2}\left(1 - \frac{1}{8(1 - R/2d)^2}\right)$$

$$= \frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(2.95\ \text{kg})(0.431\ \text{kg})}{(9.00\times10^{-2}\ \text{m})^2}\left(1 - \frac{1}{8[1-(4\times10^{-2}\ \text{m})/(2\cdot9\times10^{-2}\ \text{m})]^2}\right)$$

$$= 8.31\times10^{-9}\ \text{N}.$$

14. All the forces are being evaluated at the origin (since particle A is there), and all forces (except the net force) are along the location vectors $\vec{r}$, which point to particles B and C. We note that the angle for the location-vector pointing to particle B is 180º – 30.0º = 150º (measured counterclockwise from the $+x$ axis). The component along, say, the x axis of one of the force vectors $\vec{F}$ is simply Fx/r in this situation (where F is the magnitude of $\vec{F}$). Since the force itself (see Eq. 13-1) is inversely proportional to r^2, then the aforementioned x component would have the form $GmMx/r^3$; similarly for the other components. With m_A = 0.0060 kg, m_B = 0.0120 kg, and m_C = 0.0080 kg, we therefore have

$$F_{\text{net }x} = \frac{Gm_Am_Bx_B}{r_B^3} + \frac{Gm_Am_Cx_C}{r_C^3} = (2.77\times10^{-14}\ \text{N})\cos(-163.8^\text{o})$$

and

$$F_{\text{net }y} = \frac{Gm_Am_By_B}{r_B^3} + \frac{Gm_Am_Cy_C}{r_C^3} = (2.77\times10^{-14}\ \text{N})\sin(-163.8^\text{o})$$

where $r_B = d_{AB}$ = 0.50 m, and $(x_B, y_B) = (r_B\cos(150^\text{o}), r_B\sin(150^\text{o}))$ (with SI units understood). A fairly quick way to solve for r_C is to consider the vector difference between the net force and the force exerted by A, and then employ the Pythagorean theorem. This yields r_C = 0.40 m.

(a) By solving the above equations, the x coordinate of particle C is x_C = –0.20 m.

(b) Similarly, the y coordinate of particle C is y_C = –0.35 m.

15. All the forces are being evaluated at the origin (since particle *A* is there), and all forces are along the location vectors $\vec{r}$, which point to particles *B*, *C*, and *D*. In three dimensions, the Pythagorean theorem becomes $r = \sqrt{x^2 + y^2 + z^2}$. The component along, say, the *x* axis of one of the force-vectors $\vec{F}$ is simply *Fx/r* in this situation (where *F* is the magnitude of $\vec{F}$). Since the force itself (see Eq. 13-1) is inversely proportional to r^2 then the aforementioned *x* component would have the form $GmMx/r^3$; similarly for the other components. For example, the *z* component of the force exerted on particle *A* by particle *B* is

$$\frac{Gm_A m_B z_B}{r_B{}^3} = \frac{Gm_A(2m_A)(2d)}{((2d)^2 + d^2 + (2d)^2)^3} = \frac{4Gm_A^2}{27d^2} .$$

In this way, each component can be written as some multiple of $Gm_A{}^2/d^2$. For the *z* component of the force exerted on particle *A* by particle *C*, that multiple is $-9\sqrt{14}/196$. For the *x* components of the forces exerted on particle *A* by particles *B* and *C*, those multiples are 4/27 and $-3\sqrt{14}/196$, respectively. And for the *y* components of the forces exerted on particle *A* by particles *B* and *C*, those multiples are 2/27 and $3\sqrt{14}/98$, respectively. To find the distance *r* to particle *D* one method is to solve (using the fact that the vector add to zero)

$$\left(\frac{Gm_A m_D}{r^2}\right)^2 = \left[\left(\frac{4}{27} - \frac{3\sqrt{14}}{196}\right)^2 + \left(\frac{2}{27} + \frac{3\sqrt{14}}{98}\right)^2 + \left(\frac{4}{27} - \frac{9\sqrt{14}}{196}\right)^2\right]\left(\frac{Gm_A^2}{d^2}\right)^2 = 0.4439\left(\frac{Gm_A^2}{d^2}\right)^2$$

With $m_D = 4m_A$, we obtain

$$\left(\frac{4}{r^2}\right)^2 = \frac{0.4439}{(d^2)^2} \quad \Rightarrow \quad r = \left(\frac{16}{0.4439}\right)^{1/4} d = 4.357d .$$

The individual values of *x*, *y*, and *z* (locating the particle *D*) can then be found by considering each component of the Gm_Am_D/r^2 force separately.

(a) The *x* component of $\vec{r}$ would be

$$\frac{Gm_A m_D x}{r^3} = -\left(\frac{4}{27} - \frac{3\sqrt{14}}{196}\right)^2 \frac{Gm_A^2}{d^2} = -0.0909\frac{Gm_A^2}{d^2},$$

which yields $x = -0.0909\dfrac{m_A r^3}{m_D d^2} = -0.0909\dfrac{m_A(4.357d)^3}{(4m_A)d^2} = -1.88d$.

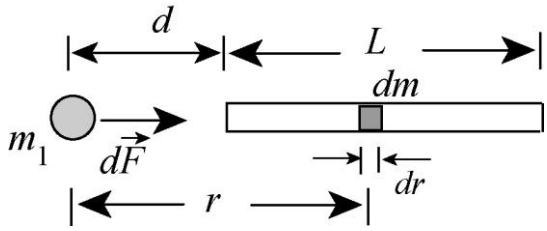
(b) Similarly, $y = -3.90d$,

(c) and $z = 0.489d$.

In this way we are able to deduce that $(x, y, z) = (1.88d, 3.90d, 0.489d)$.

16. Since the rod is an extended object, we cannot apply Equation 13-1 directly to find the force. Instead, we consider a small differential element of the rod, of mass dm of thickness dr at a distance r from m_1. The gravitational force between dm and m_1 is

$$dF = \frac{Gm_1 dm}{r^2} = \frac{Gm_1(M/L)dr}{r^2},$$



where we have substituted $dm = (M/L)dr$ since mass is uniformly distributed. The direction of $d\vec{F}$ is to the right (see figure). The total force can be found by integrating over the entire length of the rod:

$$F = \int dF = \frac{Gm_1 M}{L}\int_d^{L+d}\frac{dr}{r^2} = -\frac{Gm_1 M}{L}\left(\frac{1}{L+d}-\frac{1}{d}\right) = \frac{Gm_1 M}{d(L+d)}.$$

Substituting the values given in the problem statement, we obtain

$$F = \frac{Gm_1 M}{d(L+d)} = \frac{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(0.67\ \text{kg})(5.0\ \text{kg})}{(0.23\ \text{m})(3.0\ \text{m}+0.23\ \text{m})} = 3.0\times10^{-10}\ \text{N}.$$

17. (a) The gravitational acceleration at the surface of the Moon is $g_{\text{moon}} = 1.67\ \text{m/s}^2$ (see Appendix C). The ratio of weights (for a given mass) is the ratio of g-values, so

$$W_{\text{moon}} = (100\ \text{N})(1.67/9.8) = 17\ \text{N}.$$

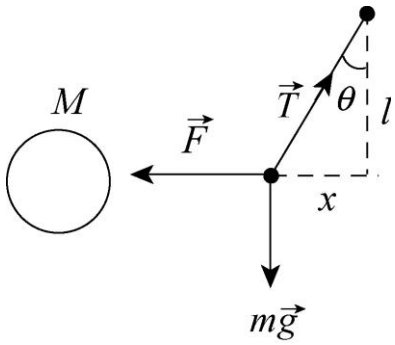
(b) For the force on that object caused by Earth's gravity to equal 17 N, then the free-fall acceleration at its location must be $a_g = 1.67\ \text{m/s}^2$. Thus,

$$a_g = \frac{Gm_E}{r^2} \Rightarrow r = \sqrt{\frac{Gm_E}{a_g}} = 1.5\times10^7\ \text{m}$$

so the object would need to be a distance of $r/R_E = 2.4$ "radii" from Earth's center.

18. The free-body diagram of the force acting on the plumb line is shown to the right. The mass of the sphere is

$$M = \rho V = \rho\left(\frac{4\pi}{3}R^3\right) = \frac{4\pi}{3}(2.6\times10^3\ \text{kg/m}^3)(2.00\times10^3\ \text{m})^3$$
$$= 8.71\times10^{13}\ \text{kg}.$$



The force between the "spherical" mountain and the plumb line is $F = GMm/r^2$. Suppose at equilibrium the line makes an angle θ with the vertical and the net force acting on the line is zero. Therefore,

$$0 = \sum F_{\text{net},\,x} = T\sin\theta - F = T\sin\theta - \frac{GMm}{r^2}$$

$$0 = \sum F_{\text{net},\,y} = T\cos - mg$$

The two equations can be combined to give $\tan\theta = \dfrac{F}{mg} = \dfrac{GM}{gr^2}$. The distance the lower end moves toward the sphere is

$$x = l\tan\theta = l\frac{GM}{gr^2} = (0.50\text{ m})\frac{(6.67\times10^{-11}\text{ m}^3/\text{kg}\cdot\text{s}^2)(8.71\times10^{13}\text{ kg})}{(9.8)(3\times2.00\times10^{3}\text{ m})^2}$$

$$= 8.2\times10^{-6}\text{ m}.$$

19. **THINK** Earth's gravitational acceleration varies with altitude.

EXPRESS The acceleration due to gravity is given by $a_g = GM/r^2$, where M is the mass of Earth and r is the distance from Earth's center. We substitute $r = R + h$, where R is the radius of Earth and h is the altitude, to obtain

$$a_g = \frac{GM}{r^2} = \frac{GM}{(R_E + h)^2}.$$

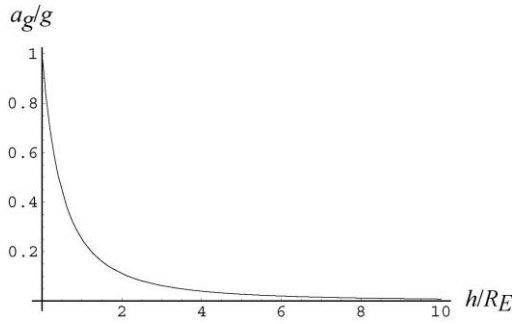
ANALYZE Solving for h, we obtain $h = \sqrt{GM/a_g} - R_E$. From Appendix C, $R_E = 6.37 \times 10^6$ m and $M = 5.98 \times 10^{24}$ kg, so

$$h = \sqrt{\frac{\left(6.67\times10^{-11}\text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(5.98\times10^{24}\text{kg}\right)}{\left(4.9\text{m}/\text{s}^2\right)}} - 6.37\times10^{6}\text{m} = 2.6\times10^{6}\text{m}.$$

LEARN We may rewrite a_g as

$$a_g = \frac{GM}{r^2} = \frac{GM/R_E^2}{(1+h/R_E)^2} = \frac{g}{(1+h/R_E)^2}$$

where $g = 9.83\,\text{m/s}^2$ is the gravitational acceleration on the Surface of the Earth. The plot below depicts how a_g decreases with increasing altitude.



20. We follow the method shown in Sample Problem 13.2 – "Difference in acceleration at head and feet." Thus,

$$a_g = \frac{GM_E}{r^2} \Rightarrow da_g = -2\frac{GM_E}{r^3}dr$$

which implies that the change in weight is

$$W_{\text{top}} - W_{\text{bottom}} \approx m\left(da_g\right).$$

However, since $W_{\text{bottom}} = GmM_E/R^2$ (where R is Earth's mean radius), we have

$$mda_g = -2\frac{GmM_E}{R^3}dr = -2W_{\text{bottom}}\frac{dr}{R} = -2\left(600\text{ N}\right)\frac{1.61\times10^3\text{ m}}{6.37\times10^6\text{ m}} = -0.303\text{ N}$$

for the weight change (the minus sign indicating that it is a decrease in W). We are not including any effects due to the Earth's rotation (as treated in Eq. 13-13).

21. From Eq. 13-14, we see the extreme case is when "g" becomes zero, and plugging in Eq. 13-15 leads to

$$0 = \frac{GM}{R^2} - R\omega^2 \Rightarrow M = \frac{R^3\omega^2}{G}.$$

Thus, with $R = 20000$ m and $\omega = 2\pi$ rad/s, we find $M = 4.7 \times 10^{24}$ kg $\approx 5 \times 10^{24}$ kg.

22. (a) Plugging $R_h = 2GM_h/c^2$ into the indicated expression, we find

$$a_g = \frac{GM_h}{\left(1.001R_h\right)^2} = \frac{GM_h}{\left(1.001\right)^2\left(2GM_h/c^2\right)^2} = \frac{c^4}{\left(2.002\right)^2 G}\frac{1}{M_h}$$

which yields $a_g = (3.02 \times 10^{43}\text{ kg}\cdot\text{m/s}^2)/M_h$.

(b) Since M_h is in the denominator of the above result, a_g decreases as M_h increases.

(c) With $M_h = (1.55 \times 10^{12})(1.99 \times 10^{30}\text{ kg})$, we obtain $a_g = 9.82\text{ m/s}^2$.

(d) This part refers specifically to the very large black hole treated in the previous part. With that mass for M in Eq. 13-16, and $r = 2.002GM/c^2$, we obtain

$$da_g = -2\frac{GM}{\left(2.002GM/c^2\right)^3}dr = -\frac{2c^6}{\left(2.002\right)^3\left(GM\right)^2}dr$$

where $dr \to 1.70$ m as in Sample Problem 13.2 – "Difference in acceleration at head and feet." This yields (in absolute value) an acceleration difference of 7.30×10^{-15} m/s^2.

(e) The miniscule result of the previous part implies that, in this case, any effects due to the differences of gravitational forces on the body are negligible.

23. (a) The gravitational acceleration is $a_g = \dfrac{GM}{R^2} = 7.6$ m/s^2.

(b) Note that the total mass is $5M$. Thus, $a_g = \dfrac{G(5M)}{(3R)^2} = 4.2$ m/s^2.

24. (a) What contributes to the GmM/r^2 force on m is the (spherically distributed) mass M contained within r (where r is measured from the center of M). At point A we see that $M_1 + M_2$ is at a smaller radius than $r = a$ and thus contributes to the force:

$$\left|F_{\text{on } m}\right| = \frac{G(M_1 + M_2)m}{a^2}.$$

(b) In the case $r = b$, only M_1 is contained within that radius, so the force on m becomes GM_1m/b^2.

(c) If the particle is at C, then no other mass is at smaller radius and the gravitational force on it is zero.

25. Using the fact that the volume of a sphere is $4\pi R^3/3$, we find the density of the sphere:

$$\rho = \frac{M_{\text{total}}}{\frac{4}{3}\pi R^3} = \frac{1.0\times10^4 \text{ kg}}{\frac{4}{3}\pi(1.0 \text{ m})^3} = 2.4\times10^3 \text{ kg/m}^3.$$

When the particle of mass m (upon which the sphere, or parts of it, are exerting a gravitational force) is at radius r (measured from the center of the sphere), then whatever mass M is at a radius less than r must contribute to the magnitude of that force (GMm/r^2).

(a) At $r = 1.5$ m, all of M_{total} is at a smaller radius and thus all contributes to the force:

$$\left|F_{\text{on } m}\right| = \frac{GmM_{\text{total}}}{r^2} = m(3.0\times10^{-7} \text{ N/kg}).$$

(b) At $r = 0.50$ m, the portion of the sphere at radius smaller than that is

$$M = \rho\left(\frac{4}{3}\pi r^3\right) = 1.3\times10^3 \text{ kg}.$$

Thus, the force on m has magnitude $GMm/r^2 = m$ (3.3×10^{-7} N/kg).

(c) Pursuing the calculation of part (b) algebraically, we find

$$\left|F_{\text{on } m}\right| = \frac{Gm\rho\left(\frac{4}{3}\pi r^3\right)}{r^2} = mr\left(6.7\times10^{-7}\,\frac{\text{N}}{\text{kg}\cdot\text{m}}\right).$$

26. (a) Since the volume of a sphere is $4\pi R^3/3$, the density is

$$\rho = \frac{M_{\text{total}}}{\frac{4}{3}\pi R^3} = \frac{3M_{\text{total}}}{4\pi R^3}.$$

When we test for gravitational acceleration (caused by the sphere, or by parts of it) at radius r (measured from the center of the sphere), the mass M, which is at radius less than r, is what contributes to the reading (GM/r^2). Since $M = \rho(4\pi r^3/3)$ for $r \le R$, then we can write this result as

$$\frac{G\left(\frac{3M_{\text{total}}}{4\pi R^3}\right)\left(\frac{4\pi r^3}{3}\right)}{r^2} = \frac{GM_{\text{total}}r}{R^3}$$

when we are considering points on or inside the sphere. Thus, the value a_g referred to in the problem is the case where $r = R$:

$$a_g = \frac{GM_{\text{total}}}{R^2},$$

and we solve for the case where the acceleration equals $a_g/3$:

$$\frac{GM_{\text{total}}}{3R^2} = \frac{GM_{\text{total}}r}{R^3} \;\Rightarrow\; r = \frac{R}{3}.$$

(b) Now we treat the case of an external test point. For points with $r > R$ the acceleration is GM_{total}/r^2, so the requirement that it equal $a_g/3$ leads to

$$\frac{GM_{\text{total}}}{3R^2} = \frac{GM_{\text{total}}}{r^2} \;\Rightarrow\; r = \sqrt{3}R.$$

27. (a) The magnitude of the force on a particle with mass m at the surface of Earth is given by $F = GMm/R^2$, where M is the total mass of Earth and R is Earth's radius. The acceleration due to gravity is

$$a_g = \frac{F}{m} = \frac{GM}{R^2} = \frac{\left(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(5.98\times10^{24}\ \text{kg}\right)}{\left(6.37\times10^{6}\ \text{m}\right)^2} = 9.83\ \text{m/s}^2.$$

(b) Now $a_g = GM/R^2$, where M is the total mass contained in the core and mantle together and R is the outer radius of the mantle (6.345×10^6 m, according to the figure). The total mass is

$$M = (1.93 \times 10^{24}\ \text{kg} + 4.01 \times 10^{24}\ \text{kg}) = 5.94 \times 10^{24}\ \text{kg}.$$

The first term is the mass of the core and the second is the mass of the mantle. Thus,

$$a_g = \frac{\left(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(5.94\times10^{24}\ \text{kg}\right)}{\left(6.345\times10^{6}\ \text{m}\right)^2} = 9.84\ \text{m/s}^2.$$

(c) A point 25 km below the surface is at the mantle–crust interface and is on the surface of a sphere with a radius of $R = 6.345 \times 10^6$ m. Since the mass is now assumed to be uniformly distributed, the mass within this sphere can be found by multiplying the mass per unit volume by the volume of the sphere: $M = (R^3 / R_e^3) M_e$, where M_e is the total mass of Earth and R_e is the radius of Earth. Thus,

$$M = \left(\frac{6.345\times10^{6}\ \text{m}}{6.37\times10^{6}\ \text{m}}\right)^3 \left(5.98\times10^{24}\ \text{kg}\right) = 5.91\times10^{24}\ \text{kg}.$$

The acceleration due to gravity is

$$a_g = \frac{GM}{R^2} = \frac{\left(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(5.91\times10^{24}\ \text{kg}\right)}{\left(6.345\times10^{6}\ \text{m}\right)^2} = 9.79\ \text{m/s}^2.$$

28. (a) Using Eq. 13-1, we set GmM/r^2 equal to $\frac{1}{2} GmM/R^2$, and we find $r = R\sqrt{2}$. Thus, the distance from the surface is $(\sqrt{2} - 1)R = 0.414R$.

(b) Setting the density ρ equal to M/V where $V = \frac{4}{3}\pi R^3$, we use Eq. 13-19:

$$F = \frac{4\pi Gmr\rho}{3} = \frac{4\pi Gmr}{3}\left(\frac{M}{4\pi R^3/3}\right) = \frac{GMmr}{R^3} = \frac{1}{2}\frac{GMm}{R^2} \quad \Rightarrow \quad r = R/2.$$

29. The equation immediately preceding Eq. 13-28 shows that $K = -U$ (with U evaluated at the planet's surface: -5.0×10^9 J) is required to "escape." Thus, $K = 5.0 \times 10^9$ J.

30. The gravitational potential energy is

$$U = -\frac{Gm(M-m)}{r} = -\frac{G}{r}\left(Mm - m^2\right)$$

which we differentiate with respect to m and set equal to zero (in order to minimize). Thus, we find $M - 2m = 0$, which leads to the ratio $m/M = 1/2$ to obtain the least potential energy.

Note that a second derivative of U with respect to m would lead to a positive result regardless of the value of m, which means its graph is everywhere concave upward and thus its extremum is indeed a minimum.

31. **THINK** Given the mass and diameter of Mars, we can calculate its mean density, gravitational acceleration and escape speed.

EXPRESS The density of a uniform sphere is given by $\rho = 3M/4\pi R^3$, where M is its mass and R is its radius. On the other hand, the value of gravitational acceleration a_g at the surface of a planet is given by $a_g = GM/R^2$. for a particle of mass m, its escape speed is given by

$$\frac{1}{2}mv^2 = G\frac{mM}{R} \quad \Rightarrow \quad v = \sqrt{\frac{2GM}{R}}.$$

ANALYZE (a) From the definition of density above, we find the ratio of the density of Mars to the density of Earth to be

$$\frac{\rho_M}{\rho_E} = \frac{M_M}{M_E}\frac{R_E^3}{R_M^3} = 0.11\left(\frac{0.65\times10^4\ \text{km}}{3.45\times10^3\ \text{km}}\right)^3 = 0.74.$$

(b) The value of gravitational acceleration for Mars is

$$a_{gM} = \frac{GM_M}{R_M^2} = \frac{M_M}{R_M^2}\cdot\frac{R_E^2}{M_E}\cdot\frac{GM_E}{R_E^2} = \frac{M_M}{M_E}\frac{R_E^2}{R_M^2}a_{gE} = 0.11\left(\frac{0.65\times10^4\ \text{km}}{3.45\times10^3\ \text{km}}\right)^2\left(9.8\ \text{m/s}^2\right) = 3.8\ \text{m/s}^2.$$

(c) For Mars, the escape speed is

$$v_M = \sqrt{\frac{2GM_M}{R_M}} = \sqrt{\frac{2(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(0.11)(5.98\times10^{24}\ \text{kg})}{3.45\times10^6\ \text{m}}} = 5.0\times10^3\ \text{m/s}.$$

LEARN The ratio of the escape speeds on Mars and on Earth is

$$\frac{v_M}{v_E} = \frac{\sqrt{2GM_M/R_M}}{\sqrt{2GM_E/R_E}} = \sqrt{\frac{M_M}{M_E}\cdot\frac{R_E}{R_M}} = \sqrt{(0.11)\cdot\frac{6.5\times10^3\ \text{km}}{3.45\times10^3\ \text{km}}} = 0.455.$$

32. (a) The gravitational potential energy is

$$U = -\frac{GMm}{r} = -\frac{\left(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(5.2\ \text{kg}\right)\left(2.4\ \text{kg}\right)}{19\ \text{m}} = -\ 4.4\times10^{-11}\ \text{J}.$$

(b) Since the change in potential energy is

$$\Delta U = -\frac{GMm}{3r} - \left(-\frac{GMm}{r}\right) = -\frac{2}{3}\left(-4.4\times10^{-11}\ \text{J}\right) = 2.9\times10^{-11}\ \text{J},$$

the work done by the gravitational force is $W = -\Delta U = -2.9 \times 10^{-11}$ J.

(c) The work done by you is $W' = \Delta U = 2.9 \times 10^{-11}$ J.

33. The amount of (kinetic) energy needed to escape is the same as the (absolute value of the) gravitational potential energy at its original position. Thus, an object of mass m on a planet of mass M and radius R needs $K = GmM/R$ in order to (barely) escape.
(a) Setting up the ratio, we find

$$\frac{K_m}{K_E} = \frac{M_m}{M_E}\frac{R_E}{R_m} = 0.0451$$

using the values found in Appendix C.

(b) Similarly, for the Jupiter escape energy (divided by that for Earth) we obtain

$$\frac{K_J}{K_E} = \frac{M_J}{M_E}\frac{R_E}{R_J} = 28.5.$$

34. (a) The potential energy U at the surface is $U_s = -5.0 \times 10^9$ J according to the graph, since U is inversely proportional to r (see Eq. 13-21), at an r-value a factor of 5/4 times what it was at the surface then U must be $4\ U_s/5$. Thus, at $r = 1.25R_s$, $U = -4.0 \times 10^9$ J. Since mechanical energy is assumed to be conserved in this problem, we have

$$K + U = -2.0 \times 10^9\ \text{J}$$

at this point. Since $U = -4.0 \times 10^9$ J here, then $K = 2.0\times10^9$ J at this point.

(b) To reach the point where the mechanical energy equals the potential energy (that is, where $U = -2.0 \times 10^9$ J) means that U must reduce (from its value at $r = 1.25R_s$) by a factor of 2, which means the r value must increase (relative to $r = 1.25R_s$) by a corresponding factor of 2. Thus, the turning point must be at $r = 2.5R_s$.

35. Let m = 0.020 kg and d = 0.600 m (the original edge-length, in terms of which the final edge-length is $d/3$). The total initial gravitational potential energy (using Eq. 13-21 and some elementary trigonometry) is

$$U_i = -\frac{4Gm^2}{d} - \frac{2Gm^2}{\sqrt{2}\,d} .$$

Since U is inversely proportional to r then reducing the size by $1/3$ means increasing the magnitude of the potential energy by a factor of 3, so

$$U_f = 3U_i \;\Rightarrow\; \Delta U = 2U_i = 2(4+\sqrt{2})\left(-\frac{Gm^2}{d}\right) = -4.82\times 10^{-13}\ \text{J} .$$

36. Energy conservation for this situation may be expressed as follows:

$$K_1 + U_1 = K_2 + U_2 \;\Rightarrow\; K_1 - \frac{GmM}{r_1} = K_2 - \frac{GmM}{r_2}$$

where $M = 5.0\times 10^{23}$ kg, $r_1 = R = 3.0\times 10^6$ m and m = 10 kg.

(a) If $K_1 = 5.0\times 10^7$ J and $r_2 = 4.0\times 10^6$ m, then the above equation leads to

$$K_2 = K_1 + GmM\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = 2.2\times 10^7\ \text{J}.$$

(b) In this case, we require $K_2 = 0$ and $r_2 = 8.0\times 10^6$ m, and solve for K_1:

$$K_1 = K_2 + GmM\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = 6.9\times 10^7\ \text{J}.$$

37. (a) The work done by you in moving the sphere of mass m_B equals the change in the potential energy of the three-sphere system. The initial potential energy is

$$U_i = -\frac{Gm_Am_B}{d} - \frac{Gm_Am_C}{L} - \frac{Gm_Bm_C}{L-d}$$

and the final potential energy is

$$U_f = -\frac{Gm_Am_B}{L-d} - \frac{Gm_Am_C}{L} - \frac{Gm_Bm_C}{d}.$$

The work done is

$$W = U_f - U_i = Gm_B\left[m_A\left(\frac{1}{d} - \frac{1}{L-d}\right) + m_C\left(\frac{1}{L-d} - \frac{1}{d}\right)\right]$$

$$= Gm_B\left[m_A\frac{L-2d}{d(L-d)} + m_C\frac{2d-L}{d(L-d)}\right] = Gm_B(m_A - m_C)\frac{L-2d}{d(L-d)}$$

$$= (6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(0.010\ \text{kg})(0.080\ \text{kg} - 0.020\ \text{kg})\frac{0.12\ \text{m} - 2(0.040\ \text{m})}{(0.040\ \text{m})(0.12 - 0.040\ \text{m})}$$

$$= +5.0\times10^{-13}\ \text{J}.$$

(b) The work done by the force of gravity is $-(U_f - U_i) = -5.0\times10^{-13}$ J.

38. (a) The initial gravitational potential energy is

$$U_i = -\frac{GM_AM_B}{r_i} = -\frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(20\ \text{kg})(10\ \text{kg})}{0.80\ \text{m}}$$

$$= -1.67\times10^{-8}\ \text{J} \approx -1.7\times10^{-8}\ \text{J}.$$

(b) We use conservation of energy (with $K_i = 0$):

$$U_i = K + U \quad\Rightarrow\quad -1.7\times10^{-8} = K - \frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(20\ \text{kg})(10\ \text{kg})}{0.60\ \text{m}}$$

which yields $K = 5.6\times10^{-9}$ J. Note that the value of r is the difference between 0.80 m and 0.20 m.

39. **THINK** The escape speed on the asteroid is related to the gravitational acceleration at the surface of the asteroid and its size.

EXPRESS We use the principle of conservation of energy. Initially the particle is at the surface of the asteroid and has potential energy $U_i = -GMm/R$, where M is the mass of the asteroid, R is its radius, and m is the mass of the particle being fired upward. The initial kinetic energy is $\frac{1}{2}mv^2$. The particle just escapes if its kinetic energy is zero when it is infinitely far from the asteroid. The final potential and kinetic energies are both zero. Conservation of energy yields

$$-GMm/R + \tfrac{1}{2}mv^2 = 0.$$

We replace GM/R with a_gR, where a_g is the acceleration due to gravity at the surface. Then, the energy equation becomes $-a_gR + \frac{1}{2}v^2 = 0$. Solving for v, we have

$$v = \sqrt{2a_gR}\ .$$

ANALYZE (a) Given that $R = 500\,\text{km}$ and $a_g = 3.0\,\text{m/s}^2$, we find the escape speed to be

$$v = \sqrt{2a_g R} = \sqrt{2(3.0\ \text{m/s}^2)(500 \times 10^3\ \text{m})} = 1.7 \times 10^3\ \text{m/s}.$$

(b) Initially the particle is at the surface; the potential energy is $U_i = -GMm/R$ and the kinetic energy is $K_i = \frac{1}{2}mv^2$. Suppose the particle is a distance h above the surface when it momentarily comes to rest. The final potential energy is $U_f = -GMm/(R + h)$ and the final kinetic energy is $K_f = 0$. Conservation of energy yields

$$-\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R+h}.$$

We replace GM with $a_g R^2$ and cancel m in the energy equation to obtain

$$-a_g R + \frac{1}{2}v^2 = -\frac{a_g R^2}{(R+h)}.$$

The solution for h is

$$h = \frac{2a_g R^2}{2a_g R - v^2} - R = \frac{2(3.0\ \text{m/s}^2)(500 \times 10^3\ \text{m})^2}{2(3.0\ \text{m/s}^2)(500 \times 10^3\ \text{m}) - (1000\ \text{m/s})^2} - (500 \times 10^3\ \text{m})$$
$$= 2.5 \times 10^5\ \text{m}.$$

(c) Initially the particle is a distance h above the surface and is at rest. Its potential energy is $U_i = -GMm/(R + h)$ and its initial kinetic energy is $K_i = 0$. Just before it hits the asteroid its potential energy is $U_f = -GMm/R$. Write $\frac{1}{2}mv_f^2$ for the final kinetic energy. Conservation of energy yields

$$-\frac{GMm}{R+h} = -\frac{GMm}{R} + \frac{1}{2}mv^2.$$

We substitute $a_g R^2$ for GM and cancel m, obtaining

$$-\frac{a_g R^2}{R+h} = -a_g R + \frac{1}{2}v^2.$$

The solution for v is

$$v = \sqrt{2a_g R - \frac{2a_g R^2}{R+h}} = \sqrt{2(3.0\ \text{m/s}^2)(500 \times 10^3\ \text{m}) - \frac{2(3.0\ \text{m/s}^2)(500 \times 10^3\ \text{m})^2}{(500 \times 10^3\ \text{m}) + (1000 \times 10^3\ \text{m})}}$$
$$= 1.4 \times 10^3\ \text{m/s}.$$

LEARN The key idea in this problem is to realize that energy is conserved in the process:

$$K_i + U_i = K_f + U_f \quad \Rightarrow \quad \Delta K + \Delta U = 0.$$

The decrease in potential energy is equal to the gain in kinetic energy, and vice versa.

40. (a) From Eq. 13-28, we see that $v_0 = \sqrt{GM/2R_E}$ in this problem. Using energy conservation, we have

$$\frac{1}{2} mv_0^2 - GMm/R_E = -GMm/r$$

which yields $r = 4R_E/3$. So the multiple of R_E is 4/3 or 1.33.

(b) Using the equation in the textbook immediately preceding Eq. 13-28, we see that in this problem we have $K_i = GMm/2R_E$, and the above manipulation (using energy conservation) in this case leads to $r = 2R_E$. So the multiple of R_E is 2.00.

(c) Again referring to the equation in the textbook immediately preceding Eq. 13-28, we see that the mechanical energy = 0 for the "escape condition."

41. **THINK** The two neutron stars are attracted toward each other due to their gravitational interaction.

EXPRESS The momentum of the two-star system is conserved, and since the stars have the same mass, their speeds and kinetic energies are the same. We use the principle of conservation of energy. The initial potential energy is $U_i = -GM^2/r_i$, where M is the mass of either star and r_i is their initial center-to-center separation. The initial kinetic energy is zero since the stars are at rest. The final potential energy is $U_f = -GM^2/r_f$, where the final separation is $r_f = r_i/2$. We write Mv^2 for the final kinetic energy of the system. This is the sum of two terms, each of which is $\frac{1}{2}Mv^2$. Conservation of energy yields

$$-\frac{GM^2}{r_i} = -\frac{2GM^2}{r_i} + Mv^2.$$

ANALYZE (a) The solution for v is

$$v = \sqrt{\frac{GM}{r_i}} = \sqrt{\frac{(6.67 \times 10^{-11}\ \text{m}^3/\text{s}^2 \cdot \text{kg})(10^{30}\ \text{kg})}{10^{10}\ \text{m}}} = 8.2 \times 10^4\ \text{m/s}.$$

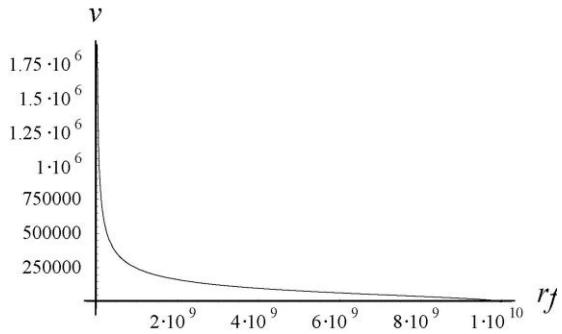
(b) Now the final separation of the centers is $r_f = 2R = 2 \times 10^5$ m, where R is the radius of either of the stars. The final potential energy is given by $U_f = -GM^2/r_f$ and the energy equation becomes

$$-GM^2/r_i = -GM^2/r_f + Mv^2.$$

The solution for v is

$$v=\sqrt{GM\left(\frac{1}{r_f}-\frac{1}{r_i}\right)}=\sqrt{(6.67\times10^{-11}\,\text{m}^3/\text{s}^2\cdot\text{kg})(10^{30}\,\text{kg})\left(\frac{1}{2\times10^5\ \text{m}}-\frac{1}{10^{10}\ \text{m}}\right)}$$
$$= 1.8\times10^7\ \text{m/s}.$$

LEARN The speed of the stars as a function of their final separation is plotted below. The decrease in gravitational potential energy is accompanied by an increase in kinetic energy, so that the total energy of the two-star system remains conserved.



42. (a) Applying Eq. 13-21 and the Pythagorean theorem leads to

$$U = -\left(\frac{GM^2}{2D}+\frac{2GmM}{\sqrt{y^2+D^2}}\right)$$

where M is the mass of particle B (also that of particle C) and m is the mass of particle A. The value given in the problem statement (for infinitely large y, for which the second term above vanishes) determines M, since D is given. Thus $M = 0.50$ kg.

(b) We estimate (from the graph) the $y = 0$ value to be $U_o = -3.5 \times 10^{-10}$ J. Using this, our expression above determines m. We obtain $m = 1.5$ kg.

43. (a) If r is the radius of the orbit then the magnitude of the gravitational force acting on the satellite is given by GMm/r^2, where M is the mass of Earth and m is the mass of the satellite. The magnitude of the acceleration of the satellite is given by v^2/r, where v is its speed. Newton's second law yields $GMm/r^2 = mv^2/r$. Since the radius of Earth is 6.37×10^6 m, the orbit radius is $r = (6.37 \times 10^6\ \text{m} + 160 \times 10^3\ \text{m}) = 6.53 \times 10^6$ m. The solution for v is

$$v=\sqrt{\frac{GM}{r}}=\sqrt{\frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(5.98\times10^{24}\,\text{kg})}{6.53\times10^6\ \text{m}}}=7.82\times10^3\ \text{m/s}.$$

(b) Since the circumference of the circular orbit is $2\pi r$, the period is

$$T=\frac{2\pi r}{v}=\frac{2\pi(6.53\times10^6\,\text{m})}{7.82\times10^3\ \text{m/s}}=5.25\times10^3\,\text{s}.$$

This is equivalent to 87.5 min.

44. Kepler's law of periods, expressed as a ratio, is

$$\left(\frac{r_s}{r_m}\right)^3 = \left(\frac{T_s}{T_m}\right)^2 \Rightarrow \left(\frac{1}{2}\right)^3 = \left(\frac{T_s}{1 \text{ lunar month}}\right)^2$$

which yields $T_s = 0.35$ lunar month for the period of the satellite.

45. The period T and orbit radius r are related by the law of periods: $T^2 = (4\pi^2/GM)r^3$, where M is the mass of Mars. The period is 7 h 39 min, which is 2.754×10^4 s. We solve for M:

$$M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 (9.4 \times 10^6 \text{ m})^3}{(6.67 \times 10^{-11} \text{m}^3/\text{s}^2 \cdot \text{kg})\left(2.754 \times 10^4 \text{ s}\right)^2} = 6.5 \times 10^{23} \text{ kg}.$$

46. From Eq. 13-37, we obtain $v = \sqrt{GM/r}$ for the speed of an object in circular orbit (of radius r) around a planet of mass M. In this case, $M = 5.98 \times 10^{24}$ kg and

$$r = (700 + 6370)\text{m} = 7070 \text{ km} = 7.07 \times 10^6 \text{ m}.$$

The speed is found to be $v = 7.51 \times 10^3$ m/s. After multiplying by 3600 s/h and dividing by 1000 m/km this becomes $v = 2.7 \times 10^4$ km/h.

(a) For a head-on collision, the relative speed of the two objects must be $2v = 5.4 \times 10^4$ km/h.

(b) A perpendicular collision is possible if one satellite is, say, orbiting above the equator and the other is following a longitudinal line. In this case, the relative speed is given by the Pythagorean theorem: $\sqrt{v^2 + v^2} = 3.8 \times 10^4$ km/h.

47. **THINK** The centripetal force on the Sun is due to the gravitational attraction between the Sun and the stars at the center of the Galaxy.

EXPRESS Let N be the number of stars in the galaxy, M be the mass of the Sun, and r be the radius of the galaxy. The total mass in the galaxy is $N\,M$ and the magnitude of the gravitational force acting on the Sun is

$$F_g = \frac{GM(NM)}{R^2} = \frac{GNM^2}{R^2}.$$

The force, pointing toward the galactic center, is the centripetal force on the Sun. Thus,

$$F_c = F_g \quad \Rightarrow \quad \frac{Mv^2}{R} = \frac{GNM^2}{R^2}.$$

The magnitude of the Sun's acceleration is $a = v^2/R$, where v is its speed. If T is the period of the Sun's motion around the galactic center then $v = 2\pi R/T$ and $a = 4\pi^2 R/T^2$. Newton's second law yields

$$GNM^2/R^2 = 4\pi^2 MR/T^2.$$

The solution for N is

$$N = \frac{4\pi^2 R^3}{GT^2 M}.$$

ANALYZE The period is 2.5×10^8 y, which is 7.88×10^{15} s, so

$$N = \frac{4\pi^2 (2.2 \times 10^{20}\ \text{m})^3}{(6.67 \times 10^{-11}\ \text{m}^3/\text{s}^2 \cdot \text{kg})(7.88 \times 10^{15}\ \text{s})^2 (2.0 \times 10^{30}\ \text{kg})} = 5.1 \times 10^{10}.$$

LEARN The number of stars in the Milky Way is between 10^{11} to 4×10^{11}. Our simplified model provides a reasonable estimate.

48. Kepler's law of periods, expressed as a ratio, is

$$\left(\frac{a_M}{a_E}\right)^3 = \left(\frac{T_M}{T_E}\right)^2 \Rightarrow (1.52)^3 = \left(\frac{T_M}{1\text{y}}\right)^2$$

where we have substituted the mean-distance (from Sun) ratio for the semi-major axis ratio. This yields $T_M = 1.87$ y. The value in Appendix C (1.88 y) is quite close, and the small apparent discrepancy is not significant, since a more precise value for the semi-major axis ratio is $a_M/a_E = 1.523$, which does lead to $T_M = 1.88$ y using Kepler's law. A question can be raised regarding the use of a ratio of mean distances for the ratio of semi-major axes, but this requires a more lengthy discussion of what is meant by a "mean distance" than is appropriate here.

49. (a) The period of the comet is 1420 years (and one month), which we convert to $T = 4.48 \times 10^{10}$ s. Since the mass of the Sun is 1.99×10^{30} kg, then Kepler's law of periods gives

$$(4.48 \times 10^{10}\ \text{s})^2 = \left(\frac{4\pi^2}{(6.67 \times 10^{-11}\ \text{m}^3/\text{kg} \cdot \text{s}^2)(1.99 \times 10^{30}\ \text{kg})}\right) a^3 \Rightarrow a = 1.89 \times 10^{13}\ \text{m}.$$

(b) Since the distance from the focus (of an ellipse) to its center is ea and the distance from center to the aphelion is a, then the comet is at a distance of

$$ea + a = (0.9932 + 1)\ (1.89 \times 10^{13}\ \text{m}) = 3.767 \times 10^{13}\ \text{m}$$

when it is farthest from the Sun. To express this in terms of Pluto's orbital radius (found in Appendix C), we set up a ratio:

$$\left(\frac{3.767\times10^{13}}{5.9\times10^{12}}\right)R_P \approx 6,4R_P.$$

50. To "hover" above Earth ($M_E = 5.98 \times 10^{24}$ kg) means that it has a period of 24 hours (86400 s). By Kepler's law of periods,

$$(86400)^2 = \left(\frac{4\pi^2}{GM_E}\right)r^3 \Rightarrow r = 4.225\times10^7 \text{ m}.$$

Its altitude is therefore $r - R_E$ (where $R_E = 6.37 \times 10^6$ m), which yields 3.58×10^7 m.

51. **THINK** The satellite moves in an elliptical orbit about Earth. An elliptical orbit can be characterized by its semi-major axis and eccentricity.

EXPRESS The greatest distance between the satellite and Earth's center (the apogee distance) and the least distance (perigee distance) are, respectively,

$$R_a = R_E + d_a = 6.37\times10^6 \text{ m} + 360\times10^3 \text{ m} = 6.73\times10^6 \text{ m}$$
$$R_p = R_E + d_p = 6.37\times10^6 \text{ m} + 180\times10^3 \text{ m} = 6.55\times10^6 \text{ m}.$$

Here $R_E = 6.37 \times 10^6$ m is the radius of Earth.

ANALYZE The semi-major axis is given by

$$a = \frac{R_a + R_p}{2} = \frac{6.73\times10^6 \text{ m} + 6.55\times10^6 \text{ m}}{2} = 6.64\times10^6 \text{ m}.$$

(b) The apogee and perigee distances are related to the eccentricity e by $R_a = a(1 + e)$ and $R_p = a(1 - e)$. Add to obtain $R_a + R_p = 2a$ and $a = (R_a + R_p)/2$. Subtract to obtain $R_a - R_p = 2ae$. Thus,

$$e = \frac{R_a - R_p}{2a} = \frac{R_a - R_p}{R_a + R_p} = \frac{6.73\times10^6 \text{ m} - 6.55\times10^6 \text{ m}}{6.73\times10^6 \text{ m} + 6.55\times10^6 \text{ m}} = 0.0136.$$

LEARN Since e is very small, the orbit is nearly circular. On the other hand, if e is close to unity, then the orbit would be a long, thin ellipse.

52. (a) The distance from the center of an ellipse to a focus is ae where a is the semi-major axis and e is the eccentricity. Thus, the separation of the foci (in the case of Earth's orbit) is

$$2ae = 2\left(1.50\times10^{11} \text{ m}\right)\left(0.0167\right) = 5.01\times10^9 \text{ m}.$$

(b) To express this in terms of solar radii (see Appendix C), we set up a ratio:

$$\frac{5.01\times10^{9}\ \text{m}}{6.96\times10^{8}\ \text{m}} = 7.20.$$

53. From Kepler's law of periods (where T = (2.4 h)(3600 s/h) = 8640 s), we find the planet's mass M:

$$(8640\,\text{s})^2 = \left(\frac{4\pi^2}{GM}\right)(8.0\times10^6\,\text{m})^3 \Rightarrow M = 4.06\times10^{24}\,\text{kg}.$$

However, we also know $a_g = GM/R^2 = 8.0\ \text{m/s}^2$ so that we are able to solve for the planet's radius:

$$R = \sqrt{\frac{GM}{a_g}} = \sqrt{\frac{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(4.06\times10^{24}\ \text{kg})}{8.0\ \text{m/s}^2}} = 5.8\times10^6\ \text{m}.$$

54. The two stars are in circular orbits, not about each other, but about the two-star system's center of mass (denoted as O), which lies along the line connecting the centers of the two stars. The gravitational force between the stars provides the centripetal force necessary to keep their orbits circular. Thus, for the visible, Newton's second law gives

$$F = \frac{Gm_1m_2}{r^2} = \frac{m_1v^2}{r_1}$$

where r is the distance between the centers of the stars. To find the relation between r and r_1, we locate the center of mass relative to m_1. Using Equation 9-1, we obtain

$$r_1 = \frac{m_1(0)+m_2r}{m_1+m_2} = \frac{m_2r}{m_1+m_2} \quad\Rightarrow\quad r = \frac{m_1+m_2}{m_2}r_1.$$

On the other hand, since the orbital speed of m_1 is $v = 2\pi r_1/T$, then $r_1 = vT/2\pi$ and the expression for r can be rewritten as

$$r = \frac{m_1+m_2}{m_2}\frac{vT}{2\pi}.$$

Substituting r and r_1 into the force equation, we obtain

$$F = \frac{4\pi^2Gm_1m_2^3}{(m_1+m_2)^2v^2T^2} = \frac{2\pi m_1v}{T}$$

or

$$\frac{m_2^3}{(m_1+m_2)^2} = \frac{v^3T}{2\pi G} = \frac{(2.7\times10^5\ \text{m/s})^3(1.70\ \text{days})(86400\ \text{s/day})}{2\pi(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)} = 6.90\times10^{30}\ \text{kg}$$
$$= 3.467M_s,$$

where $M_s = 1.99\times10^{30}$ kg is the mass of the sun. With $m_1 = 6M_s$, we write $m_2 = \alpha M_s$ and solve the following cubic equation for α:

$$\frac{\alpha^3}{(6+\alpha)^2} - 3.467 = 0 .$$

The equation has one real solution: $\alpha = 9.3$, which implies $m_2 / M_s \approx 9$.

55. (a) If we take the logarithm of Kepler's law of periods, we obtain

$$2\log(T) = \log(4\pi^2/GM) + 3\log(a) \Rightarrow \log(a) = \frac{2}{3}\log(T) - \frac{1}{3}\log(4\pi^2/GM)$$

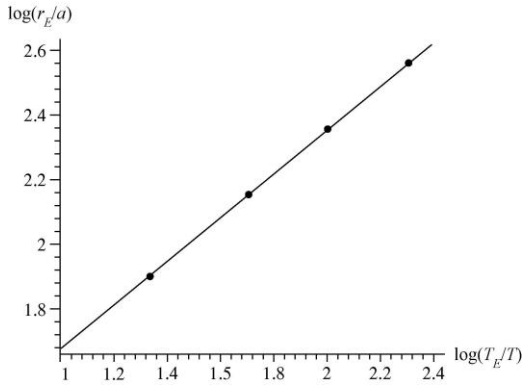
where we are ignoring an important subtlety about units (the arguments of logarithms cannot have units, since they are transcendental functions). Although the problem can be continued in this way, we prefer to set it up without units, which requires taking a ratio. If we divide Kepler's law (applied to the Jupiter–moon system, where M is mass of Jupiter) by the law applied to Earth orbiting the Sun (of mass M_o), we obtain

$$(T/T_E)^2 = \left(\frac{M_o}{M}\right)\left(\frac{a}{r_E}\right)^3$$

where T_E = 365.25 days is Earth's orbital period and r_E = 1.50 × 10^{11} m is its mean distance from the Sun. In this case, it is perfectly legitimate to take logarithms and obtain

$$\log\left(\frac{r_E}{a}\right) = \frac{2}{3}\log\left(\frac{T_E}{T}\right) + \frac{1}{3}\log\left(\frac{M_o}{M}\right)$$

(written to make each term positive), which is the way we plot the data (log (r_E/a) on the vertical axis and log (T_E/T) on the horizontal axis).



(b) When we perform a least-squares fit to the data, we obtain

$$\log (r_E/a) = 0.666 \log (T_E/T) + 1.01,$$

which confirms the expectation of slope = 2/3 based on the above equation.

(c) And the 1.01 intercept corresponds to the term 1/3 log (M_o/M), which implies

$$\frac{M_o}{M} = 10^{3.03} \Rightarrow M = \frac{M_o}{1.07 \times 10^3}.$$

Plugging in $M_o = 1.99 \times 10^{30}$ kg (see Appendix C), we obtain $M = 1.86 \times 10^{27}$ kg for Jupiter's mass. This is reasonably consistent with the value 1.90×10^{27} kg found in Appendix C.

56. (a) The period is $T = 27(3600) = 97200$ s, and we are asked to assume that the orbit is circular (of radius $r = 100000$ m). Kepler's law of periods provides us with an approximation to the asteroid's mass:

$$(97200)^2 = \left(\frac{4\pi^2}{GM}\right)(100000)^3 \Rightarrow M = 6.3 \times 10^{16} \text{ kg}.$$

(b) Dividing the mass M by the given volume yields an average density equal to

$$\rho = (6.3 \times 10^{16} \text{ kg})/(1.41 \times 10^{13} \text{ m}^3) = 4.4 \times 10^3 \text{ kg/m}^3,$$

which is about 20% less dense than Earth.

57. In our system, we have $m_1 = m_2 = M$ (the mass of our Sun, 1.99×10^{30} kg). With $r = 2r_1$ in this system (so r_1 is one-half the Earth-to-Sun distance r), and $v = \pi r/T$ for the speed, we have

$$\frac{Gm_1m_2}{r^2} = m_1 \frac{(\pi r/T)^2}{r/2} \Rightarrow T = \sqrt{\frac{2\pi^2 r^3}{GM}}.$$

With $r = 1.5 \times 10^{11}$ m, we obtain $T = 2.2 \times 10^7$ s. We can express this in terms of Earth-years, by setting up a ratio:

$$T = \left(\frac{T}{1\text{y}}\right)(1\text{y}) = \left(\frac{2.2 \times 10^7 \text{ s}}{3.156 \times 10^7 \text{ s}}\right)(1 \text{ y}) = 0.71 \text{ y}.$$

58. (a) We make use of

$$\frac{m_2^3}{(m_1 + m_2)^2} = \frac{v^3 T}{2\pi G}$$

where $m_1 = 0.9M_{\text{Sun}}$ is the estimated mass of the star. With $v = 70$ m/s and $T = 1500$ days (or $1500 \times 86400 = 1.3 \times 10^8$ s), we find

$$\frac{m_2^3}{(0.9M_{\text{Sun}}+m_2)^2} = 1.06\times 10^{23}\ \text{kg}\ .$$

Since $M_{\text{Sun}} \approx 2.0 \times 10^{30}$ kg, we find $m_2 \approx 7.0 \times 10^{27}$ kg. Dividing by the mass of Jupiter (see Appendix C), we obtain $m \approx 3.7m_J$.

(b) Since $v = 2\pi r_1/T$ is the speed of the star, we find

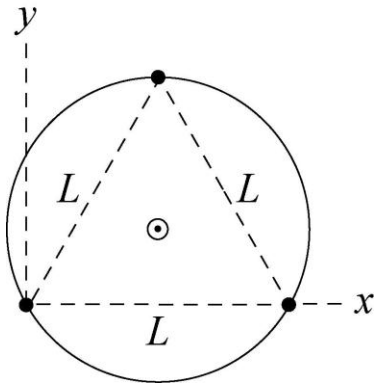
$$r_1 = \frac{vT}{2\pi} = \frac{(70\,\text{m/s})(1.3\times 10^8\,\text{s})}{2\pi} = 1.4\times 10^9\ \text{m}$$

for the star's orbital radius. If r is the distance between the star and the planet, then $r_2 = r - r_1$ is the orbital radius of the planet, and is given by

$$r_2 = r_1\left(\frac{m_1+m_2}{m_2}-1\right) = r_1\frac{m_1}{m_2} = 3.7\times 10^{11}\ \text{m}\ .$$

Dividing this by 1.5×10^{11} m (Earth's orbital radius, r_E) gives $r_2 = 2.5r_E$.

59. Each star is attracted toward each of the other two by a force of magnitude GM^2/L^2, along the line that joins the stars. The net force on each star has magnitude $2(GM^2/L^2)\cos 30°$ and is directed toward the center of the triangle. This is a centripetal force and keeps the stars on the same circular orbit if their speeds are appropriate. If R is the radius of the orbit, Newton's second law yields $(GM^2/L^2)\cos 30° = Mv^2/R$.



The stars rotate about their center of mass (marked by a circled dot on the diagram above) at the intersection of the perpendicular bisectors of the triangle sides, and the radius of the orbit is the distance from a star to the center of mass of the three-star system. We take the coordinate system to be as shown in the diagram, with its origin at the left-most star. The altitude of an equilateral triangle is $\left(\sqrt{3}/2\right)L$, so the stars are located at $x = 0$, $y = 0$; $x = L$, $y = 0$; and $x = L/2$, $y = \sqrt{3}L/2$. The x coordinate of the center of mass is $x_c = (L +$

$L/2)/3 = L/2$ and the y coordinate is $y_c = \left(\sqrt{3}L/2\right)/3 = L/2\sqrt{3}$. The distance from a star to the center of mass is

$$R = \sqrt{x_c^2 + y_c^2} = \sqrt{\left(L^2/4\right) + \left(L^2/12\right)} = L/\sqrt{3}.$$

Once the substitution for R is made, Newton's second law then becomes $\left(2GM^2/L^2\right)\cos 30° = \sqrt{3}Mv^2/L$. This can be simplified further by recognizing that $\cos 30° = \sqrt{3}/2$. Divide the equation by M then gives $GM/L^2 = v^2/L$, or $v = \sqrt{GM/L}$.

60. (a) From Eq. 13-40, we see that the energy of each satellite is $-GM_Em/2r$. The total energy of the two satellites is twice that result:

$$E = E_A + E_B = -\frac{GM_Em}{r} = -\frac{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(5.98\times10^{24}\text{kg})(125\ \text{kg})}{7.87\times10^6\ \text{m}}$$
$$= -6.33\times10^9\ \text{J}.$$

(b) We note that the speed of the wreckage will be zero (immediately after the collision), so it has no kinetic energy at that moment. Replacing m with $2m$ in the potential energy expression, we therefore find the total energy of the wreckage at that instant is

$$E = -\frac{GM_E(2m)}{2r} = -\frac{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(5.98\times10^{24}\text{kg})2(125\ \text{kg})}{2(7.87\times10^6\ \text{m})} = -6.33\times10^9\ \text{J}.$$

(c) An object with zero speed at that distance from Earth will simply fall toward the Earth, its trajectory being toward the center of the planet.

61. The energy required to raise a satellite of mass m to an altitude h (at rest) is given by

$$E_1 = \Delta U = GM_Em\left(\frac{1}{R_E} - \frac{1}{R_E + h}\right),$$

and the energy required to put it in circular orbit once it is there is

$$E_2 = \frac{1}{2}mv_{\text{orb}}^2 = \frac{GM_Em}{2\left(R_E + h\right)}.$$

Consequently, the energy difference is

$$\Delta E = E_1 - E_2 = GM_Em\left[\frac{1}{R_E} - \frac{3}{2(R_E + h)}\right].$$

(a) Solving the above equation, the height h_0 at which $\Delta E = 0$ is given by

$$\frac{1}{R_E} - \frac{3}{2(R_E + h_0)} = 0 \quad \Rightarrow \; h_0 = \frac{R_E}{2} = 3.19\times 10^6 \text{ m}.$$

(b) For greater height $h > h_0$, $\Delta E > 0$, implying $E_1 > E_2$. Thus, the energy of lifting is greater.

62. Although altitudes are given, it is the orbital radii that enter the equations. Thus, r_A = (6370 + 6370) km = 12740 km, and r_B = (19110 + 6370) km = 25480 km.

(a) The ratio of potential energies is

$$\frac{U_B}{U_A} = \frac{-GmM / r_B}{-GmM / r_A} = \frac{r_A}{r_B} = \frac{1}{2}.$$

(b) Using Eq. 13-38, the ratio of kinetic energies is

$$\frac{K_B}{K_A} = \frac{GmM / 2r_B}{GmM / 2r_A} = \frac{r_A}{r_B} = \frac{1}{2}.$$

(c) From Eq. 13-40, it is clear that the satellite with the largest value of r has the smallest value of $|E|$ (since r is in the denominator). And since the values of E are negative, then the smallest value of $|E|$ corresponds to the largest energy E. Thus, satellite B has the largest energy.

(d) The difference is

$$\Delta E = E_B - E_A = -\frac{GmM}{2}\left(\frac{1}{r_B} - \frac{1}{r_A}\right).$$

Being careful to convert the r values to meters, we obtain $\Delta E = 1.1 \times 10^8$ J. The mass M of Earth is found in Appendix C.

63. **THINK** We apply Kepler's laws to analyze the motion of the asteroid.

EXPRESS We use the law of periods: $T^2 = (4\pi^2/GM)r^3$, where M is the mass of the Sun (1.99×10^{30} kg) and r is the radius of the orbit. On the other hand, the kinetic energy of any asteroid or planet in a circular orbit of radius r is given by $K = GMm/2r$, where m is the mass of the asteroid or planet. We note that it is proportional to m and inversely proportional to r.

ANALYZE (a) The radius of the orbit is twice the radius of Earth's orbit: $r = 2r_{SE}$ = $2(150 \times 10^9 \text{ m}) = 300 \times 10^9$ m. Thus, the period of the asteroid is

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}} = \sqrt{\frac{4\pi^2 (300\times 10^9 \text{ m})^3}{(6.67\times 10^{-11}\text{m}^3/\text{s}^2\cdot\text{kg})(1.99\times 10^{30}\text{kg})}} = 8.96\times 10^7 \text{ s}.$$

Dividing by (365 d/y) (24 h/d) (60 min/h) (60 s/min), we obtain T = 2.8 y.

(b) The ratio of the kinetic energy of the asteroid to the kinetic energy of Earth is

$$\frac{K}{K_E} = \frac{GMm/(2r)}{GMM_E/(2r_{SE})} = \frac{m}{M_E}\cdot\frac{r_{SE}}{r} = (2.0\times 10^{-4})\left(\frac{1}{2}\right) = 1.0\times 10^{-4}.$$

LEARN An alternative way to calculate the ratio of kinetic energies is to use $K = mv^2/2$ and note that $v = 2\pi r/T$. This gives

$$\frac{K}{K_E} = \frac{mv^2/2}{M_E v_E^2/2} = \frac{m}{M_E}\left(\frac{v}{v_E}\right)^2 = \frac{m}{M_E}\left(\frac{r/T}{r_{SE}/T_E}\right)^2 = \frac{m}{M_E}\left(\frac{r}{r_{SE}}\cdot\frac{T_E}{T}\right)^2$$
$$= (2.0\times 10^{-4})\left(2\cdot\frac{1.0 \text{ y}}{2.8 \text{ y}}\right)^2 = 1.0\times 10^{-4}$$

in agreement with what we found in (b).

64. (a) Circular motion requires that the force in Newton's second law provide the necessary centripetal acceleration:

$$\frac{GmM}{r^2} = m\frac{v^2}{r}.$$

Since the left-hand side of this equation is the force given as 80 N, then we can solve for the combination mv^2 by multiplying both sides by $r = 2.0\times 10^7$ m. Thus, $mv^2 = (2.0\times 10^7$ m) (80 N) = 1.6×10^9 J. Therefore,

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(1.6\times 10^9 \text{ J}\right) = 8.0\times 10^8 \text{ J}.$$

(b) Since the gravitational force is inversely proportional to the square of the radius, then

$$\frac{F'}{F} = \left(\frac{r}{r'}\right)^2.$$

Thus, $F' = (80 \text{ N})(2/3)^2 = 36$ N.

65. (a) From Kepler's law of periods, we see that T is proportional to $r^{3/2}$.

(b) Equation 13-38 shows that K is inversely proportional to r.

(c) and (d) From the previous part, knowing that K is proportional to v^2, we find that v is proportional to $1/\sqrt{r}$. Thus, by Eq. 13-31, the angular momentum (which depends on the product rv) is proportional to $r/\sqrt{r} = \sqrt{r}$.

66. (a) The pellets will have the same speed v but opposite direction of motion, so the *relative speed* between the pellets and satellite is $2v$. Replacing v with $2v$ in Eq. 13-38 is equivalent to multiplying it by a factor of 4. Thus,

$$K_{\text{rel}} = 4\left(\frac{GM_E m}{2r}\right) = \frac{2(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)\left(5.98\times10^{24}\ \text{kg}\right)\left(0.0040\ \text{kg}\right)}{(6370+500)\times10^3\ \text{m}}$$
$$= 4.6\times10^5\ \text{J}.$$

(b) We set up the ratio of kinetic energies:

$$\frac{K_{\text{rel}}}{K_{\text{bullet}}} = \frac{4.6\times10^5\ \text{J}}{\frac{1}{2}\left(0.0040\,\text{kg}\right)\left(950\,\text{m/s}\right)^2} = 2.6\times10^2.$$

67. (a) The force acting on the satellite has magnitude GMm/r^2, where M is the mass of Earth, m is the mass of the satellite, and r is the radius of the orbit. The force points toward the center of the orbit. Since the acceleration of the satellite is v^2/r, where v is its speed, Newton's second law yields $GMm/r^2 = mv^2/r$ and the speed is given by $v = \sqrt{GM/r}$. The radius of the orbit is the sum of Earth's radius and the altitude of the satellite:

$$r = (6.37\times10^6 + 640\times10^3)\ \text{m} = 7.01\times10^6\ \text{m}.$$

Thus,

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})\left(5.98\times10^{24}\ \text{kg}\right)}{7.01\times10^6\ \text{m}}} = 7.54\times10^3\ \text{m/s}.$$

(b) The period is

$$T = 2\pi r/v = 2\pi(7.01\times10^6\ \text{m})/(7.54\times10^3\ \text{m/s}) = 5.84\times10^3\ \text{s} \approx 97\ \text{min}.$$

(c) If E_0 is the initial energy then the energy after n orbits is $E = E_0 - nC$, where $C = 1.4\times10^5$ J/orbit. For a circular orbit the energy and orbit radius are related by $E = -GMm/2r$, so the radius after n orbits is given by $r = -GMm/2E$.
The initial energy is

$$E_0 = -\frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})\left(5.98\times10^{24}\ \text{kg}\right)\left(220\ \text{kg}\right)}{2(7.01\times10^6\ \text{m})} = -6.26\times10^9\ \text{J},$$

the energy after 1500 orbits is

$$E = E_0 - nC = -6.26 \times 10^9 \text{ J} - (1500 \text{ orbit})(1.4 \times 10^5 \text{ J/orbit}) = -6.47 \times 10^9 \text{ J},$$

and the orbit radius after 1500 orbits is

$$r = -\frac{(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg})(5.98 \times 10^{24} \text{ kg})(220 \text{ kg})}{2(-6.47 \times 10^9 \text{ J})} = 6.78 \times 10^6 \text{ m}.$$

The altitude is

$$h = r - R = (6.78 \times 10^6 \text{ m} - 6.37 \times 10^6 \text{ m}) = 4.1 \times 10^5 \text{ m}.$$

Here R is the radius of Earth. This torque is internal to the satellite–Earth system, so the angular momentum of that system is conserved.

(d) The speed is

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg})(5.98 \times 10^{24} \text{ kg})}{6.78 \times 10^6 \text{ m}}} = 7.67 \times 10^3 \text{ m/s} \approx 7.7 \text{ km/s}.$$

(e) The period is

$$T = \frac{2\pi r}{v} = \frac{2\pi(6.78 \times 10^6 \text{ m})}{7.67 \times 10^3 \text{ m/s}} = 5.6 \times 10^3 \text{ s} \approx 93 \text{ min}.$$

(f) Let F be the magnitude of the average force and s be the distance traveled by the satellite. Then, the work done by the force is $W = -Fs$. This is the change in energy: $-Fs = \Delta E$. Thus, $F = -\Delta E/s$. We evaluate this expression for the first orbit. For a complete orbit $s = 2\pi r = 2\pi(7.01 \times 10^6 \text{ m}) = 4.40 \times 10^7$ m, and $\Delta E = -1.4 \times 10^5$ J. Thus,

$$F = -\frac{\Delta E}{s} = \frac{1.4 \times 10^5 \text{ J}}{4.40 \times 10^7 \text{ m}} = 3.2 \times 10^{-3} \text{ N}.$$

(g) The resistive force exerts a torque on the satellite, so its angular momentum is not conserved.

(h) The satellite–Earth system is essentially isolated, so its momentum is very nearly conserved.

68. The orbital radius is $r = R_E + h = 6370 \text{ km} + 400 \text{ km} = 6770 \text{ km} = 6.77 \times 10^6 \text{ m}$.

(a) Using Kepler's law given in Eq. 13-34, we find the period of the ships to be

$$T_0 = \sqrt{\frac{4\pi^2 r^3}{GM}} = \sqrt{\frac{4\pi^2 (6.77\times10^6\ \text{m})^3}{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(5.98\times10^{24}\ \text{kg})}} = 5.54\times10^3\ \text{s} \approx 92.3\ \text{min}.$$

(b) The speed of the ships is

$$v_0 = \frac{2\pi r}{T_0} = \frac{2\pi(6.77\times10^6\ \text{m})}{5.54\times10^3\ \text{s}} = 7.68\times10^3\ \text{m/s}^2.$$

(c) The new kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m(0.99v_0)^2 = \frac{1}{2}(2000\ \text{kg})(0.99)^2(7.68\times10^3\ \text{m/s})^2 = 5.78\times10^{10}\ \text{J}.$$

(d) Immediately after the burst, the potential energy is the same as it was before the burst. Therefore,

$$U = -\frac{GMm}{r} = -\frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(5.98\times10^{24}\ \text{kg})(2000\ \text{kg})}{6.77\times10^6\ \text{m}} = -1.18\times10^{11}\ \text{J}.$$

(e) In the new elliptical orbit, the total energy is

$$E = K + U = 5.78\times10^{10}\ \text{J} + (-1.18\times10^{11}\ \text{J}) = -6.02\times10^{10}\ \text{J}.$$

(f) For elliptical orbit, the total energy can be written as (see Eq. 13-42) $E = -GMm/2a$, where a is the semi-major axis. Thus,

$$a = -\frac{GMm}{2E} = -\frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(5.98\times10^{24}\ \text{kg})(2000\ \text{kg})}{2(-6.02\times10^{10}\ \text{J})} = 6.63\times10^6\ \text{m}.$$

(g) To find the period, we use Eq. 13-34 but replace r with a. The result is

$$T = \sqrt{\frac{4\pi^2 a^3}{GM}} = \sqrt{\frac{4\pi^2 (6.63\times10^6\ \text{m})^3}{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(5.98\times10^{24}\ \text{kg})}} = 5.37\times10^3\ \text{s} \approx 89.5\ \text{min}.$$

(h) The orbital period T for Picard's elliptical orbit is shorter than Igor's by

$$\Delta T = T_0 - T = 5540\ \text{s} - 5370\ \text{s} = 170\ \text{s}.$$

Thus, Picard will arrive back at point P ahead of Igor by 170 s – 90 s = 80 s.

69. We define the "effective gravity" in his environment as g_{eff} = 220/60 = 3.67 m/s^2. Thus, using equations from Chapter 2 (and selecting downward as the positive direction), we find the "fall-time" to be

$$\Delta y = v_0 t + \frac{1}{2} g_{eff} t^2 \;\Rightarrow t = \sqrt{\frac{2(2.1\text{ m})}{3.67\text{ m/s}^2}} = 1.1\text{ s}.$$

70. (a) The gravitational acceleration a_g is defined in Eq. 13-11. The problem is concerned with the difference between a_g evaluated at $r = 50R_\text{h}$ and a_g evaluated at $r = 50R_\text{h} + h$ (where h is the estimate of your height). Assuming h is much smaller than $50R_\text{h}$ then we can approximate h as the dr that is present when we consider the differential of Eq. 13-11:

$$|da_g| = \frac{2GM}{r^3}\,dr \;\approx\; \frac{2GM}{50^3 R_\text{h}^3}\,h \;= \frac{2GM}{50^3 (2GM/c^2)^3}\,h\,.$$

If we approximate $|da_g| = 10\text{ m/s}^2$ and $h \approx 1.5$ m, we can solve this for M. Giving our results in terms of the Sun's mass means dividing our result for M by 2×10^{30} kg. Thus, admitting some tolerance into our estimate of h we find the "critical" black hole mass should in the range of 105 to 125 solar masses.

(b) Interestingly, this turns out to be lower limit (which will surprise many students) since the above expression shows $|da_g|$ is inversely proportional to M. It should perhaps be emphasized that a distance of $50R_\text{h}$ from a small black hole is much smaller than a distance of $50R_\text{h}$ from a large black hole.

71. (a) All points on the ring are the same distance ($r = \sqrt{x^2 + R^2}$) from the particle, so the gravitational potential energy is simply $U = -GMm/\sqrt{x^2 + R^2}$, from Eq. 13-21. The corresponding force (by symmetry) is expected to be along the x axis, so we take a (negative) derivative of U (with respect to x) to obtain it (see Eq. 8-20). The result for the magnitude of the force is $GMmx(x^2 + R^2)^{-3/2}$.

(b) Using our expression for U, the change in potential energy as the particle falls to the center is

$$\Delta U = -GMm\left(\frac{1}{R} - \frac{1}{\sqrt{x^2 + R^2}}\right)$$

By conservation of mechanical energy, this must "turn into" kinetic energy, $\Delta K = -\Delta U = mv^2/2$. We solve for the speed and obtain

$$\frac{1}{2}mv^2 = GMm\left(\frac{1}{R} - \frac{1}{\sqrt{x^2 + R^2}}\right) \;\Rightarrow\; v = \sqrt{2GM\left(\frac{1}{R} - \frac{1}{\sqrt{x^2 + R^2}}\right)}.$$

72. (a) With $M = 2.0\times10^{30}$ kg and $r = 10000$ m, we find $a_g = \dfrac{GM}{r^2} = 1.3\times10^{12}\text{ m/s}^2$.

(b) Although a close answer may be gotten by using the constant acceleration equations of Chapter 2, we show the more general approach (using energy conservation):

$$K_o + U_o = K + U$$

where $K_o = 0$, $K = \frac{1}{2}mv^2$, and U is given by Eq. 13-21. Thus, with $r_o = 10001$ m, we find

$$v = \sqrt{2GM\left(\frac{1}{r} - \frac{1}{r_o}\right)} = 1.6 \times 10^6 \text{ m/s} .$$

73. Using energy conservation (and Eq. 13-21) we have

$$K_1 - \frac{GMm}{r_1} = K_2 - \frac{GMm}{r_2} .$$

(a) Plugging in two pairs of values (for (K_1, r_1) and (K_2, r_2)) from the graph and using the value of G and M (for Earth) given in the book, we find $m \approx 1.0 \times 10^3$ kg.

(b) Similarly, $v = (2K/m)^{1/2} \approx 1.5 \times 10^3$ m/s (at $r = 1.945 \times 10^7$ m).

74. We estimate the planet to have radius $r = 10$ m. To estimate the mass m of the planet, we require its density equal that of Earth (and use the fact that the volume of a sphere is $4\pi r^3/3$):

$$\frac{m}{4\pi r^3/3} = \frac{M_E}{4\pi R_E^3/3} \quad \Rightarrow \quad m = M_E\left(\frac{r}{R_E}\right)^3$$

which yields (with $M_E \approx 6 \times 10^{24}$ kg and $R_E \approx 6.4 \times 10^6$ m) $m = 2.3 \times 10^7$ kg.

(a) With the above assumptions, the acceleration due to gravity is

$$a_g = \frac{Gm}{r^2} = \frac{\left(6.7\times10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg}\right)\left(2.3\times10^7 \text{ kg}\right)}{(10 \text{ m})^2} = 1.5\times10^{-5} \text{ m/s}^2 \approx 2\times10^{-5} \text{ m/s}^2 .$$

(b) Equation 13-28 gives the escape speed: $v = \sqrt{\frac{2Gm}{r}} \approx 0.02$ m/s .

75. We use m_1 for the 20 kg of the sphere at $(x_1, y_1) = (0.5, 1.0)$ (SI units understood), m_2 for the 40 kg of the sphere at $(x_2, y_2) = (-1.0, -1.0)$, and m_3 for the 60 kg of the sphere at $(x_3, y_3) = (0, -0.5)$. The mass of the 20 kg object at the origin is simply denoted m. We note that $r_1 = \sqrt{1.25}$, $r_2 = \sqrt{2}$, and $r_3 = 0.5$ (again, with SI units understood). The force $\vec{F}_n$ that the n^{th} sphere exerts on m has magnitude $Gm_n m / r_n^2$ and is directed from the origin toward m_n, so that it is conveniently written as

$$\vec{F}_n = \frac{Gm_n m}{r_n^2}\left(\frac{x_n}{r_n}\hat{\mathrm{i}} + \frac{y_n}{r_n}\hat{\mathrm{j}}\right) = \frac{Gm_n m}{r_n^3}\left(x_n\hat{\mathrm{i}} + y_n\hat{\mathrm{j}}\right).$$

Consequently, the vector addition to obtain the net force on m becomes

$$\vec{F}_{\text{net}} = \sum_{n=1}^{3}\vec{F}_n = Gm\left(\left(\sum_{n=1}^{3}\frac{m_n x_n}{r_n^3}\right)\hat{\mathrm{i}} + \left(\sum_{n=1}^{3}\frac{m_n y_n}{r_n^3}\right)\hat{\mathrm{j}}\right) = (-9.3\times10^{-9}\ \text{N})\hat{\mathrm{i}} - (3.2\times10^{-7}\ \text{N})\hat{\mathrm{j}}\ .$$

Therefore, we find the net force magnitude is $\left|\vec{F}_{\text{net}}\right| = 3.2\times10^{-7}\,\text{N}$.

76. **THINK** We apply Newton's law of gravitation to calculate the force between the meteor and the satellite.

EXPRESS We use $F = Gm_s m_m/r^2$, where m_s is the mass of the satellite, m_m is the mass of the meteor, and r is the distance between their centers. The distance between centers is $r = R + d = 15\ \text{m} + 3\ \text{m} = 18\ \text{m}$. Here R is the radius of the satellite and d is the distance from its surface to the center of the meteor.

ANALYZE The gravitational force between the meteor and the satellite is

$$F = \frac{Gm_s m_s}{r^2} = \frac{\left(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)(20\text{kg})(7.0\text{kg})}{(18\text{m})^2} = 2.9\times10^{-11}\,\text{N}.$$

LEARN The force of gravitation is inversely proportional to r^2.

77. We note that r_A (the distance from the origin to sphere A, which is the same as the separation between A and B) is 0.5, $r_C = 0.8$, and $r_D = 0.4$ (with SI units understood). The force $\vec{F}_k$ that the k^{th} sphere exerts on m_B has magnitude $Gm_k m_B / r_k^2$ and is directed from the origin toward m_k so that it is conveniently written as

$$\vec{F}_k = \frac{Gm_k m_B}{r_k^2}\left(\frac{x_k}{r_k}\hat{\mathrm{i}} + \frac{y_k}{r_k}\hat{\mathrm{j}}\right) = \frac{Gm_k m_B}{r_k^3}\left(x_k\hat{\mathrm{i}} + y_k\hat{\mathrm{j}}\right).$$

Consequently, the vector addition (where k equals A, B, and D) to obtain the net force on m_B becomes

$$\vec{F}_{\text{net}} = \sum_k \vec{F}_k = Gm_B\left(\left(\sum_k \frac{m_k x_k}{r_k^3}\right)\hat{\mathrm{i}} + \left(\sum_k \frac{m_k y_k}{r_k^3}\right)\hat{\mathrm{j}}\right) = (3.7\times10^{-5}\,\text{N})\hat{\mathrm{j}}.$$

78. (a) We note that r_C (the distance from the origin to sphere C, which is the same as the separation between C and B) is 0.8, $r_D = 0.4$, and the separation between spheres C and D is $r_{CD} = 1.2$ (with SI units understood). The total potential energy is therefore

$$-\frac{GM_BM_C}{r_C^2}-\frac{GM_BM_D}{r_D^2}-\frac{GM_CM_D}{r_{CD}^2}=-1.3\times10^{-4}\ \text{J}$$

using the mass-values given in the previous problem.

(b) Since any gravitational potential energy term (of the sort considered in this chapter) is necessarily negative ($-GmM/r^2$ where all variables are positive) then having another mass to include in the computation can only lower the result (that is, make the result more negative).

(c) The observation in the previous part implies that the work I do in removing sphere *A* (to obtain the case considered in part (a)) must lead to an increase in the system energy; thus, I do positive work.

(d) To put sphere *A* back in, I do negative work, since I am causing the system energy to become more negative.

79. **THINK** Since the orbit is circular, the net gravitational force on the smaller star is equal to the centripetal force.

EXPRESS The magnitude of the net gravitational force on one of the smaller stars (of mass *m*) is

$$F=\frac{GMm}{r^2}+\frac{Gmm}{(2r)^2}=\frac{Gm}{r^2}\left(M+\frac{m}{4}\right).$$

This supplies the centripetal force needed for the motion of the star:

$$\frac{Gm}{r^2}\left(M+\frac{m}{4}\right)=m\frac{v^2}{r}$$

where $v=2\pi r/T$. Combining the two expressions allows us to solve for *T*.

ANALYZE Plugging in for speed *v*, we arrive at an equation for the period *T*:

$$T=\frac{2\pi r^{3/2}}{\sqrt{G(M+m/4)}}.$$

LEARN In the limit where $m \ll M$, we recover the expected result $T=\dfrac{2\pi r^{3/2}}{\sqrt{GM}}$ for two bodies.

80. If the angular velocity were any greater, loose objects on the surface would not go around with the planet but would travel out into space.

(a) The magnitude of the gravitational force exerted by the planet on an object of mass m at its surface is given by $F = GmM / R^2$, where M is the mass of the planet and R is its radius. According to Newton's second law this must equal mv^2 / R, where v is the speed of the object. Thus,

$$\frac{GM}{R^2} = \frac{v^2}{R}.$$

With $M = 4\pi\rho R^3/3$ where ρ is the density of the planet, and $v = 2\pi R/T$, where T is the period of revolution, we find

$$\frac{4\pi}{3} G\rho R = \frac{4\pi^2 R}{T^2}.$$

We solve for T and obtain

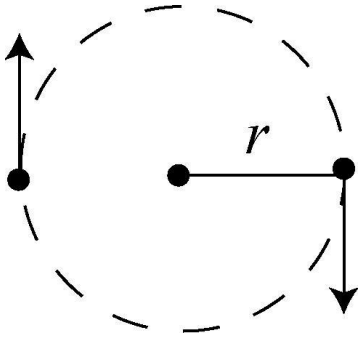
$$T = \sqrt{\frac{3\pi}{G\rho}}.$$

(b) The density is 3.0×10^3 kg/m^3. We evaluate the equation for T:

$$T = \sqrt{\frac{3\pi}{\left(6.67\times10^{-11}\,\mathrm{m^3/s^2\cdot kg}\right)\left(3.0\times10^3\,\mathrm{kg/m^3}\right)}} = 6.86\times10^3\,\mathrm{s} = 1.9\,\mathrm{h}.$$

81. **THINK** In a two-star system, the stars rotate about their common center of mass.

EXPRESS The situation is depicted on the right. The gravitational force between the two stars (each having a mass M) is



$$F_g = \frac{GM^2}{(2r)^2} = \frac{GM^2}{4r^2}$$

The gravitational force between the stars provides the centripetal force necessary to keep their orbits circular. Thus, writing the centripetal acceleration as $r\omega^2$ where ω is the angular speed, we have

$$F_g = F_c \quad \Rightarrow \quad \frac{GM^2}{4r^2} = Mr\omega^2.$$

ANALYZE (a) Substituting the values given, we find the common angular speed to be

$$\omega = \frac{1}{2}\sqrt{\frac{GM}{r^3}} = \frac{1}{2}\sqrt{\frac{(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})(3.0\times10^{30}\ \mathrm{kg})}{(1.0\times10^{11}\ \mathrm{m})^3}} = 2.2\times10^{-7}\,\mathrm{rad/s}.$$

(b) To barely escape means to have total energy equal to zero (see discussion prior to Eq. 13-28). If m is the mass of the meteoroid, then

$$\frac{1}{2}mv^2 - \frac{GmM}{r} - \frac{GmM}{r} = 0 \;\Rightarrow\; v = \sqrt{\frac{4GM}{r}} = 8.9\times10^4 \text{ m/s}.$$

LEARN Comparing with Eq. 13-28, we see that the escape speed of the two-star system is the same as that of a star with mass $2M$.

82. The key point here is that angular momentum is conserved:

$$I_p\omega_p = I_a\omega_a$$

which leads to $\omega_p = (r_a / r_p)^2 \omega_a$, but $r_p = 2a - r_a$ where a is determined by Eq. 13-34 (particularly, see the paragraph after that equation in the textbook). Therefore,

$$\omega_p = \frac{r_a{}^2\,\omega_a}{(2(GMT^2/4\pi^2)^{1/3} - r_a)^2} = 9.24\times10^{-5} \text{ rad/s}.$$

83. **THINK** The orbit of the shuttle goes from circular to elliptical after changing its speed by firing the thrusters.

EXPRESS We first use the law of periods: $T^2 = (4\pi^2/GM)r^3$, where M is the mass of the planet and r is the radius of the orbit. After the orbit of the shuttle turns elliptical by firing the thrusters to reduce its speed, the semi-major axis is $a = -GMm/2E$, where $E = K + U$ is the mechanical energy of the shuttle and its new period becomes $T' = \sqrt{4\pi^2 a^3 / GM}$.

ANALYZE (a) Using Kepler's law of periods, we find the period to be

$$T = \sqrt{\left(\frac{4\pi^2}{GM}\right) r^3} = \sqrt{\frac{4\pi^2(4.20\times10^7 \text{ m})^3}{(6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(9.50\times10^{25} \text{ kg})}} = 2.15\times10^4 \text{ s}.$$

(b) The speed is constant (before she fires the thrusters), so

$$v_0 = \frac{2\pi r}{T} = \frac{2\pi(4.20\times10^7 \text{m})}{2.15\times10^4 \text{ s}} = 1.23\times10^4 \text{ m/s}.$$

(c) A two percent reduction in the previous value gives

$$v = 0.98v_0 = 0.98(1.23\times10^4 \text{ m/s}) = 1.20\times10^4 \text{ m/s}.$$

(d) The kinetic energy is $K = \frac{1}{2}mv^2 = \frac{1}{2}(3000\,\text{kg})(1.20\times10^4\ \text{m/s})^2 = 2.17\times10^{11}\ \text{J}$.

(e) Immediately after the firing, the potential energy is the same as it was before firing the thruster:

$$U = -\frac{GMm}{r} = -\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(9.50\times10^{25}\ \text{kg})(3000\,\text{kg})}{4.20\times10^7\ \text{m}} = -4.53\times10^{11}\ \text{J}.$$

(f) Adding these two results gives the total mechanical energy:

$$E = K + U = 2.17\times10^{11}\ \text{J} + (-4.53\times10^{11}\ \text{J}) = -2.35\times10^{11}\ \text{J}.$$

(g) Using Eq. 13-42, we find the semi-major axis to be

$$a = -\frac{GMm}{2E} = -\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(9.50\times10^{25}\ \text{kg})(3000\,\text{kg})}{2(-2.35\times10^{11}\ \text{J})} = 4.04\times10^7\ \text{m}.$$

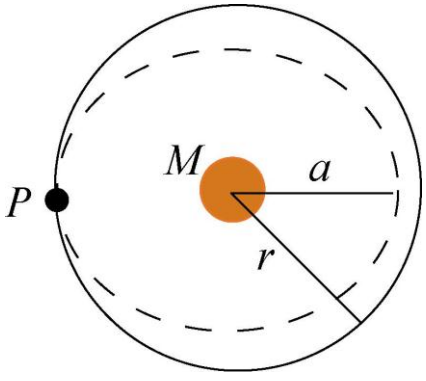
(h) Using Kepler's law of periods for elliptical orbits (using a instead of r) we find the new period to be

$$T' = \sqrt{\left(\frac{4\pi^2}{GM}\right)a^3} = \sqrt{\frac{4\pi^2(4.04\times10^7\ \text{m})^3}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(9.50\times10^{25}\ \text{kg})}} = 2.03\times10^4\ \text{s}.$$

This is smaller than our result for part (a) by $T - T' = 1.22\times10^3$ s.

(i) Comparing the results in (a) and (h), we see that elliptical orbit has a smaller period.

LEARN The orbits of the shuttle before and after firing the thruster are shown below. Point P corresponds to the location where the thruster was fired.



84. The difference between free-fall acceleration g and the gravitational acceleration a_g at the equator of the star is (see Equation 13.14):

$$a_g - g = \omega^2 R$$

where

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.041\,\text{s}} = 153\,\text{rad/s}$$

is the angular speed of the star. The gravitational acceleration at the equator is

$$a_g = \frac{GM}{R^2} = \frac{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(1.98\times10^{30}\ \text{kg})}{(1.2\times10^4\ \text{m})^2} = 9.17\times10^{11}\ \text{m/s}^2.$$

Therefore, the percentage difference is

$$\frac{a_g - g}{a_g} = \frac{\omega^2 R}{a_g} = \frac{(153\,\text{rad/s})^2(1.2\times10^4\ \text{m})}{9.17\times10^{11}\ \text{m/s}^2} = 3.06\times10^{-4} \approx 0.031\%.$$

85. Energy conservation for this situation may be expressed as follows:

$$K_1 + U_1 = K_2 + U_2 \quad \Rightarrow \quad \frac{1}{2}mv_1^2 - \frac{GmM}{r_1} = \frac{1}{2}mv_2^2 - \frac{GmM}{r_2}$$

where $M = 5.98 \times 10^{24}$ kg, $r_1 = R = 6.37 \times 10^6$, m and $v_1 = 10000$ m/s. Setting $v_2 = 0$ to find the maximum of its trajectory, we solve the above equation (noting that m cancels in the process) and obtain $r_2 = 3.2 \times 10^7$ m. This implies that its *altitude* is

$$h = r_2 - R = 2.5 \times 10^7 \text{ m}.$$

86. We note that, since $v = 2\pi r/T$, the centripetal acceleration may be written as $a = 4\pi^2 r/T^2$. To express the result in terms of g, we divide by 9.8 m/s^2.

(a) The acceleration associated with Earth's spin (T = 24 h = 86400 s) is

$$a = g\frac{4\pi^2(6.37\times10^6\ \text{m})}{(86400\,\text{s})^2(9.8\,\text{m/s}^2)} = 3.4\times10^{-3}\,g\ .$$

(b) The acceleration associated with Earth's motion around the Sun (T = 1 y = 3.156×10^7 s) is

$$a = g\frac{4\pi^2(1.5\times10^{11}\ \text{m})}{(3.156\times10^7\ \text{s})^2(9.8\,\text{m/s}^2)} = 6.1\times10^{-4}\,g\ .$$

(c) The acceleration associated with the Solar System's motion around the galactic center ($T = 2.5 \times 10^8$ y = 7.9×10^{15} s) is

$$a = g\frac{4\pi^2(2.2\times10^{20}\text{ m})}{(7.9\times10^{15}\text{ s})^2(9.8\text{ m/s}^2)} = 1.4\times10^{-11}g\ .$$

87. (a) It is possible to use $v^2 = v_0^2 + 2a\Delta y$ as we did for free-fall problems in Chapter 2 because the acceleration can be considered approximately constant over this interval. However, our approach will not assume constant acceleration; we use energy conservation:

$$\frac{1}{2}mv_0^2 - \frac{GMm}{r_0} = \frac{1}{2}mv^2 - \frac{GMm}{r} \ \Rightarrow\ v = \sqrt{\frac{2GM(r_0 - r)}{r_0 r}}$$

which yields $v = 1.4 \times 10^6$ m/s.

(b) We estimate the height of the apple to be $h = 7$ cm $= 0.07$ m. We may find the answer by evaluating Eq. 13-11 at the surface (radius r in part (a)) and at radius $r + h$, being careful not to round off, and then taking the difference of the two values, or we may take the differential of that equation — setting dr equal to h. We illustrate the latter procedure:

$$|da_g| = \left|-2\frac{GM}{r^3}dr\right| \approx 2\frac{GM}{r^3}h = 3\times10^6\text{ m/s}^2\ .$$

88. We apply the work-energy theorem to the object in question. It starts from a point at the surface of the Earth with zero initial speed and arrives at the center of the Earth with final speed v_f. The corresponding increase in its kinetic energy, $\frac{1}{2}mv_f^2$, is equal to the work done on it by Earth's gravity: $\int F\,dr = \int(-Kr)dr$. Thus,

$$\frac{1}{2}mv_f^2 = \int_R^0 F\,dr = \int_R^0(-Kr)\,dr = \frac{1}{2}KR^2$$

where R is the radius of Earth. Solving for the final speed, we obtain $v_f = R\sqrt{K/m}$. We note that the acceleration of gravity $a_g = g = 9.8$ m/s^2 on the surface of Earth is given by

$$a_g = GM/R^2 = G(4\pi R^3/3)\rho/R^2,$$

where ρ is Earth's average density. This permits us to write $K/m = 4\pi G\rho/3 = g/R$. Consequently,

$$v_f = R\sqrt{\frac{K}{m}} = R\sqrt{\frac{g}{R}} = \sqrt{gR} = \sqrt{(9.8\text{ m/s}^2)(6.37\times10^6\text{ m})} = 7.9\times10^3\text{ m/s}\ .$$

89. **THINK** To compare the kinetic energy, potential energy, and the speed of the Earth at aphelion (farthest distance) and perihelion (closest distance), we apply both conservation of energy and conservation of angular momentum.

EXPRESS As Earth orbits about the Sun, its total energy is conserved:

$$\frac{1}{2}mv_a^2 - \frac{GM_SM_E}{R_a} = \frac{1}{2}mv_p^2 - \frac{GM_SM_E}{R_p}.$$

In addition, angular momentum conservation implies $v_aR_a = v_pR_p$.

ANALYZE (a) The total energy is conserved, so there is no difference between its values at aphelion and perihelion.

(b) The difference in potential energy is

$$\Delta U = U_a - U_p = -GM_SM_E\left(\frac{1}{R_a} - \frac{1}{R_p}\right)$$

$$= -(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})(5.98\times10^{24}\ \text{kg})\left(\frac{1}{1.52\times10^{11}\ \text{m}} - \frac{1}{1.47\times10^{11}\ \text{m}}\right)$$

$$\approx 1.8\times10^{32}\ \text{J}.$$

(c) Since $\Delta K + \Delta U = 0$, $\Delta K = K_a - K_p = -\Delta U \approx -1.8\times10^{32}$ J.

(d) With $v_aR_a = v_pR_p$, the change in kinetic energy may be written as

$$\Delta K = K_a - K_p = \frac{1}{2}M_E\left(v_a^2 - v_p^2\right) = \frac{1}{2}M_Ev_a^2\left(1 - \frac{R_a^2}{R_p^2}\right)$$

from which we find the speed at the aphelion to be

$$v_a = \sqrt{\frac{2(\Delta K)}{M_E(1 - R_a^2/R_p^2)}} = 2.95\times10^4\ \text{m/s}.$$

Thus, the variation in speed is

$$\Delta v = v_a - v_p = \left(1 - \frac{R_a}{R_p}\right)v_a = \left(1 - \frac{1.52\times10^{11}\ \text{m}}{1.47\times10^{11}\ \text{m}}\right)(2.95\times10^4\ \text{m/s})$$

$$= -0.99\times10^3\ \text{m/s} = -0.99\ \text{km/s}.$$

The speed at the aphelion is smaller than that at the perihelion.

LEARN Since the changes are small, the problem could also be solved by using differentials:

$$dU = \left(\frac{GM_E M_S}{r^2}\right) dr \approx \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\ \text{kg}\right)\left(5.98\times10^{24}\ \text{kg}\right)}{\left(1.5\times10^{11}\ \text{m}\right)^2}\left(5\times10^{9}\ \text{m}\right).$$

This yields $\Delta U \approx 1.8 \times 10^{32}$ J. Similarly, with $\Delta K \approx dK = M_E v\, dv$, where $v \approx 2\pi R/T$, we have

$$1.8\times10^{32}\ \text{J} \approx \left(5.98\times10^{24}\ \text{kg}\right)\left(\frac{2\pi\left(1.5\times10^{11}\ \text{m}\right)}{3.156\times10^{7}\ \text{s}}\right)\Delta v$$

which yields a difference of $\Delta v \approx 0.99$ km/s in Earth's speed (relative to the Sun) between aphelion and perihelion.

90. (a) Because it is moving in a circular orbit, F/m must equal the centripetal acceleration:

$$\frac{80\ \text{N}}{50\ \text{kg}} = \frac{v^2}{r}.$$

However, $v = 2\pi r/T$, where $T = 21600$ s, so we are led to

$$1.6\,\text{m/s}^2 = \frac{4\pi^2}{T^2} r$$

which yields $r = 1.9 \times 10^7$ m.

(b) From the above calculation, we infer $v^2 = (1.6\ \text{m/s}^2)r$, which leads to $v^2 = 3.0 \times 10^7\ \text{m}^2/\text{s}^2$. Thus, $K = \frac{1}{2}mv^2 = 7.6 \times 10^8$ J.

(c) As discussed in Section 13-4, F/m also tells us the gravitational acceleration:

$$a_g = 1.6\ \text{m/s}^2 = \frac{GM}{r^2}.$$

We therefore find $M = 8.6 \times 10^{24}$ kg.

91. (a) Their initial potential energy is $-Gm^2/R_i$ and they started from rest, so energy conservation leads to

$$-\frac{Gm^2}{R_i} = K_{\text{total}} - \frac{Gm^2}{0.5R_i} \Rightarrow K_{\text{total}} = \frac{Gm^2}{R_i}.$$

(b) They have equal mass, and this is being viewed in the center-of-mass frame, so their speeds are identical and their kinetic energies are the same. Thus,

$$K = \frac{1}{2}K_{\text{total}} = \frac{Gm^2}{2R_i}.$$

(c) With $K = \frac{1}{2}mv^2$, we solve the above equation and find $v = \sqrt{Gm/R_i}$.

(d) Their relative speed is $2v = 2\sqrt{Gm/R_i}$. This is the (instantaneous) rate at which the gap between them is closing.

(e) The premise of this part is that we assume we are not moving (that is, that body A acquires no kinetic energy in the process). Thus, $K_{\text{total}} = K_B$, and the logic of part (a) leads to $K_B = Gm^2/R_i$.

(f) And $\frac{1}{2}mv_B^2 = K_B$ yields $v_B = \sqrt{2Gm/R_i}$.

(g) The answer to part (f) is incorrect, due to having ignored the accelerated motion of "our" frame (that of body A). Our computations were therefore carried out in a noninertial frame of reference, for which the energy equations of Chapter 8 are not directly applicable.

92. (a) We note that the altitude of the rocket is $h = R - R_E$ where $R_E = 6.37\times10^6$ m. With $M = 5.98\times10^{24}$ kg, $R_0 = R_E + h_0 = 6.57 \times 10^6$ m and $R = 7.37 \times 10^6$ m, we have

$$K_i + U_i = K + U \Rightarrow \frac{1}{2}m\,(3.70\times10^3\ \text{m/s})^2 - \frac{GmM}{R_0} = K - \frac{GmM}{R},$$

which yields $K = 3.83 \times 10^7$ J.

(b) Again, we use energy conservation.

$$K_i + U_i = K_f + U_f \Rightarrow \frac{1}{2}m\,(3.70\times10^3)^2 - \frac{GmM}{R_0} = 0 - \frac{GmM}{R_f}$$

Therefore, we find $R_f = 7.40 \times 10^6$ m. This corresponds to a distance of 1034.9 km ≈ 1.03 $\times 10^3$ km above the Earth's surface.

93. Energy conservation for this situation may be expressed as follows:

$$K_1 + U_1 = K_2 + U_2 \Rightarrow \frac{1}{2}mv_1^2 - \frac{GmM}{r_1} = \frac{1}{2}mv_2^2 - \frac{GmM}{r_2}$$

where $M = 7.0 \times 10^{24}$ kg, $r_2 = R = 1.6 \times 10^6$ m, and $r_1 = \infty$ (which means that $U_1 = 0$). We are told to assume the meteor starts at rest, so $v_1 = 0$. Thus, $K_1 + U_1 = 0$, and the above equation is rewritten as

$$\frac{1}{2}mv_2^2 - \frac{GmM}{r_2} \quad \Rightarrow \quad v_2 = \sqrt{\frac{2GM}{R}} = 2.4 \times 10^4 \text{ m/s}.$$

94. The initial distance from each fixed sphere to the ball is $r_0 = \infty$, which implies the initial gravitational potential energy is zero. The distance from each fixed sphere to the ball when it is at $x = 0.30$ m is $r = 0.50$ m, by the Pythagorean theorem.

(a) With $M = 20$ kg and $m = 10$ kg, energy conservation leads to

$$K_i + U_i = K + U \Rightarrow 0 + 0 = K - 2\frac{GmM}{r}$$

which yields $K = 2GmM/r = 5.3 \times 10^{-8}$ J.

(b) Since the y-component of each force will cancel, the net force points in the $-x$ direction, with a magnitude

$$2F_x = 2\,(GmM/r^2)\cos\theta,$$

where $\theta = \tan^{-1}(4/3) = 53°$. Thus, the result is $\vec{F}_{\text{net}} = (-6.4 \times 10^{-8} \text{ N})\hat{\text{i}}$.

95. The magnitudes of the individual forces (acting on m_C, exerted by m_A and m_B, respectively) are

$$F_{AC} = \frac{Gm_A m_C}{r_{AC}^2} = 2.7 \times 10^{-8} \text{ N} \quad \text{and} \quad F_{BC} = \frac{Gm_B m_C}{r_{BC}^2} = 3.6 \times 10^{-8} \text{ N}$$

where $r_{AC} = 0.20$ m and $r_{BC} = 0.15$ m. With $r_{AB} = 0.25$ m, the angle $\vec{F}_A$ makes with the x axis can be obtained as

$$\theta_A = \pi + \cos^{-1}\left(\frac{r_{AC}^2 + r_{AB}^2 - r_{BC}^2}{2r_{AC}r_{AB}}\right) = \pi + \cos^{-1}(0.80) = 217°.$$

Similarly, the angle $\vec{F}_B$ makes with the x axis can be obtained as

$$\theta_B = -\cos^{-1}\left(\frac{r_{AB}^2 + r_{BC}^2 - r_{AC}^2}{2r_{AB}r_{BC}}\right) = -\cos^{-1}(0.60) = -53°.$$

The net force acting on m_C then becomes

$$\begin{aligned}\vec{F}_C &= F_{AC}(\cos\theta_A\,\hat{\mathrm{i}} + \sin\theta_A\,\hat{\mathrm{j}}) + F_{BC}(\cos\theta_B\,\hat{\mathrm{i}} + \sin\theta_B\,\hat{\mathrm{j}}) \\ &= (F_{AC}\cos\theta_A + F_{BC}\cos\theta_B)\hat{\mathrm{i}} + (F_{AC}\sin\theta_A + F_{BC}\sin\theta_B)\hat{\mathrm{j}} \\ &= (-4.4\times10^{-8}\ \mathrm{N})\hat{\mathrm{j}}.\end{aligned}$$

96. (a) From Chapter 2, we have $v^2 = v_0^2 + 2a\Delta x$, where a may be interpreted as an average acceleration in cases where the acceleration is not uniform. With $v_0 = 0$, $v =$ 11000 m/s, and $\Delta x = 220$ m, we find $a = 2.75 \times 10^5$ m/s^2. Therefore,

$$a = \left(\frac{2.75\times10^5\ \mathrm{m/s^2}}{9.8\ \mathrm{m/s^2}}\right) g = 2.8\times10^4\, g\,.$$

(b) The acceleration is certainly deadly enough to kill the passengers.

(c) Again using $v^2 = v_0^2 + 2a\Delta x$, we find

$$a = \frac{(7000\ \mathrm{m/s})^2}{2(3500\ \mathrm{m})} = 7000\ \mathrm{m/s^2} = 714g\ .$$

(d) Energy conservation gives the craft's speed v (in the absence of friction and other dissipative effects) at altitude $h = 700$ km after being launched from $R = 6.37 \times 10^6$ m (the surface of Earth) with speed $v_0 = 7000$ m/s. That altitude corresponds to a distance from Earth's center of $r = R + h = 7.07 \times 10^6$ m.

$$\frac{1}{2}mv_0^2 - \frac{GMm}{R} = \frac{1}{2}mv^2 - \frac{GMm}{r}\ .$$

With $M = 5.98 \times 10^{24}$ kg (the mass of Earth) we find $v = 6.05 \times 10^3$ m/s. However, to orbit at that radius requires (by Eq. 13-37)

$$v' = \sqrt{GM/r}\ = 7.51\times10^3\ \mathrm{m/s}.$$

The difference between these two speeds is $v' - v = 1.46 \times 10^3$ m/s $\approx 1.5\times10^3$ m/s, which presumably is accounted for by the action of the rocket engine.

97. We integrate Eq. 13-1 with respect to r from $3R_E$ to $4R_E$ and obtain the work equal to

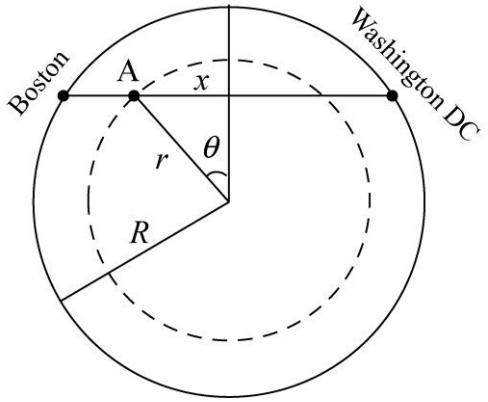
$$W = -\Delta U = -GM_E m\left(\frac{1}{4R_E} - \frac{1}{3R_E}\right) = \frac{GM_E m}{12R_E}\ .$$

98. The gravitational force at a radial distance r inside Earth (e.g., point A in the figure) is

$$F_g = -\frac{GMm}{R^3} r$$

The component of the force along the tunnel is

$$F_x = F_g \sin\theta = \left(-\frac{GMm}{R^3} r\right)\frac{x}{r} = -\frac{GMm}{R^3} x$$



which can be rewritten as

$$a_x = \frac{d^2x}{dt^2} - \frac{GM}{R^3} x = -\omega^2 x$$

where $\omega^2 = GM/R^3$. The equation is similar to Hooke's law, in that the force on the train is proportional to the displacement of the train but oppositely directed. Without exiting the tunnel, the motion of the train would be periodic would a period given by $T = 2\pi/\omega$. The travel time required from Boston to Washington DC is only half that (one-way):

$$\Delta t = \frac{T}{2} = \frac{\pi}{\omega} = \pi\sqrt{\frac{R^3}{GM}} = \pi\sqrt{\frac{(6.37\times10^6\,\text{m})^3}{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(5.98\times10^{24}\ \text{kg})}} = 2529\ \text{s} = 42.1\ \text{min}$$

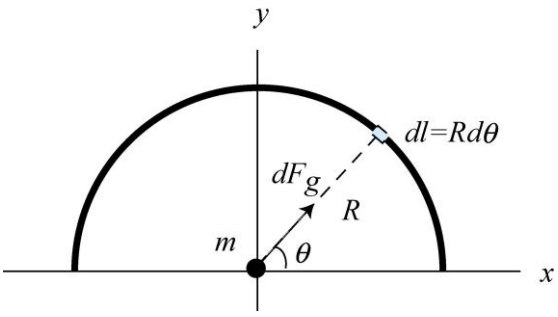
Note that the result is independent of the distance between the two cities.

99. The gravitational force exerted on *m* due to a mass element *dM* from the thin rod is

$$dF_g = \frac{Gm(dM)}{R^2}$$

By symmetry, the force is along the *y*-direction. With

$$dM = \lambda dl = \left(\frac{M}{\pi R}\right) R d\theta = \frac{M}{\pi} d\theta$$



where $\lambda = M/\pi R$ is the mass density (mass per unit length), we have

$$dF_{g,y} = dF_g \sin\theta = \frac{Gm}{R^2}\left(\frac{Md\theta}{\pi}\right)\sin\theta = \frac{GMm}{\pi R^2}\sin\theta d\theta$$

Integrating over θ gives

$$F_{g,y} = \int_0^\pi \frac{GMm}{\pi R^2}\sin\theta d\theta = \frac{GMm}{\pi R^2}\int_0^\pi \sin\theta d\theta = \frac{2GMm}{\pi R^2}$$

Substituting the values given leads to

$$F_{g,y} = \frac{2GMm}{\pi R^2} = \frac{2(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(5.0\ \text{kg})(3.0\times10^{-3}\ \text{kg})}{\pi(0.650\ \text{m})^2} = 1.51\times10^{-12}\ \text{N}$$

If the rod were a complete circle, by symmetry, the net force on the particle would be zero.

100. The gravitational acceleration at a distance r from the center of Earth is

$$a_g = \frac{GM}{r^2}$$

Thus, the weight difference between the two objects is

$$\Delta w = m(g - a_g) = \frac{GMm}{R^2} - \frac{GMm}{(R+h)^2} = \frac{GMm}{R^2}\left[1-(1+h/R)^{-2}\right] \approx \frac{GMm}{R^2}\cdot\frac{2h}{R} = \frac{2GMmh}{R^3}$$

With $M = \frac{4}{3}\pi R^3\rho,$ the above expression can be rewritten as

$$\Delta w = \frac{2GMmh}{R^3} = \frac{2Gmh}{R^3}\cdot\left(\frac{4\pi}{3}R^3\rho\right) = \frac{8\pi\rho Gmh}{3}$$

Substituting the values given, we obtain

$$\Delta w = \frac{8\pi\rho Gmh}{3} = \frac{8\pi}{3}(5.5\times10^3\ \text{kg/m}^3)(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(2.00\ \text{kg})(0.050\ \text{m})$$
$$= 3.07\times10^{-7}\ \text{N}$$

101. Let the distance from Earth to the spaceship be *r*. $R_{em} = 3.82 \times 10^8$ m is the distance from Earth to the moon. Thus,

$$F_m = \frac{GM_m m}{\left(R_{em}-r\right)^2} = F_E = \frac{GM_e m}{r^2},$$

where *m* is the mass of the spaceship. Solving for *r*, we obtain

$$r = \frac{R_{em}}{\sqrt{M_m/M_e}+1} = \frac{3.82\times10^8\,\text{m}}{\sqrt{(7.36\times10^{22}\,\text{kg})/(5.98\times10^{24}\,\text{kg})}+1} = 3.44\times10^8\,\text{m}.$$