Chapter 11

1. The velocity of the car is a constant

$$\vec{v} = +(80 \text{ km/h})(1000 \text{ m/km})(1 \text{ h/3600 s})\,\hat{\text{i}} = (+22\,\text{m/s})\hat{\text{i}},$$

and the radius of the wheel is r = 0.66/2 = 0.33 m.

(a) In the car's reference frame (where the lady perceives herself to be at rest) the road is moving toward the rear at $\vec{v}_{\text{road}} = -v = -22\,\text{m/s}$, and the motion of the tire is purely rotational. In this frame, the center of the tire is "fixed" so $v_{\text{center}} = 0$.

(b) Since the tire's motion is only rotational (not translational) in this frame, Eq. 10-18 gives $\vec{v}_{\text{top}} = (+22\,\text{m/s})\hat{\text{i}}$.

(c) The bottom-most point of the tire is (momentarily) in firm contact with the road (not skidding) and has the same velocity as the road: $\vec{v}_{\text{bottom}} = (-22\,\text{m/s})\hat{\text{i}}$. This also follows from Eq. 10-18.

(d) This frame of reference is not accelerating, so "fixed" points within it have zero acceleration; thus, $a_{\text{center}} = 0$.

(e) Not only is the motion purely rotational in this frame, but we also have ω = constant, which means the only acceleration for points on the rim is radial (centripetal). Therefore, the magnitude of the acceleration is

$$a_{\text{top}} = \frac{v^2}{r} = \frac{(22 \text{ m/s})^2}{0.33 \text{ m}} = 1.5\times10^3 \text{ m/s}^2 .$$

(f) The magnitude of the acceleration is the same as in part (d): $a_{\text{bottom}} = 1.5 \times 10^3$ m/s^2.

(g) Now we examine the situation in the road's frame of reference (where the road is "fixed" and it is the car that appears to be moving). The center of the tire undergoes purely translational motion while points at the rim undergo a combination of translational and rotational motions. The velocity of the center of the tire is $\vec{v} = (+22\,\text{m/s})\hat{\text{i}}$.

(h) In part (b), we found $\vec{v}_{\text{top,car}} = +v$ and we use Eq. 4-39:

$$\vec{v}_{\text{top, ground}} = \vec{v}_{\text{top, car}} + \vec{v}_{\text{car, ground}} = v\,\hat{\text{i}} + v\,\hat{\text{i}} = 2v\,\hat{\text{i}}$$

which yields $2v$ = +44 m/s.

(i) We can proceed as in part (h) or simply recall that the bottom-most point is in firm contact with the (zero-velocity) road. Either way, the answer is zero.

(j) The translational motion of the center is constant; it does not accelerate.

(k) Since we are transforming between constant-velocity frames of reference, the accelerations are unaffected. The answer is as it was in part (e): 1.5×10^3 m/s^2.

(1) As explained in part (k), $a = 1.5 \times 10^3$ m/s^2.

2. The initial speed of the car is

$$v = (80 \text{ km/h})(1000 \text{ m/km})(1 \text{ h}/3600 \text{ s}) = 22.2 \text{ m/s}.$$

The tire radius is R = 0.750/2 = 0.375 m.

(a) The initial speed of the car is the initial speed of the center of mass of the tire, so Eq. 11-2 leads to

$$\omega_0 = \frac{v_{\text{com}0}}{R} = \frac{22.2 \text{ m/s}}{0.375 \text{ m}} = 59.3 \text{ rad/s}.$$

(b) With θ = (30.0)(2π) = 188 rad and ω = 0, Eq. 10-14 leads to

$$\omega^2 = \omega_0^2 + 2\alpha\theta \;\Rightarrow\; |\alpha| = \frac{(59.3 \text{ rad/s})^2}{2(188 \text{ rad})} = 9.31 \text{ rad/s}^2.$$

(c) Equation 11-1 gives $R\theta$ = 70.7 m for the distance traveled.

3. **THINK** The work required to stop the hoop is the negative of the initial kinetic energy of the hoop.

EXPRESS From Eq. 11-5, the initial kinetic energy of the hoop is $K_i = \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2$, where $I = mR^2$ is its rotational inertia about the center of mass. Eq. 11-2 relates the angular speed to the speed of the center of mass: $\omega = v/R$. Thus,

$$K_i = \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2 = \frac{1}{2}(mR^2)\left(\frac{v}{R}\right)^2 + \frac{1}{2} mv^2 = mv^2$$

ANALYZE With m = 140 kg, and the speed of its center of mass v = 0.150 m/s, we find the initial kinetic energy to be

$$K_i = mv^2 = (140\text{ kg})(0.150\text{ m/s})^2 = 3.15\text{ J}$$

which implies that the work required is $W = \Delta K = K_f - K_i = -K_i = -3.15\text{ J}$.

LEARN By the work-kinetic energy theorem, the work done is negative since it decreases the kinetic energy. A rolling body has two types of kinetic energy: rotational and translational.

4. We use the results from section 11.3.

(a) We substitute $I = \frac{2}{5} M R^2$ (Table 10-2(f)) and $a = -0.10g$ into Eq. 11-10:

$$-0.10g = -\frac{g\sin\theta}{1+\left(\frac{2}{5} MR^2\right)\big/ MR^2} = -\frac{g\sin\theta}{7/5}$$

which yields $\theta = \sin^{-1}(0.14) = 8.0°$.

(b) The acceleration would be more. We can look at this in terms of forces or in terms of energy. In terms of forces, the uphill static friction would then be absent so the downhill acceleration would be due only to the downhill gravitational pull. In terms of energy, the rotational term in Eq. 11-5 would be absent so that the potential energy it started with would simply become $\frac{1}{2}mv^2$ (without it being "shared" with another term) resulting in a greater speed (and, because of Eq. 2-16, greater acceleration).

5. Let M be the mass of the car (presumably including the mass of the wheels) and v be its speed. Let I be the rotational inertia of one wheel and ω be the angular speed of each wheel. The kinetic energy of rotation is

$$K_{\text{rot}} = 4\left(\frac{1}{2} I\omega^2\right),$$

where the factor 4 appears because there are four wheels. The total kinetic energy is given by

$$K = \tfrac{1}{2} Mv^2 + 4(\tfrac{1}{2} I\omega^2).$$

The fraction of the total energy that is due to rotation is

$$\text{fraction} = \frac{K_{\text{rot}}}{K} = \frac{4I\omega^2}{Mv^2 + 4I\omega^2}.$$

For a uniform disk (relative to its center of mass) $I = \frac{1}{2}mR^2$ (Table 10-2(c)). Since the wheels roll without sliding $\omega = v/R$ (Eq. 11-2). Thus the numerator of our fraction is

$$4I\omega^2 = 4\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 = 2mv^2$$

and the fraction itself becomes

$$\text{fraction} = \frac{2mv^2}{Mv^2 + 2mv^2} = \frac{2m}{M + 2m} = \frac{2(10)}{1000} = \frac{1}{50} = 0.020.$$

The wheel radius cancels from the equations and is not needed in the computation.

6. We plug $a = -3.5$ m/s^2 (where the magnitude of this number was estimated from the "rise over run" in the graph), $\theta = 30°$, $M = 0.50$ kg, and $R = 0.060$ m into Eq. 11-10 and solve for the rotational inertia. We find $I = 7.2 \times 10^{-4}$ kg·m^2.

7. (a) We find its angular speed as it leaves the roof using conservation of energy. Its initial kinetic energy is $K_i = 0$ and its initial potential energy is $U_i = Mgh$ where $h = 6.0\sin 30° = 3.0$ m (we are using the edge of the roof as our reference level for computing U). Its final kinetic energy (as it leaves the roof) is (Eq. 11-5)

$$K_f = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\omega^2.$$

Here we use v to denote the speed of its center of mass and ω is its angular speed — at the moment it leaves the roof. Since (up to that moment) the ball rolls without sliding we can set $v = R\omega = v$ where $R = 0.10$ m. Using $I = \tfrac{1}{2}MR^2$ (Table 10-2(c)), conservation of energy leads to

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}MR^2\omega^2 + \frac{1}{4}MR^2\omega^2 = \frac{3}{4}MR^2\omega^2.$$

The mass M cancels from the equation, and we obtain

$$\omega = \frac{1}{R}\sqrt{\frac{4}{3}gh} = \frac{1}{0.10\text{ m}}\sqrt{\frac{4}{3}(9.8\text{ m/s}^2)(3.0\text{ m})} = 63\text{ rad/s}.$$

(b) Now this becomes a projectile motion of the type examined in Chapter 4. We put the origin at the position of the center of mass when the ball leaves the track (the "initial" position for this part of the problem) and take $+x$ leftward and $+y$ downward. The result of part (a) implies $v_0 = R\omega = 6.3$ m/s, and we see from the figure that (with these positive direction choices) its components are

$$v_{0x} = v_0\cos 30° = 5.4\text{ m/s}$$
$$v_{0y} = v_0\sin 30° = 3.1\text{ m/s}.$$

The projectile motion equations become

$$x = v_{0x}t \quad \text{and} \quad y = v_{0y}t + \frac{1}{2}gt^2 .$$

We first find the time when $y = H = 5.0$ m from the second equation (using the quadratic formula, choosing the positive root):

$$t = \frac{-v_{0y} + \sqrt{v_{0y}^2 + 2gH}}{g} = 0.74\,\text{s}.$$

Then we substitute this into the x equation and obtain $x = (5.4\ \text{m/s})(0.74\ \text{s}) = 4.0\ \text{m}.$

8. (a) Let the turning point be designated P. By energy conservation, the mechanical energy at $x = 7.0$ m is equal to the mechanical energy at P. Thus, with Eq. 11-5, we have

$$75\ \text{J} = \tfrac{1}{2}mv_p^2 + \tfrac{1}{2}I_{\text{com}}\,\omega_p^2 + U_p .$$

Using item (f) of Table 10-2 and Eq. 11-2 (which means, if this is to be a turning point, that $\omega_p = v_p = 0$), we find $U_p = 75$ J. On the graph, this seems to correspond to $x = 2.0$ m, and we conclude that there is a turning point (and this is it). The ball, therefore, does not reach the origin.

(b) We note that there is no point (on the graph, to the right of $x = 7.0$ m) taht is shown "higher" than 75 J, so we suspect that there is no turning point in this direction, and we seek the velocity v_p at $x = 13$ m. If we obtain a real, nonzero answer, then our suspicion is correct (that it does reach this point P at $x = 13$ m). By energy conservation, the mechanical energy at $x = 7.0$ m is equal to the mechanical energy at P. Therefore,

$$75\ \text{J} = \tfrac{1}{2}mv_p^2 + \tfrac{1}{2}I_{\text{com}}\,\omega_p^2 + U_p .$$

Again, using item (f) of Table 11-2, Eq. 11-2 (less trivially this time) and $U_p = 60$ J (from the graph), as well as the numerical data given in the problem, we find $v_p = 7.3$ m/s.

9. To find where the ball lands, we need to know its speed as it leaves the track (using conservation of energy). Its initial kinetic energy is $K_i = 0$ and its initial potential energy is $U_i = M\,gH$. Its final kinetic energy (as it leaves the track) is given by Eq. 11-5:

$$K_f = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\omega^2$$

and its final potential energy is $M\,gh$. Here we use v to denote the speed of its center of mass and ω is its angular speed — at the moment it leaves the track. Since (up to that moment) the ball rolls without sliding we can set $\omega = v/R$. Using $I = \tfrac{2}{5}MR^2$ (Table 10-2(f)), conservation of energy leads to

$$MgH = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 + Mgh = \frac{1}{2}Mv^2 + \frac{2}{10}Mv^2 + Mgh$$
$$= \frac{7}{10}Mv^2 + Mgh.$$

The mass M cancels from the equation, and we obtain

$$v = \sqrt{\frac{10}{7}g(H-h)} = \sqrt{\frac{10}{7}\left(9.8\ \text{m/s}^2\right)\left(6.0\ \text{m} - 2.0\ \text{m}\right)} = 7.48\ \text{m/s}.$$

Now this becomes a projectile motion of the type examined in Chapter 4. We put the origin at the position of the center of mass when the ball leaves the track (the "initial" position for this part of the problem) and take $+x$ rightward and $+y$ downward. Then (since the initial velocity is purely horizontal) the projectile motion equations become

$$x = vt, \quad y = -\frac{1}{2}gt^2.$$

Solving for x at the time when $y = h$, the second equation gives $t = \sqrt{2h/g}$. Then, substituting this into the first equation, we find

$$x = v\sqrt{\frac{2h}{g}} = (7.48\ \text{m/s})\sqrt{\frac{2(2.0\ \text{m})}{9.8\ \text{m/s}^2}} = 4.8\ \text{m}.$$

10. From $I = \frac{2}{3}MR^2$ (Table 10-2(g)) we find

$$M = \frac{3I}{2R^2} = \frac{3\left(0.040\ \text{kg}\cdot\text{m}^2\right)}{2\left(0.15\ \text{m}\right)^2} = 2.7\ \text{kg}.$$

It also follows from the rotational inertia expression that $\frac{1}{2}I\omega^2 = \frac{1}{3}MR^2\omega^2$. Furthermore, it rolls without slipping, $v_{com} = R\omega$, and we find

$$\frac{K_{rot}}{K_{com} + K_{rot}} = \frac{\frac{1}{3}MR^2\omega^2}{\frac{1}{2}mR^2\omega^2 + \frac{1}{3}MR^2\omega^2}.$$

(a) Simplifying the above ratio, we find $K_{rot}/K = 0.4$. Thus, 40% of the kinetic energy is rotational, or

$$K_{rot} = (0.4)(20\ \text{J}) = 8.0\ \text{J}.$$

(b) From $K_{rot} = \frac{1}{3}M\,R^2\omega^2 = 8.0\,\text{J}$ (and using the above result for M) we find

$$\omega = \frac{1}{0.15\text{ m}}\sqrt{\frac{3(8.0\text{ J})}{2.7\text{ kg}}} = 20\text{ rad/s}$$

which leads to v_{com} = (0.15 m)(20 rad/s) = 3.0 m/s.

(c) We note that the inclined distance of 1.0 m corresponds to a height h = 1.0 sin 30° = 0.50 m. Mechanical energy conservation leads to

$$K_i = K_f + U_f \quad \Rightarrow \quad 20\text{ J} = K_f + Mgh$$

which yields (using the values of M and h found above) K_f = 6.9 J.

(d) We found in part (a) that 40% of this must be rotational, so

$$\frac{1}{3}MR^2\omega_f^2 = (0.40)K_f \quad \Rightarrow \quad \omega_f = \frac{1}{0.15\text{ m}}\sqrt{\frac{3(0.40)(6.9\text{ J})}{2.7\text{ kg}}}$$

which yields ω_f = 12 rad/s and leads to

$$v_{\text{com}\,f} = R\omega_f = (0.15\text{ m})(12\text{ rad/s}) = 1.8\text{ m/s}.$$

11. With $\vec{F}_{app} = (10\text{ N})\hat{\text{i}}$, we solve the problem by applying Eq. 9-14 and Eq. 11-37.

(a) Newton's second law in the x direction leads to

$$F_{app} - f_s = ma \quad \Rightarrow f_s = 10\text{N} - (10\text{kg})\left(0.60\text{ m/s}^2\right) = 4.0\text{ N}.$$

In unit vector notation, we have $\vec{f}_s = (-4.0\text{ N})\hat{\text{i}}$, which points leftward.

(b) With R = 0.30 m, we find the magnitude of the angular acceleration to be

$$|\alpha| = |a_{com}| / R = 2.0\text{ rad/s}^2,$$

from Eq. 11-6. The only force not directed toward (or away from) the center of mass is $\vec{f}_s$, and the torque it produces is clockwise:

$$|\tau| = I\,|\alpha| \quad \Rightarrow \quad (0.30\text{m})(4.0\text{N}) = I\left(2.0\text{rad/s}^2\right)$$

which yields the wheel's rotational inertia about its center of mass: $I = 0.60\,\text{kg}\cdot\text{m}^2$.

12. Using the floor as the reference position for computing potential energy, mechanical energy conservation leads to

$$U_{\text{release}} = K_{\text{top}} + U_{\text{top}} \quad \Rightarrow \quad mgh = \frac{1}{2}mv_{\text{com}}^2 + \frac{1}{2}I\omega^2 + mg\left(2R\right).$$

Substituting $I = \frac{2}{5}mr^2$ (Table 10-2(f)) and $\omega = v_{\text{com}}/r$ (Eq. 11-2), we obtain

$$mgh = \frac{1}{2}mv_{\text{com}}^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v_{\text{com}}}{r}\right)^2 + 2mgR \quad \Rightarrow \quad gh = \frac{7}{10}v_{\text{com}}^2 + 2gR$$

where we have canceled out mass *m* in that last step.

(a) To be on the verge of losing contact with the loop (at the top) means the normal force is nearly zero. In this case, Newton's second law along the vertical direction (+*y* downward) leads to

$$mg = ma_r \Rightarrow g = \frac{v_{\text{com}}^2}{R-r}$$

where we have used Eq. 10-23 for the radial (centripetal) acceleration (of the center of mass, which at this moment is a distance $R - r$ from the center of the loop). Plugging the result $v_{\text{com}}^2 = g(R-r)$ into the previous expression stemming from energy considerations gives

$$gh = \frac{7}{10}(g)(R-r) + 2gR$$

which leads to $h = 2.7R - 0.7r \approx 2.7R.$ With R = 14.0 cm , we have

$$h = (2.7)(14.0 \text{ cm}) = 37.8 \text{ cm}.$$

(b) The energy considerations shown above (now with $h = 6R$) can be applied to point *Q* (which, however, is only at a height of *R*) yielding the condition

$$g(6R) = \frac{7}{10}v_{\text{com}}^2 + gR$$

which gives us $v_{\text{com}}^2 = 50gR/7$. Recalling previous remarks about the radial acceleration, Newton's second law applied to the horizontal axis at *Q* leads to

$$N = m\frac{v_{\text{com}}^2}{R-r} = m\frac{50gR}{7(R-r)}$$

which (for $R \gg r$) gives

$$N \approx \frac{50mg}{7} = \frac{50(2.80\times10^{-4}\ \text{kg})(9.80\ \text{m/s}^2)}{7} = 1.96\times10^{-2}\ \text{N}.$$

(b) The direction is toward the center of the loop.

13. The physics of a rolling object usually requires a separate and very careful discussion (above and beyond the basics of rotation discussed in Chapter 10); this is done in the first three sections of Chapter 11. Also, the normal force on something (which is here the center of mass of the ball) following a circular trajectory is discussed in Section 6-6. Adapting Eq. 6-19 to the consideration of forces at the *bottom* of an arc, we have

$$F_N - Mg = Mv^2/r$$

which tells us (since we are given $F_N = 2Mg$) that the center of mass speed (squared) is $v^2 = gr$, where r is the arc radius (0.48 m) Thus, the ball's angular speed (squared) is

$$\omega^2 = v^2/R^2 = gr/R^2,$$

where R is the ball's radius. Plugging this into Eq. 10-5 and solving for the rotational inertia (about the center of mass), we find

$$I_{\text{com}} = 2MhR^2/r - MR^2 = MR^2[2(0.36/0.48) - 1]\ .$$

Thus, using the β notation suggested in the problem, we find

$$\beta = 2(0.36/0.48) - 1 = 0.50.$$

14. To find the center of mass speed v on the plateau, we use the projectile motion equations of Chapter 4. With $v_{oy} = 0$ (and using "h" for h_2) Eq. 4-22 gives the time-of-flight as $t = \sqrt{2h/g}$. Then Eq. 4-21 (squared, and using d for the horizontal displacement) gives $v^2 = gd^2/2h$. Now, to find the speed v_p at point P, we apply energy conservation, that is, mechanical energy on the plateau is equal to the mechanical energy at P. With Eq. 11-5, we obtain

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}I_{\text{com}}\,\omega^2 + mgh_1 = \tfrac{1}{2}mv_p^{\ 2} + \tfrac{1}{2}I_{\text{com}}\,\omega_p^{\ 2}\ .$$

Using item (f) of Table 10-2, Eq. 11-2, and our expression (above) $v^2 = gd^2/2h$, we obtain

$$gd^2/2h + 10gh_1/7 = v_p^{\ 2}$$

which yields (using the values stated in the problem) $v_p = 1.34$ m/s.

15. (a) We choose clockwise as the negative rotational sense and rightward as the positive translational direction. Thus, since this is the moment when it begins to roll smoothly, Eq. 11-2 becomes

$$v_{\text{com}} = -R\omega = (-0.11\ \text{m})\omega.$$

This velocity is positive-valued (rightward) since ω is negative-valued (clockwise) as shown in the figure.

(b) The force of friction exerted on the ball of mass *m* is $-\mu_k mg$ (negative since it points left), and setting this equal to ma_{com} leads to

$$a_{com} = -\mu g = -(0.21)\left(9.8 \text{ m/s}^2\right) = -2.1 \text{ m/s}^2$$

where the minus sign indicates that the center of mass acceleration points left, opposite to its velocity, so that the ball is decelerating.

(c) Measured about the center of mass, the torque exerted on the ball due to the frictional force is given by $\tau = -\mu mgR$. Using Table 10-2(f) for the rotational inertia, the angular acceleration becomes (using Eq. 10-45)

$$\alpha = \frac{\tau}{I} = \frac{-\mu mgR}{2mR^2/5} = \frac{-5\mu g}{2R} = \frac{-5(0.21)\left(9.8 \text{ m/s}^2\right)}{2(0.11 \text{ m})} = -47 \text{ rad/s}^2$$

where the minus sign indicates that the angular acceleration is clockwise, the same direction as ω (so its angular motion is "speeding up'').

(d) The center of mass of the sliding ball decelerates from $v_{com,0}$ to v_{com} during time t according to Eq. 2-11: $v_{com} = v_{com,0} - \mu gt$. During this time, the angular speed of the ball increases (in magnitude) from zero to $|\omega|$ according to Eq. 10-12:

$$|\omega| = |\alpha| t = \frac{5\mu gt}{2R} = \frac{v_{com}}{R}$$

where we have made use of our part (a) result in the last equality. We have two equations involving v_{com}, so we eliminate that variable and find

$$t = \frac{2v_{com,0}}{7\mu g} = \frac{2(8.5 \text{ m/s})}{7(0.21)\left(9.8 \text{ m/s}^2\right)} = 1.2 \text{ s}.$$

(e) The skid length of the ball is (using Eq. 2-15)

$$\Delta x = v_{com,0}t - \frac{1}{2}(\mu g)t^2 = (8.5 \text{ m/s})(1.2 \text{ s}) - \frac{1}{2}(0.21)\left(9.8 \text{ m/s}^2\right)(1.2 \text{ s})^2 = 8.6 \text{ m}.$$

(f) The center of mass velocity at the time found in part (d) is

$$v_{com} = v_{com,0} - \mu gt = 8.5\ \text{m/s} - (0.21)(9.8\ \text{m/s}^2)(1.2\ \text{s}) = 6.1\ \text{m/s}.$$

16. Using energy conservation with Eq. 11-5 and solving for the rotational inertia (about the center of mass), we find

$$I_{com} = 2MhR^2/r - MR^2 = MR^2[2g(H-h)/v^2 - 1]\ .$$

Thus, using the β notation suggested in the problem, we find

$$\beta = 2g(H-h)/v^2 - 1.$$

To proceed further, we need to find the center of mass speed v, which we do using the projectile motion equations of Chapter 4. With $v_{oy} = 0$, Eq. 4-22 gives the time-of-flight as $t = \sqrt{2h/g}$. Then Eq. 4-21 (squared, and using d for the horizontal displacement) gives $v^2 = gd^2/2h$. Plugging this into our expression for β gives

$$2g(H-h)/v^2 - 1 = 4h(H-h)/d^2 - 1.$$

Therefore, with the values given in the problem, we find β = 0.25.

17. **THINK** The yo-yo has both translational and rotational types of motion.

EXPRESS The derivation of the acceleration is given by Eq. 11-13:

$$a_{com} = -\frac{g}{1 + I_{com}/MR_0^2}$$

where M is the mass of the yo-yo, I_{cm} is the rotational inertia and R_0 is the radius of the axel. The positive direction is upward. The time it takes for the yo-yo to reach the end of the string can be found by solving the kinematic equation $y_{com} = \frac{1}{2}a_{com}t^2$.

ANALYZE (a) With $I_{com} = 950\ \text{g}\cdot\text{cm}^2$, M =120 g, R_0 = 0.320 cm and g = 980 cm/s^2, we obtain

$$|a_{com}| = \frac{980\ \text{cm/s}^2}{1 + (950\ \text{g}\cdot\text{cm}^2)/(120\ \text{g})(0.32\ \text{cm})^2} = 12.5\ \text{cm/s}^2 \approx 13\ \text{cm/s}^2.$$

(b) Taking the coordinate origin at the initial position, Eq. 2-15 leads to $y_{com} = \frac{1}{2}a_{com}t^2$. Thus, we set y_{com} = – 120 cm and find

$$t = \sqrt{\frac{2y_{com}}{a_{com}}} = \sqrt{\frac{2(-120\,\text{cm})}{-12.5\ \text{cm/s}^2}} = 4.38\ \text{s} \approx 4.4\ \text{s}.$$

(c) As the yo-yo reaches the end of the string, its center of mass velocity is given by Eq. 2-11:

$$v_{com} = a_{com}t = \left(-12.5 \text{ cm/s}^2\right)\left(4.38\text{s}\right) = -54.8 \text{ cm/s},$$

so its linear speed then is approximately $|v_{com}| = 55$ cm/s.

(d) The translational kinetic energy of the yo-yo is

$$K_{trans} = \frac{1}{2}mv_{com}^2 = \frac{1}{2}\left(0.120 \text{ kg}\right)\left(0.548 \text{ m/s}\right)^2 = 1.8\times10^{-2} \text{ J}.$$

(e) The angular velocity is $\omega = -v_{com}/R_0$, so the rotational kinetic energy is

$$K_{rot} = \frac{1}{2}I_{com}\omega^2 = \frac{1}{2}I_{com}\left(\frac{v_{com}}{R_0}\right)^2 = \frac{1}{2}(9.50\times10^{-5} \text{ kg}\cdot\text{m}^2)\left(\frac{0.548 \text{ m/s}}{3.2\times10^{-3} \text{ m}}\right)^2$$
$$= 1.393 \text{ J} \approx 1.4 \text{ J}$$

(f) The angular speed is

$$\omega = \frac{|v_{com}|}{R_0} = \frac{0.548 \text{ m/s}}{3.2\times10^{-3} \text{ m}} = 1.7\times10^2 \text{ rad/s} = 27 \text{ rev/s}.$$

LEARN As the yo-yo rolls down, its gravitational potential energy gets converted into both translational kinetic energy as well as rotational kinetic energy of the wheel. To show that the total energy remains conserved, we note that the initial energy is

$$U_i = Mgy_i = (0.120 \text{ kg})(9.80 \text{ m/s}^2)(1.20 \text{ m}) = 1.411 \text{ J}$$

which is equal to the sum of K_{trans} (= 0.018 J) and K_{rot} (= 1.393 J).

18. (a) The derivation of the acceleration is found in § 11-4; Eq. 11-13 gives

$$a_{com} = -\frac{g}{1 + I_{com}/MR_0^2}$$

where the positive direction is upward. We use $I_{com} = MR^2/2$ where the radius is R = 0.32 m and M = 116 kg is the *total* mass (thus including the fact that there are two disks) and obtain

$$a = -\frac{g}{1 + (MR^2/2)/MR_0^2} = \frac{g}{1 + \left(R/R_0\right)^2/2}$$

which yields $a = -g/51$ upon plugging in $R_0 = R/10 = 0.032$ m. Thus, the magnitude of the center of mass acceleration is 0.19 m/s^2.

(b) As observed in §11-4, our result in part (a) applies to both the descending and the rising yo-yo motions.

(c) The external forces on the center of mass consist of the cord tension (upward) and the pull of gravity (downward). Newton's second law leads to

$$T - Mg = ma \Rightarrow T = M\left(g - \frac{g}{51}\right) = 1.1 \times 10^3 \text{ N}.$$

(d) Our result in part (c) indicates that the tension is well below the ultimate limit for the cord.

(e) As we saw in our acceleration computation, all that mattered was the ratio R/R_0 (and, of course, g). So if it's a scaled-up version, then such ratios are unchanged and we obtain the same result.

(f) Since the tension also depends on mass, then the larger yo-yo will involve a larger cord tension.

19. If we write $\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$\left(yF_z - zF_y\right)\hat{\mathrm{i}} + \left(zF_x - xF_z\right)\hat{\mathrm{j}} + \left(xF_y - yF_x\right)\hat{\mathrm{k}}.$$

With (using SI units) $x = 0$, $y = -4.0$, $z = 5.0$, $F_x = 0$, $F_y = -2.0$, and $F_z = 3.0$ (these latter terms being the individual forces that contribute to the net force), the expression above yields

$$\vec{\tau} = \vec{r} \times \vec{F} = (-2.0\,\mathrm{N \cdot m})\hat{\mathrm{i}}.$$

20. If we write $\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$\left(yF_z - zF_y\right)\hat{\mathrm{i}} + \left(zF_x - xF_z\right)\hat{\mathrm{j}} + \left(xF_y - yF_x\right)\hat{\mathrm{k}}.$$

(a) In the above expression, we set (with SI units understood) $x = -2.0$, $y = 0$, $z = 4.0$, $F_x = 6.0$, $F_y = 0$, and $F_z = 0$. Then we obtain $\vec{\tau} = \vec{r} \times \vec{F} = (24\,\mathrm{N \cdot m})\hat{\mathrm{j}}$.

(b) The values are just as in part (a) with the exception that now $F_x = -6.0$. We find $\vec{\tau} = \vec{r} \times \vec{F} = (-24\,\mathrm{N \cdot m})\hat{\mathrm{j}}$.

(c) In the above expression, we set $x = -2.0$, $y = 0$, $z = 4.0$, $F_x = 0$, $F_y = 0$, and $F_z = 6.0$. We get $\vec{\tau} = \vec{r} \times \vec{F} = (12\,\mathrm{N \cdot m})\hat{\mathrm{j}}$.

(d) The values are just as in part (c) with the exception that now F_z = –6.0. We find $\vec{\tau} = \vec{r} \times \vec{F} = (-12\,\text{N}\cdot\text{m})\hat{\text{j}}$.

21. If we write $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$\left(yF_z - zF_y\right)\hat{\text{i}} + \left(zF_x - xF_z\right)\hat{\text{j}} + \left(xF_y - yF_x\right)\hat{\text{k}}.$$

(a) In the above expression, we set (with SI units understood) x = 0, y = – 4.0, z = 3.0, F_x = 2.0, F_y = 0, and F_z = 0. Then we obtain

$$\vec{\tau} = \vec{r} \times \vec{F} = \left(6.0\hat{\text{j}} + 8.0\hat{\text{k}}\right)\text{N}\cdot\text{m}.$$

This has magnitude $\sqrt{(6.0\ \text{N}\cdot\text{m})^2 + (8.0\ \text{N}\cdot\text{m})^2} = 10\ \text{N}\cdot\text{m}$ and is seen to be parallel to the yz plane. Its angle (measured counterclockwise from the +y direction) is $\tan^{-1}(8/6) = 53°$.

(b) In the above expression, we set x = 0, y = – 4.0, z = 3.0, F_x = 0, F_y = 2.0, and F_z = 4.0. Then we obtain $\vec{\tau} = \vec{r} \times \vec{F} = (-22\,\text{N}\cdot\text{m})\hat{\text{i}}$. The torque has magnitude $22\ \text{N}\cdot\text{m}$ and points in the –x direction.

22. Equation 11-14 (along with Eq. 3-30) gives

$$\vec{\tau} = \vec{r} \times \vec{F} = 4.00\hat{\text{i}} + (12.0 + 2.00F_x)\hat{\text{j}} + (14.0 + 3.00F_x)\hat{\text{k}}$$

with SI units understood. Comparing this with the known expression for the torque (given in the problem statement), we see that F_x must satisfy two conditions:

$$12.0 + 2.00F_x = 2.00 \quad \text{and} \quad 14.0 + 3.00F_x = -1.00.$$

The answer (F_x = –5.00 N) satisfies both conditions.

23. We use the notation $\vec{r}\,'$ to indicate the vector pointing from the axis of rotation directly to the position of the particle. If we write $\vec{r}\,' = x'\,\hat{\text{i}} + y'\,\hat{\text{j}} + z'\,\hat{\text{k}}$, then (using Eq. 3-30) we find $\vec{r}\,' \times \vec{F}$ is equal to

$$\left(y'F_z - z'F_y\right)\hat{\text{i}} + \left(z'F_x - x'F_z\right)\hat{\text{j}} + \left(x'F_y - y'F_x\right)\hat{\text{k}}.$$

(a) Here, $\vec{r}\,' = \vec{r}$. Dropping the primes in the above expression, we set (with SI units understood) x = 0, y = 0.5, z = –2.0, F_x = 2.0, F_y = 0, and F_z = –3.0. Then we obtain

$$\vec{\tau} = \vec{r} \times \vec{F} = \left(-1.5\,\hat{\mathrm{i}} - 4.0\,\hat{\mathrm{j}} - 1.0\,\hat{\mathrm{k}}\right)\ \mathrm{N\cdot m}.$$

(b) Now $\vec{r}\,' = \vec{r} - \vec{r}_{\mathrm{o}}$ where $\vec{r}_{\mathrm{o}} = 2.0\,\hat{\mathrm{i}} - 3.0\,\hat{\mathrm{k}}$. Therefore, in the above expression, we set $x' = -2.0, y' = 0.5, z' = 1.0, F_x = 2.0, F_y = 0$, and $F_z = -3.0$. Thus, we obtain

$$\vec{\tau} = \vec{r}\,' \times \vec{F} = \left(-1.5\,\hat{\mathrm{i}} - 4.0\,\hat{\mathrm{j}} - 1.0\,\hat{\mathrm{k}}\right)\ \mathrm{N\cdot m}.$$

24. If we write $\vec{r}\,' = x'\,\hat{\mathrm{i}} + y'\,\hat{\mathrm{j}} + z'\,\hat{\mathrm{k}}$, then (using Eq. 3-30) we find $\vec{r}\,' \times \vec{F}$ is equal to

$$\left(y'F_z - z'F_y\right)\hat{\mathrm{i}} + \left(z'F_x - x'F_z\right)\hat{\mathrm{j}} + \left(x'F_y - y'F_x\right)\hat{\mathrm{k}}.$$

(a) Here, $\vec{r}\,' = \vec{r}$ where $\vec{r} = 3.0\,\hat{\mathrm{i}} - 2.0\,\hat{\mathrm{j}} + 4.0\,\hat{\mathrm{k}}$, and $\vec{F} = \vec{F}_1$. Thus, dropping the prime in the above expression, we set (with SI units understood) $x = 3.0$, $y = -2.0$, $z = 4.0$, $F_x = 3.0$, $F_y = -4.0$, and $F_z = 5.0$. Then we obtain

$$\vec{\tau} = \vec{r} \times \vec{F}_1 = \left(6.0\,\hat{\mathrm{i}} - 3.0\,\hat{\mathrm{j}} - 6.0\,\hat{\mathrm{k}}\right)\ \mathrm{N\cdot m}.$$

(b) This is like part (a) but with $\vec{F} = \vec{F}_2$. We plug in $F_x = -3.0$, $F_y = -4.0$, and $F_z = -5.0$ and obtain

$$\vec{\tau} = \vec{r} \times \vec{F}_2 = \left(26\,\hat{\mathrm{i}} + 3.0\,\hat{\mathrm{j}} - 18\,\hat{\mathrm{k}}\right)\ \mathrm{N\cdot m}.$$

(c) We can proceed in either of two ways. We can add (vectorially) the answers from parts (a) and (b), or we can first add the two force vectors and then compute $\vec{\tau} = \vec{r} \times \left(\vec{F}_1 + \vec{F}_2\right)$ (these total force components are computed in the next part). The result is

$$\vec{\tau} = \vec{r} \times \left(\vec{F}_1 + \vec{F}_2\right) = \left(32\,\hat{\mathrm{i}} - 24\,\hat{\mathrm{k}}\right)\ \mathrm{N\cdot m}.$$

(d) Now $\vec{r}\,' = \vec{r} - \vec{r}_{\mathrm{o}}$ where $\vec{r}_{\mathrm{o}} = 3.0\,\hat{\mathrm{i}} + 2.0\,\hat{\mathrm{j}} + 4.0\,\hat{\mathrm{k}}$. Therefore, in the above expression, we set $x' = 0, y' = -4.0, z' = 0$, and

$$F_x = 3.0 - 3.0 = 0$$
$$F_y = -4.0 - 4.0 = -8.0$$
$$F_z = 5.0 - 5.0 = 0.$$

We get $\vec{\tau} = \vec{r}\,' \times \left(\vec{F}_1 + \vec{F}_2\right) = 0.$

25. **THINK** We take the cross product of $\vec{r}$ and $\vec{F}$ to find the torque $\vec{\tau}$ on a particle.

EXPRESS If we write $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}}$ and $\vec{F} = F_x\hat{\text{i}} + F_y\hat{\text{j}} + F_z\hat{\text{k}}$, then (using Eq. 3-30) the general expression for torque can be written as

$$\vec{\tau} = \vec{r} \times \vec{F} = \left(yF_z - zF_y\right)\hat{\text{i}} + \left(zF_x - xF_z\right)\hat{\text{j}} + \left(xF_y - yF_x\right)\hat{\text{k}}.$$

ANALYZE (a) With $\vec{r} = (3.0\text{ m})\hat{\text{i}} + (4.0\text{ m})\hat{\text{j}}$ and $\vec{F} = (-8.0\text{ N})\hat{\text{i}} + (6.0\text{ N})\hat{\text{j}}$, we have

$$\vec{\tau} = \left[(3.0\text{m})(6.0\text{N}) - (4.0\text{m})(-8.0\text{N})\right]\hat{\text{k}} = (50\,\text{N}\cdot\text{m})\,\hat{\text{k}}.$$

(b) To find the angle ϕ between $\vec{r}$ and $\vec{F}$, we use Eq. 3-27: $|\vec{r} \times \vec{F}| = rF\sin\phi$. Now $r = \sqrt{x^2 + y^2} = 5.0\text{ m}$ and $F = \sqrt{F_x^2 + F_y^2} = 10\text{ N}$. Thus,

$$rF = (5.0\text{ m})(10\text{ N}) = 50\text{ N}\cdot\text{m},$$

the same as the magnitude of the vector product calculated in part (a). This implies sin ϕ = 1 and ϕ = 90°.

LEARN Our result (ϕ = 90°) implies that $\vec{r}$ and $\vec{F}$ are perpendicular to each other. A useful check is to show that their dot product is zero. This is indeed the case:

$$\begin{aligned}\vec{r}\cdot\vec{F} &= [(3.0\text{ m})\hat{\text{i}} + (4.0\text{ m})\hat{\text{j}}]\cdot[(-8.0\text{ N})\hat{\text{i}} + (6.0\text{ N})\hat{\text{j}}]\\ &= (3.0\text{ m})(-8.0\text{ N}) + (4.0\text{ m})(6.0\text{ N}) = 0.\end{aligned}$$

26. We note that the component of $\vec{v}$ perpendicular to $\vec{r}$ has magnitude $v\sin\theta_2$ where θ_2= 30°. A similar observation applies to $\vec{F}$.

(a) Eq. 11-20 leads to

$$\ell = rmv_\perp = (3.0\text{ m})(2.0\text{ kg})(4.0\text{ m/s})\sin 30° = 12\text{ kg}\cdot\text{m}^2/\text{s}.$$

(b) Using the right-hand rule for vector products, we find $\vec{r} \times \vec{p}$ points out of the page, or along the +z axis, perpendicular to the plane of the figure.

(c) Similarly, Eq. 10-38 leads to

$$\tau = rF\sin\theta_2 = (3.0\text{ m})(2.0\text{ N})\sin 30° = 3.0\,\text{N}\cdot\text{m}.$$

(d) Using the right-hand rule for vector products, we find $\vec{r} \times \vec{F}$ is also out of the page, or along the +z axis, perpendicular to the plane of the figure.

27. **THINK** We evaluate the cross product $\vec{\ell} = m\vec{r}\times\vec{v}$ to find the angular momentum $\vec{\ell}$ on the object, and the cross product of $\vec{r}\times\vec{F}$ for the torque $\vec{\tau}$.

EXPRESS Let $\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}$ be the position vector of the object, $\vec{v} = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} + v_z\hat{\mathrm{k}}$ its velocity vector, and m its mass. The cross product of $\vec{r}$ and $\vec{v}$ is (using Eq. 3-30)

$$\vec{r}\times\vec{v} = \left(yv_z - zv_y\right)\hat{\mathrm{i}} + \left(zv_x - xv_z\right)\hat{\mathrm{j}} + \left(xv_y - yv_x\right)\hat{\mathrm{k}}.$$

Since only the x and z components of the position and velocity vectors are nonzero (i.e., $y = 0$ and $v_y = 0$), the above expression becomes $\vec{r}\times\vec{v} = \left(-xv_z + zv_z\right)\hat{\mathrm{j}}.$ As for the torque, writing $\vec{F} = F_x\hat{\mathrm{i}} + F_y\hat{\mathrm{j}} + F_z\hat{\mathrm{k}},$ we find $\vec{r}\times\vec{F}$ to be

$$\vec{\tau} = \vec{r}\times\vec{F} = \left(yF_z - zF_y\right)\hat{\mathrm{i}} + \left(zF_x - xF_z\right)\hat{\mathrm{j}} + \left(xF_y - yF_x\right)\hat{\mathrm{k}}.$$

ANALYZE (a) With $\vec{r} = (2.0\text{ m})\hat{\mathrm{i}} - (2.0\text{ m})\hat{\mathrm{k}}$ and $\vec{v} = (-5.0\text{ m/s})\hat{\mathrm{i}} + (5.0\text{ m/s})\hat{\mathrm{k}},$ in unit-vector notation, the angular momentum of the object is

$$\vec{\ell} = m\left(-xv_z + zv_x\right)\hat{\mathrm{j}} = \left(0.25\text{ kg}\right)\left(-\left(2.0\text{ m}\right)\left(5.0\text{ m/s}\right) + \left(-2.0\text{ m}\right)\left(-5.0\text{ m/s}\right)\right)\hat{\mathrm{j}} = 0.$$

(b) With x = 2.0 m, z = –2.0 m, F_y = 4.0 N and all other components zero, the expression above yields

$$\vec{\tau} = \vec{r}\times\vec{F} = (8.0\text{ N}\cdot\text{m})\hat{\mathrm{i}} + (8.0\text{ N}\cdot\text{m})\hat{\mathrm{k}}\,.$$

LEARN The fact that $\vec{\ell} = 0$ implies that $\vec{r}$ and $\vec{v}$ are parallel to each other ($\vec{r}\times\vec{v} = 0$). Using $\tau = |\vec{r}\times\vec{F}| = rF\sin\phi$, we find the angle between $\vec{r}$ and $\vec{F}$ to be

$$\sin\phi = \frac{\tau}{rF} = \frac{8\sqrt{2}\text{ N}\cdot\text{m}}{(2\sqrt{2}\text{ m})(4.0\text{ N})} = 1 \;\Rightarrow\; \phi = 90°$$

That is, $\vec{r}$ and $\vec{F}$ are perpendicular to each other.

28. If we write $\vec{r}\,' = x'\,\hat{\mathrm{i}} + y'\,\hat{\mathrm{j}} + z'\,\hat{\mathrm{k}},$ then (using Eq. 3-30) we find $\vec{r}\,' = \vec{v}$ is equal to

$$\left(y'v_z - z'v_y\right)\hat{\mathrm{i}} + \left(z'v_x - x'v_z\right)\hat{\mathrm{j}} + \left(x'v_y - y'v_x\right)\hat{\mathrm{k}}.$$

(a) Here, $\vec{r}\,' = \vec{r}$ where $\vec{r} = 3.0\hat{\mathrm{i}} - 4.0\hat{\mathrm{j}}.$ Thus, dropping the primes in the above expression, we set (with SI units understood) $x = 3.0, y = -4.0, z = 0, v_x = 30, v_y = 60$, and v_z = 0. Then (with m = 2.0 kg) we obtain

$$\vec{\ell} = m(\vec{r}\times\vec{v}) = (6.0\times10^2\ \text{kg}\cdot\text{m}^2/\text{s})\hat{\text{k}}.$$

(b) Now $\vec{r}\,' = \vec{r} - \vec{r}_o$ where $\vec{r}_o = -2.0\hat{\text{i}} - 2.0\hat{\text{j}}$. Therefore, in the above expression, we set $x' = 5.0, y' = -2.0, z' = 0, v_x = 30, v_y = 60$, and $v_z = 0$. We get

$$\vec{\ell} = m(\vec{r}\,'\times\vec{v}) = (7.2\times10^2\ \text{kg}\cdot\text{m}^2/\text{s})\hat{\text{k}}.$$

29. For the 3.1 kg particle, Eq. 11-21 yields

$$\ell_1 = r_{\perp 1} m v_1 = (2.8\ \text{m})(3.1\ \text{kg})(3.6\ \text{m/s}) = 31.2\ \text{kg}\cdot\text{m}^2/\text{s}.$$

Using the right-hand rule for vector products, we find this $(\vec{r}_1\times\vec{p}_1)$ is out of the page, or along the +*z* axis, perpendicular to the plane of Fig. 11-41. And for the 6.5 kg particle, we find

$$\ell_2 = r_{\perp 2} m v_2 = (1.5\ \text{m})(6.5\ \text{kg})(2.2\ \text{m/s}) = 21.4\ \text{kg}\cdot\text{m}^2/\text{s}.$$

And we use the right-hand rule again, finding that this $(\vec{r}_2\times\vec{p}_2)$ is into the page, or in the –*z* direction.

(a) The two angular momentum vectors are in opposite directions, so their vector sum is the *difference* of their magnitudes: $L = \ell_1 - \ell_2 = 9.8\ \text{kg}\cdot\text{m}^2/\text{s}$.

(b) The direction of the net angular momentum is along the +*z* axis.

30. (a) The acceleration vector is obtained by dividing the force vector by the (scalar) mass:

$$\vec{a} = \vec{F}/m = (3.00\ \text{m/s}^2)\hat{\text{i}} - (4.00\ \text{m/s}^2)\hat{\text{j}} + (2.00\ \text{m/s}^2)\hat{\text{k}}.$$

(b) Use of Eq. 11-18 leads directly to

$$\vec{L} = (42.0\ \text{kg}\cdot\text{m}^2/\text{s})\hat{\text{i}} + (24.0\ \text{kg}\cdot\text{m}^2/\text{s})\hat{\text{j}} + (60.0\ \text{kg}\cdot\text{m}^2/\text{s})\hat{\text{k}}.$$

(c) Similarly, the torque is

$$\vec{\tau} = \vec{r}\times\vec{F} = (-8.00\ \text{N}\cdot\text{m})\hat{\text{i}} - (26.0\ \text{N}\cdot\text{m})\hat{\text{j}} - (40.0\ \text{N}\cdot\text{m})\hat{\text{k}}.$$

(d) We note (using the Pythagorean theorem) that the magnitude of the velocity vector is 7.35 m/s and that of the force is 10.8 N. The dot product of these two vectors is $\vec{v}\cdot\vec{F} = -48$ (in SI units). Thus, Eq. 3-20 yields

$$\theta = \cos^{-1}[-48.0/(7.35\times10.8)] = 127°.$$

31. (a) Since the speed is (momentarily) zero when it reaches maximum height, the angular momentum is zero then.

(b) With the convention (used in several places in the book) that clockwise sense is to be associated with the negative sign, we have $L = -r_\perp m v$ where $r_\perp = 2.00$ m, $m = 0.400$ kg, and v is given by free-fall considerations (as in Chapter 2). Specifically, y_{max} is determined by Eq. 2-16 with the speed at max height set to zero; we find $y_{max} = v_o^2/2g$ where $v_o = 40.0$ m/s. Then with $y = \frac{1}{2}y_{max}$, Eq. 2-16 can be used to give $v = v_o/\sqrt{2}$. In this way we arrive at $L = -22.6\ \text{kg}\cdot\text{m}^2/\text{s}$.

(c) As mentioned in the previous part, we use the minus sign in writing $\tau = -r_\perp F$ with the force F being equal (in magnitude) to mg. Thus, $\tau = -7.84\ \text{N}\cdot\text{m}$.

(d) Due to the way $r_\perp$ is defined it does not matter how far up the ball is. The answer is the same as in part (c), $\tau = -7.84\ \text{N}\cdot\text{m}$.

32. The rate of change of the angular momentum is

$$\frac{d\vec{\ell}}{dt} = \vec{\tau}_1 + \vec{\tau}_2 = (2.0\ \text{N}\cdot\text{m})\hat{\text{i}} - (4.0\ \text{N}\cdot\text{m})\hat{\text{j}}.$$

Consequently, the vector $d\vec{\ell}/dt$ has a magnitude $\sqrt{(2.0\ \text{N}\cdot\text{m})^2 + (-4.0\ \text{N}\cdot\text{m})^2} = 4.5\ \text{N}\cdot\text{m}$ and is at an angle θ (in the xy plane, or a plane parallel to it) measured from the positive x axis, where

$$\theta = \tan^{-1}\left(\frac{-4.0\ \text{N}\cdot\text{m}}{2.0\ \text{N}\cdot\text{m}}\right) = -63°,$$

the negative sign indicating that the angle is measured clockwise as viewed "from above" (by a person on the $+z$ axis).

33. **THINK** We evaluate the cross product $\vec{\ell} = m\vec{r}\times\vec{v}$ to find the angular momentum $\vec{\ell}$ on the particle, and the cross product of $\vec{r}\times\vec{F}$ for the torque $\vec{\tau}$.

EXPRESS Let $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}}$ be the position vector of the object, $\vec{v} = v_x\hat{\text{i}} + v_y\hat{\text{j}} + v_z\hat{\text{k}}$ its velocity vector, and m its mass. The cross product of $\vec{r}$ and $\vec{v}$ is

$$\vec{r}\times\vec{v} = (yv_z - zv_y)\hat{\text{i}} + (zv_x - xv_z)\hat{\text{j}} + (xv_y - yv_x)\hat{\text{k}}.$$

The angular momentum is given by the vector product $\vec{\ell} = m\vec{r}\times\vec{v}$. As for the torque, writing $\vec{F} = F_x\hat{\text{i}} + F_y\hat{\text{j}} + F_z\hat{\text{k}}$, then we find $\vec{r}\times\vec{F}$ to be

$$\vec{\tau} = \vec{r} \times \vec{F} = \left(yF_z - zF_y\right)\hat{\text{i}} + \left(zF_x - xF_z\right)\hat{\text{j}} + \left(xF_y - yF_x\right)\hat{\text{k}}.$$

ANALYZE (a) Substituting m = 3.0 kg, x = 3.0 m, y = 8.0 m, z = 0, v_x = 5.0 m/s, $v_y = -6.0$ m/s and v_z = 0 into the above expression, we obtain

$$\vec{\ell} = (3.0\text{ kg})\,[(3.0\text{ m})(-6.0\text{ m/s}) - (8.0\text{ m})(5.0\text{ m/s})]\hat{\text{k}} = (-174\text{ kg}\cdot\text{m}^2/\text{s})\hat{\text{k}}.$$

(b) Given that $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}}$ and $\vec{F} = F_x\hat{\text{i}}$, the corresponding torque is

$$\vec{\tau} = \left(x\hat{\text{i}} + y\hat{\text{j}}\right) \times \left(F_x\hat{\text{i}}\right) = -yF_x\hat{\text{k}}.$$

Substituting the values given, we find

$$\vec{\tau} = -(8.0\text{m})(-7.0\text{N})\,\hat{\text{k}} = (56\,\text{N}\cdot\text{m})\hat{\text{k}}.$$

(c) According to Newton's second law $\vec{\tau} = d\vec{\ell}/dt$, so the rate of change of the angular momentum is 56 kg·m^2/s^2, in the positive z direction.

LEARN The direction of $\vec{\ell}$ is in the –z-direction, which is perpendicular to both $\vec{r}$ and $\vec{v}$. Similarly, the torque $\vec{\tau}$ is perpendicular to both $\vec{r}$ and $\vec{F}$ (i.e, $\vec{\tau}$ is in the direction normal to the plane formed by $\vec{r}$ and $\vec{F}$).

34. We use a right-handed coordinate system with $\hat{\text{k}}$ directed out of the xy plane so as to be consistent with counterclockwise rotation (and the right-hand rule). Thus, all the angular momenta being considered are along the $-\hat{\text{k}}$ direction; for example, in part (b) $\vec{\ell} = -4.0t^2\,\hat{\text{k}}$ in SI units. We use Eq. 11-23.

(a) The angular momentum is constant so its derivative is zero. There is no torque in this instance.

(b) Taking the derivative with respect to time, we obtain the torque:

$$\vec{\tau} = \frac{d\vec{\ell}}{dt} = \left(-4.0\hat{\text{k}}\right)\frac{dt^2}{dt} = (-8.0t\ \text{N}\cdot\text{m})\hat{\text{k}}\,.$$

This vector points in the $-\hat{\text{k}}$ direction (causing the clockwise motion to speed up) for all t > 0.

(c) With $\vec{\ell} = (-4.0\sqrt{t})\hat{\text{k}}$ in SI units, the torque is

$$\vec{\tau} = \left(-4.0\hat{k}\right)\frac{d\sqrt{t}}{dt} = \left(-4.0\hat{k}\right)\left(\frac{1}{2\sqrt{t}}\right) = \left(-\frac{2.0}{\sqrt{t}}\hat{k}\right)\text{N}\cdot\text{m}.$$

This vector points in the $-\hat{k}$ direction (causing the clockwise motion to speed up) for all $t > 0$ (and it is undefined for $t < 0$).

(d) Finally, we have

$$\vec{\tau} = \left(-4.0\hat{k}\right)\frac{dt^{-2}}{dt} = \left(-4.0\hat{k}\right)\left(\frac{-2}{t^3}\right) = \left(\frac{8.0}{t^3}\hat{k}\right)\text{N}\cdot\text{m}.$$

This vector points in the $+\hat{k}$ direction (causing the initially clockwise motion to slow down) for all $t > 0$.

35. (a) We note that

$$\vec{v} = \frac{d\vec{r}}{dt} = 8.0t\,\hat{i} - (2.0 + 12t)\hat{j}$$

with SI units understood. From Eq. 11-18 (for the angular momentum) and Eq. 3-30, we find the particle's angular momentum is $8t^2\hat{k}$. Using Eq. 11-23 (relating its time-derivative to the (single) torque) then yields $\vec{\tau} = (48t\,\hat{k})\ \text{N}\cdot\text{m}$.

(b) From our (intermediate) result in part (a), we see the angular momentum increases in proportion to t^2.

36. We relate the motions of the various disks by examining their linear speeds (using Eq. 10-18). The fact that the linear speed at the rim of disk *A* must equal the linear speed at the rim of disk *C* leads to $\omega_A = 2\omega_C$. The fact that the linear speed at the hub of disk *A* must equal the linear speed at the rim of disk *B* leads to $\omega_A = \frac{1}{2}\omega_B$. Thus, $\omega_B = 4\omega_C$. The ratio of their angular momenta depend on these angular velocities as well as their rotational inertias (see item (c) in Table 11-2), which themselves depend on their masses. If *h* is the thickness and ρ is the density of each disk, then each mass is $\rho\pi R^2 h$. Therefore,

$$\frac{L_C}{L_B} = \frac{(½)\rho\pi R_C^2\, h\, R_C^2\, \omega_C}{(½)\rho\pi R_B^2\, h\, R_B^2\, \omega_B} = 1024.$$

37. (a) A particle contributes mr_2 to the rotational inertia. Here *r* is the distance from the origin *O* to the particle. The total rotational inertia is

$$I = m(3d)^2 + m(2d)^2 + m(d)^2 = 14md^2 = 14(2.3\times10^{-2}\,\text{kg})(0.12\text{ m})^2$$
$$= 4.6\times10^{-3}\ \text{kg}\cdot\text{m}^2.$$

(b) The angular momentum of the middle particle is given by $L_m = I_m\omega$, where $I_m = 4md^2$ is its rotational inertia. Thus

$$L_m = 4md^2\omega = 4(2.3\times10^{-2}\,\text{kg})(0.12\text{ m})^2(0.85\text{ rad/s}) = 1.1\times10^{-3}\text{ kg}\cdot\text{m}^2/\text{s}.$$

(c) The total angular momentum is

$$I\omega = 14md^2\omega = 14(2.3\times10^{-2}\,\text{kg})(0.12\text{ m})^2(0.85\text{ rad/s}) = 3.9\times10^{-3}\text{ kg}\cdot\text{m}^2/\text{s}.$$

38. (a) Equation 10-34 gives $\alpha = \tau/I$ and Eq. 10-12 leads to $\omega = \alpha t = \tau t/I$. Therefore, the angular momentum at $t = 0.033$ s is

$$I\omega = \tau\, t = (16\,\text{N}\cdot\text{m})(0.033\text{s}) = 0.53\,\text{kg}\cdot\text{m}^2/\text{s}$$

where this is essentially a derivation of the angular version of the impulse-momentum theorem.

(b) We find

$$\omega = \frac{\tau\, t}{I} = \frac{(16\text{ N}\cdot\text{m})(0.033\text{ s})}{1.2\times10^{-3}\text{ kg}\cdot\text{m}^2} = 440\,\text{rad/s}$$

which we convert as follows:

$$\omega = (440\text{ rad/s})(60\text{ s/min})(1\text{ rev}/2\pi\text{ rad}) \approx 4.2\times10^3\text{ rev/min}.$$

39. **THINK** A non-zero torque is required to change the angular momentum of the flywheel. We analyze the rotational motion of the wheel using the equations given in Table 10-1.

EXPRESS Since the torque is equal to the rate of change of angular momentum, $\tau = dL/dt$, the average torque acting during any interval Δt is simply given by $\tau_{\text{avg}} = (L_f - L_i)/\Delta t$, where L_i is the initial angular momentum and L_f is the final angular momentum. For uniform angular acceleration, the angle turned is $\theta = \omega_0 t + \alpha t^2/2$, and the work done on the wheel is $W = \tau\theta$.

ANALYZE (a) Substituting the values given, the average torque is

$$\tau_{\text{avg}} = \frac{L_f - L_i}{\Delta t} = \frac{(0.800\text{ kg}\cdot\text{m}^2/\text{s}) - (3.00\text{ kg}\cdot\text{m}^2/\text{s})}{1.50\text{ s}} = -1.47\text{ N}\cdot\text{m},$$

or $|\tau_{\text{avg}}| = 1.47\text{ N}\cdot\text{m}$. In this case the negative sign indicates that the direction of the torque is opposite the direction of the initial angular momentum, implicitly taken to be positive.

(b) If the angular acceleration α is uniform, so is the torque and $\alpha = \tau/I$. Furthermore, $\omega_0 = L_i/I$, and we obtain

$$\theta = \frac{L_i t + \tau\, t^2/2}{I} = \frac{\left(3.00\,\text{kg}\cdot\text{m}^2/\text{s}\right)\left(1.50\ \text{s}\right) + \left(-1.467\,\text{N}\cdot\text{m}\right)\left(1.50\ \text{s}\right)^2/2}{0.140\,\text{kg}\cdot\text{m}^2} = 20.4\,\text{rad}.$$

(c) Using the values of τ and θ found above, we find the work done on the wheel to be

$$W = \tau\theta = \left(-1.47\,\text{N}\cdot\text{m}\right)\left(20.4\,\text{rad}\right) = -29.9\ \text{J}.$$

(d) The average power is the work done by the flywheel (the negative of the work done on the flywheel) divided by the time interval:

$$P_{\text{avg}} = -\frac{W}{\Delta t} = -\frac{-29.9\ \text{J}}{1.50\,\text{s}} = 19.9\,\text{W}.$$

LEARN An alternative way to calculate the work done on the wheel is to apply the work-kinetic energy theorem:

$$W = \Delta K = K_f - K_i = \frac{1}{2}I(\omega_f^2 - \omega_i^2) = \frac{1}{2}I\left[\left(\frac{L_f}{I}\right)^2 - \left(\frac{L_i}{I}\right)^2\right] = \frac{L_f^2 - L_i^2}{2I}$$

Substituting the values given, we have

$$W = \frac{L_f^2 - L_i^2}{2I} = \frac{(0.800\ \text{kg}\cdot\text{m}^2/\text{s})^2 - (3.00\ \text{kg}\cdot\text{m}^2/\text{s})^2}{2(0.140\,\text{kg}\cdot\text{m}^2)} = -29.9\ \text{J}$$

which agrees with that calculated in part (c).

40. Torque is the time derivative of the angular momentum. Thus, the change in the angular momentum is equal to the time integral of the torque. With $\tau = (5.00 + 2.00t)\ \text{N}\cdot\text{m}$, the angular momentum (in units $\text{kg}\cdot\text{m}^2/\text{s}$) as a function of time is

$$L(t) = \int \tau dt = \int (5.00 + 2.00t)dt = L_0 + 5.00t + 1.00t^2.$$

Since $L = 5.00\ \text{kg}\cdot\text{m}^2/\text{s}$ when $t = 1.00\,\text{s}$, the integration constant is $L_0 = -1$. Thus, the complete expression of the angular momentum is

$$L(t) = -1 + 5.00t + 1.00t^2.$$

At $t = 3.00\,\text{s}$, we have $L(t = 3.00) = -1 + 5.00(3.00) + 1.00(3.00)^2 = 23.0\ \text{kg}\cdot\text{m}^2/\text{s}.$

41. (a) For the hoop, we use Table 10-2(h) and the parallel-axis theorem to obtain

$$I_1 = I_{\text{com}} + mh^2 = \frac{1}{2}mR^2 + mR^2 = \frac{3}{2}mR^2 .$$

Of the thin bars (in the form of a square), the member along the rotation axis has (approximately) no rotational inertia about that axis (since it is thin), and the member farthest from it is very much like it (by being parallel to it) except that it is displaced by a distance *h*; it has rotational inertia given by the parallel axis theorem:

$$I_2 = I_{\text{com}} + mh^2 = 0 + mR^2 = mR^2 .$$

Now the two members of the square perpendicular to the axis have the same rotational inertia (that is $I_3 = I_4$). We find I_3 using Table 10-2(e) and the parallel-axis theorem:

$$I_3 = I_{\text{com}} + mh^2 = \frac{1}{12}mR^2 + m\left(\frac{R}{2}\right)^2 = \frac{1}{3}mR^2 .$$

Therefore, the total rotational inertia is

$$I_1 + I_2 + I_3 + I_4 = \frac{19}{6}mR^2 = 1.6\,\text{kg}\cdot\text{m}^2 .$$

(b) The angular speed is constant:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{2.5} = 2.5\,\text{rad/s}.$$

Thus, $L = I_{\text{total}}\omega = 4.0\,\text{kg}\cdot\text{m}^2/\text{s}$.

42. The results may be found by integrating Eq. 11-29 with respect to time, keeping in mind that $\vec{L}_i = 0$ and that the integration may be thought of as "adding the areas" under the line-segments (in the plot of the torque versus time, with "areas" under the time axis contributing negatively). It is helpful to keep in mind, also, that the area of a triangle is $\frac{1}{2}$ (base)(height).

(a) We find that $\vec{L} = 24\ \text{kg}\cdot\text{m}^2/\text{s}$ at $t = 7.0$ s.

(b) Similarly, $\vec{L} = 1.5\ \text{kg}\cdot\text{m}^2/\text{s}$ at $t = 20$ s.

43. We assume that from the moment of grabbing the stick onward, they maintain rigid postures so that the system can be analyzed as a symmetrical rigid body with center of mass midway between the skaters.

(a) The total linear momentum is zero (the skaters have the same mass and equal and opposite velocities). Thus, their center of mass (the middle of the 3.0 m long stick) remains fixed and they execute circular motion (of radius $r = 1.5$ m) about it.

(b) Using Eq. 10-18, their angular velocity (counterclockwise as seen in Fig. 11-47) is

$$\omega = \frac{v}{r} = \frac{1.4 \text{ m/s}}{1.5 \text{ m}} = 0.93 \text{ rad/s}.$$

(c) Their rotational inertia is that of two particles in circular motion at r = 1.5 m, so Eq. 10-33 yields

$$I = \sum mr^2 = 2(50 \text{ kg})(1.5 \text{ m})^2 = 225 \text{ kg}\cdot\text{m}^2.$$

Therefore, Eq. 10-34 leads to

$$K = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(225 \text{ kg}\cdot\text{m}^2\right)(0.93 \text{rad/s})^2 = 98 \text{ J}.$$

(d) Angular momentum is conserved in this process. If we label the angular velocity found in part (a) ω_i and the rotational inertia of part (b) as I_i, we have

$$I_i\omega_i = \left(225 \text{ kg}\cdot\text{m}^2\right)(0.93 \text{rad/s}) = I_f\omega_f.$$

The final rotational inertia is $\sum mr_f^2$ where r_f = 0.5 m so $I_f = 25 \text{ kg}\cdot\text{m}^2$. Using this value, the above expression gives $\omega_f = 8.4$ rad/s.

(e) We find

$$K_f = \frac{1}{2} I_f\omega_f^2 = \frac{1}{2}\left(25 \text{ kg}\cdot\text{m}^2\right)(8.4 \text{rad/s})^2 = 8.8\times10^2 \text{ J}.$$

(f) We account for the large increase in kinetic energy (part (e) minus part (c)) by noting that the skaters do a great deal of work (converting their internal energy into mechanical energy) as they pull themselves closer — “fighting” what appears to them to be large “centrifugal forces” trying to keep them apart.

44. So that we don’t get confused about ± signs, we write the angular *speed* to the lazy Susan as $|\omega|$ and reserve the ω symbol for the angular velocity (which, using a common convention, is negative-valued when the rotation is clockwise). When the roach “stops” we recognize that it comes to rest relative to the lazy Susan (not relative to the ground).

(a) Angular momentum conservation leads to

$$mvR + I\omega_0 = \left(mR^2 + I\right)\omega_f$$

which we can write (recalling our discussion about angular speed versus angular velocity) as

$$mvR - I|\omega_0| = -(mR^2 + I)|\omega_f|.$$

We solve for the final angular speed of the system:

$$|\omega_f| = \frac{mvR - I|\omega_0|}{mR^2 + I} = \frac{(0.17\text{ kg})(2.0\text{ m/s})(0.15\text{ m}) - (5.0\times10^{-3}\text{ kg}\cdot\text{m}^2)(2.8\text{ rad/s})}{(5.0\times10^{-3}\text{ kg}\cdot\text{m}^2) + (0.17\text{ kg})(0.15\text{ m})^2}$$
$$= 4.2\text{ rad/s.}$$

(b) No, $K_f \neq K_i$ and — if desired — we can solve for the difference:

$$K_i - K_f = \frac{mI}{2}\frac{v^2 + \omega_0^2 R^2 + 2Rv|\omega_0|}{mR^2 + I}$$

which is clearly positive. Thus, some of the initial kinetic energy is "lost" — that is, transferred to another form. And the culprit is the roach, who must find it difficult to stop (and "internalize" that energy).

45. **THINK** No external torque acts on the system consisting of the man, bricks, and platform, so the total angular momentum of the system is conserved.

EXPRESS Let I_i be the initial rotational inertia of the system and let I_f be the final rotational inertia. Then $I_i\omega_i = I_f\omega_f$ by angular momentum conservation. The kinetic energy (of rotational nature) is given by $K = I\omega^2/2$.

ANALYZE (a) The final angular momentum of the system is

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{6.0\text{ kg}\cdot\text{m}^2}{2.0\text{ kg}\cdot\text{m}^2}\right)(1.2\,\text{rev/s}) = 3.6\,\text{rev/s}.$$

(b) The initial kinetic energy is $K_i = \frac{1}{2}I_i\omega_i^2$, and the final kinetic energy is $K_f = \frac{1}{2}I_f\omega_f^2$, so that their ratio is

$$\frac{K_f}{K_i} = \frac{I_f\omega_f^2/2}{I_i\omega_i^2/2} = \frac{(2.0\,\text{kg}\cdot\text{m}^2)(3.6\,\text{rev/s})^2/2}{(6.0\,\text{kg}\cdot\text{m}^2)(1.2\,\text{rev/s})^2/2} = 3.0.$$

(c) The man did work in decreasing the rotational inertia by pulling the bricks closer to his body. This energy came from the man's internal energy.

LEARN The work done by the person is equal to the change in kinetic energy:

$$W = K_f - K_i = 3K_i - K_i = 2K_i = I_i\omega_i^2 = (6.0\,\text{kg}\cdot\text{m}^2)(2\pi\cdot 1.2\,\text{rad/s})^2 = 341\text{ J}.$$

46. Angular momentum conservation $I_i\omega_i = I_f\omega_f$ leads to

$$\frac{\omega_f}{\omega_i} = \frac{I_i}{I_f}\omega_i = 3$$

which implies

$$\frac{K_f}{K_i} = \frac{I_f\omega_f^2/2}{I_i\omega_i^2/2} = \frac{I_f}{I_i}\left(\frac{\omega_f}{\omega_i}\right)^2 = 3.$$

47. **THINK** No external torque acts on the system consisting of the train and wheel, so the total angular momentum of the system (which is initially zero) remains zero.

EXPRESS Let $I = MR^2$ be the rotational inertia of the wheel (which we treat as a hoop). Its angular momentum is

$$\vec{L}_{\text{wheel}} = (I\omega)\hat{\text{k}} = -M\,R^2\,|\omega|\,\hat{\text{k}},$$

where $\hat{\text{k}}$ is *up* in Fig. 11-48 and that last step (with the minus sign) is done in recognition that the wheel's clockwise rotation implies a negative value for ω. The linear speed of a point on the track is $-|\omega|R$ and the speed of the train (going counterclockwise in Fig. 11-48 with speed v' relative to an outside observer) is therefore $v' = v - |\omega|R$ where v is its speed relative to the tracks. Consequently, the angular momentum of the train is $\vec{L}_{\text{train}} = m\left(v - |\omega|R\right)R\,\hat{\text{k}}$. Conservation of angular momentum yields

$$0 = \vec{L}_{\text{wheel}} + \vec{L}_{\text{train}} = -MR^2\,|\omega|\,\hat{\text{k}} + m\left(v - |\omega|R\right)R\hat{\text{k}}$$

which we can use to solve for $|\omega|$.

ANALYZE Solving for the angular speed, the result is

$$|\omega| = \frac{mvR}{(M+m)R^2} = \frac{v}{(M/m+1)R} = \frac{0.15\text{ m/s}}{(1.1+1)(0.43\text{ m})} = 0.17\text{ rad/s}.$$

LEARN By angular momentum conservation, we must have $\vec{L}_{\text{wheel}} = -\vec{L}_{\text{train}}$, which means that train and the wheel must have opposite senses of rotation.

48. Using Eq. 11-31 with angular momentum conservation, $\vec{L}_i = \vec{L}_f$ (Eq. 11-33) leads to the ratio of rotational inertias being inversely proportional to the ratio of angular velocities. Thus, $I_f/I_i = 6/5 = 1.0 + 0.2$. We interpret the "1.0" as the ratio of disk rotational inertias (which does not change in this problem) and the "0.2" as the ratio of the roach rotational inertial to that of the disk. Thus, the answer is 0.20.

49. (a) We apply conservation of angular momentum:

$$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega.$$

The angular speed after coupling is therefore

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} = \frac{(3.3\,\text{kg}\cdot\text{m}^2)(450\text{ rev/min}) + (6.6\,\text{kg}\cdot\text{m}^2)(900\text{ rev/min})}{3.3\,\text{kg}\cdot\text{m}^2 + 6.6\,\text{kg}\cdot\text{m}^2}$$
$$= 750\text{ rev/min}.$$

(b) In this case, we obtain

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} = \frac{(3.3\text{ kg}\cdot\text{m}^2)(450\text{ rev/min}) + (6.6\text{ kg}\cdot\text{m}^2)(-900\text{ rev/min})}{3.3\text{ kg}\cdot\text{m}^2 + 6.6\text{ kg}\cdot\text{m}^2}$$
$$= -450\text{ rev/min}$$

or $|\omega| = 450\text{ rev/min}$.

(c) The minus sign indicates that $\vec{\omega}$ is clockwise, that is, in the direction of the second disk's initial angular velocity.

50. We use conservation of angular momentum:

$$I_m\omega_m = I_p\omega_p.$$

The respective angles θ_m and θ_p by which the motor and probe rotate are therefore related by

$$\int I_m\omega_m dt = I_m\theta_m = \int I_p\omega_p dt = I_p\theta_p$$

which gives

$$\theta_m = \frac{I_p\theta_p}{I_m} = \frac{(12\text{ kg}\cdot\text{m}^2)(30°)}{2.0\times10^{-3}\text{ kg}\cdot\text{m}^2} = 180000°.$$

The number of revolutions for the rotor is then

$$N = (1.8 \times 10^5)°/(360°/\text{rev}) = 5.0 \times 10^2\text{ rev}.$$

51. **THINK** No external torques act on the system consisting of the two wheels, so its total angular momentum is conserved.

EXPRESS Let I_1 be the rotational inertia of the wheel that is originally spinning $(\text{at } \omega_i)$ and I_2 be the rotational inertia of the wheel that is initially at rest. Then by angular momentum conservation, $L_i = L_f$, or $I_1\omega_i = (I_1 + I_2)\omega_f$ and

$$\omega_f = \frac{I_1}{I_1 + I_2}\omega_i$$

where ω_f is the common final angular velocity of the wheels.

ANALYZE (a) Substituting $I_2 = 2I_1$ and $\omega_i = 800$ rev/min, we obtain

$$\omega_f = \frac{I_1}{I_1 + I_2}\omega_i = \frac{I_1}{I_1 + 2(I_1)}(800\,\text{rev/min}) = \frac{1}{3}(800\,\text{rev/min}) = 267\,\text{rev/min}.$$

(b) The initial kinetic energy is $K_i = \frac{1}{2}I_1\omega_i^2$ and the final kinetic energy is $K_f = \frac{1}{2}(I_1 + I_2)\omega_f^2$. We rewrite this as

$$K_f = \frac{1}{2}(I_1 + 2I_1)\left(\frac{I_1\omega_i}{I_1 + 2I_1}\right)^2 = \frac{1}{6}I\omega_i^2.$$

Therefore, the fraction lost is

$$\frac{\Delta K}{K_i} = \frac{K_i - K_f}{K_i} = 1 - \frac{K_f}{K_i} = 1 - \frac{I\omega_i^2/6}{I\omega_i^2/2} = \frac{2}{3} = 0.667.$$

LEARN The situation here is analogous to the case of completely inelastic collision, in which some energy is lost but momentum remains conserved.

52. We denote the cockroach with subscript 1 and the disk with subscript 2. The cockroach has a mass $m_1 = m$, while the mass of the disk is $m_2 = 4.00\ m$.

(a) Initially the angular momentum of the system consisting of the cockroach and the disk is

$$L_i = m_1 v_{1i} r_{1i} + I_2\omega_{2i} = m_1\omega_0 R^2 + \frac{1}{2}m_2\omega_0 R^2.$$

After the cockroach has completed its walk, its position (relative to the axis) is $r_{1f} = R/2$ so the final angular momentum of the system is

$$L_f = m_1\omega_f\left(\frac{R}{2}\right)^2 + \frac{1}{2}m_2\omega_f R^2.$$

Then from $L_f = L_i$ we obtain

$$\omega_f\left(\frac{1}{4}m_1R^2+\frac{1}{2}m_2R\right)=\omega_0\left(m_1R^2+\frac{1}{2}m_2R^2\right).$$

Thus,

$$\omega_f=\left(\frac{m_1R^2+m_2R^2/2}{m_1R^2/4+m_2R^2/2}\right)\omega_0=\left(\frac{1+(m_2/m_1)/2}{1/4+(m_2/m_1)/2}\right)\omega_0=\left(\frac{1+2}{1/4\ +\ 2}\right)\omega_0=1.33\omega_0.$$

With ω_0 = 0.260 rad/s, we have ω_f =0.347 rad/s.

(b) We substitute $I = L/\omega$ into $K=\frac{1}{2}I\omega^2$ and obtain $K=\frac{1}{2}L\omega$. Since we have $L_i = L_f$, the kinetic energy ratio becomes

$$\frac{K}{K_0}=\frac{L_f\omega_f/2}{L_i\omega_i/2}=\frac{\omega_f}{\omega_0}=1.33.$$

(c) The cockroach does positive work while walking toward the center of the disk, increasing the total kinetic energy of the system.

53. The axis of rotation is in the middle of the rod, with r = 0.25 m from either end. By Eq. 11-19, the initial angular momentum of the system (which is just that of the bullet, before impact) is $rmv\sin\theta$ where m = 0.003 kg and θ = 60°. Relative to the axis, this is counterclockwise and thus (by the common convention) positive. After the collision, the moment of inertia of the system is

$$I = I_{\text{rod}} + mr^2$$

where $I_{\text{rod}} = ML^2/12$ by Table 10-2(e), with M = 4.0 kg and L = 0.5 m. Angular momentum conservation leads to

$$rmv\sin\theta=\left(\frac{1}{12}ML^2+mr^2\right)\omega.$$

Thus, with ω = 10 rad/s, we obtain

$$v=\frac{\left(\frac{1}{12}(4.0\text{ kg})(0.5\text{ m})^2+(0.003\text{ kg})(0.25\text{ m})^2\right)(10\text{ rad/s})}{(0.25\text{ m})(0.003\text{ kg})\sin 60°}=1.3\times10^3\text{ m/s}.$$

54. We denote the cat with subscript 1 and the ring with subscript 2. The cat has a mass $m_1 = M/4$, while the mass of the ring is $m_2 = M$ = 8.00 kg. The moment of inertia of the ring is $I_2=m_2(R_1^2+R_2^2)/2$ (Table 10-2), and $I_1 = m_1r^2$ for the cat, where r is the perpendicular distance from the axis of rotation.

Initially the angular momentum of the system consisting of the cat (at $r = R_2$) and the ring is

$$L_i = m_1 v_{1i} r_{1i} + I_2\omega_{2i} = m_1\omega_0 R_2^2 + \frac{1}{2} m_2 (R_1^2 + R_2^2)\omega_0 = m_1 R_2^2\,\omega_0 \left[1 + \frac{1}{2}\frac{m_2}{m_1}\left(\frac{R_1^2}{R_2^2} + 1\right)\right].$$

After the cat has crawled to the inner edge at $r = R_1$ the final angular momentum of the system is

$$L_f = m_1\omega_f R_1^2 + \frac{1}{2} m_2 (R_1^2 + R_2^2)\omega_f = m_1 R_1^2\,\omega_f \left[1 + \frac{1}{2}\frac{m_2}{m_1}\left(1 + \frac{R_2^2}{R_1^2}\right)\right].$$

Then from $L_f = L_i$ we obtain

$$\frac{\omega_f}{\omega_0} = \left(\frac{R_2}{R_1}\right)^2 \frac{1 + \frac{1}{2}\frac{m_2}{m_1}\left(\frac{R_1^2}{R_2^2} + 1\right)}{1 + \frac{1}{2}\frac{m_2}{m_1}\left(1 + \frac{R_2^2}{R_1^2}\right)} = (2.0)^2\,\frac{1 + 2(0.25 + 1)}{1 + 2(1 + 4)} = 1.273.$$

Thus, $\omega_f = 1.273\omega_0$. Using ω_0 =8.00 rad/s, we have ω_f =10.2 rad/s. By substituting $I = L/\omega$ into $K = I\omega^2/2$, we obtain $K = L\omega/2$. Since $L_i = L_f$, the kinetic energy ratio becomes

$$\frac{K_f}{K_i} = \frac{L_f\omega_f/2}{L_i\omega_i/2} = \frac{\omega_f}{\omega_0} = 1.273.$$

which implies $\Delta K = K_f - K_i = 0.273 K_i$. The cat does positive work while walking toward the center of the ring, increasing the total kinetic energy of the system.

Since the initial kinetic energy is given by

$$\begin{aligned} K_i &= \frac{1}{2}\left[m_1 R_2^2 + \frac{1}{2} m_2 (R_1^2 + R_2^2)\right]\omega_0^2 = \frac{1}{2} m_1 R_2^2 \omega_0^2 \left[1 + \frac{1}{2}\,\frac{m_2}{m_1}\left(\frac{R_1^2}{R_2^2} + 1\right)\right] \\ &= \frac{1}{2}(2.00\text{ kg})(0.800\text{ m})^2(8.00\text{ rad/s})^2[1+(1/2)(4)(0.5^2+1)] \\ &= 143.36\text{ J}, \end{aligned}$$

the increase in kinetic energy is

$$\Delta K = (0.273)(143.36\text{ J}) = 39.1\text{ J}.$$

55. For simplicity, we assume the record is turning freely, without any work being done by its motor (and without any friction at the bearings or at the stylus trying to slow it down). Before the collision, the angular momentum of the system (presumed positive) is $I_i\omega_i$ where $I_i = 5.0\times10^{-4}\ \text{kg}\cdot\text{m}^2$ and $\omega_i = 4.7$ rad/s. The rotational inertia afterward is

$$I_f = I_i + mR^2$$

where m = 0.020 kg and R = 0.10 m. The mass of the record (0.10 kg), although given in the problem, is not used in the solution. Angular momentum conservation leads to

$$I_i \omega_i = I_f \omega_f \Rightarrow \omega_f = \frac{I_i \omega_i}{I_i + mR^2} = 3.4 \text{ rad/s}.$$

56. Table 10-2 gives the rotational inertia of a thin rod rotating about a perpendicular axis through its center. The angular speeds of the two arms are, respectively,

$$\omega_1 = \frac{(0.500 \text{ rev})(2\pi \text{ rad/rev})}{0.700 \text{ s}} = 4.49 \text{ rad/s}$$
$$\omega_2 = \frac{(1.00 \text{ rev})(2\pi \text{ rad/rev})}{0.700 \text{ s}} = 8.98 \text{ rad/s}.$$

Treating each arm as a thin rod of mass 4.0 kg and length 0.60 m, the angular momenta of the two arms are

$$L_1 = I\omega_1 = mr^2\omega_1 = (4.0 \text{ kg})(0.60 \text{ m})^2(4.49 \text{ rad/s}) = 6.46 \text{ kg}\cdot\text{m}^2/\text{s}$$
$$L_2 = I\omega_2 = mr^2\omega_2 = (4.0 \text{ kg})(0.60 \text{ m})^2(8.98 \text{ rad/s}) = 12.92 \text{ kg}\cdot\text{m}^2/\text{s}.$$

From the athlete's reference frame, one arm rotates clockwise, while the other rotates counterclockwise. Thus, the total angular momentum about the common rotation axis though the shoulders is

$$L = L_2 - L_1 = 12.92 \text{ kg}\cdot\text{m}^2/\text{s} - 6.46 \text{ kg}\cdot\text{m}^2/\text{s} = 6.46 \text{ kg}\cdot\text{m}^2/\text{s}.$$

57. Their angular velocities, when they are stuck to each other, are equal, regardless of whether they share the same central axis. The initial rotational inertia of the system is, using Table 10-2(c),

$$I_0 = I_{\text{bigdisk}} + I_{\text{smalldisk}}$$

where $I_{\text{bigdisk}} = MR^2/2$. Similarly, since the small disk is initially concentric with the big one, $I_{\text{smalldisk}} = \frac{1}{2}mr^2$. After it slides, the rotational inertia of the small disk is found from the parallel axis theorem (using $h = R - r$). Thus, the new rotational inertia of the system is

$$I = \frac{1}{2}MR^2 + \frac{1}{2}mr^2 + m\left(R - r\right)^2.$$

(a) Angular momentum conservation, $I_0\omega_0 = I\omega$, leads to the new angular velocity:

$$\omega = \omega_0 \frac{(MR^2/2) + (mr^2/2)}{(MR^2/2) + (mr^2/2) + m\left(R - r\right)^2}.$$

Substituting $M = 10m$ and $R = 3r$, this becomes $\omega = \omega_0(91/99)$. Thus, with $\omega_0 = 20$ rad/s, we find $\omega = 18$ rad/s.

(b) From the previous part, we know that

$$\frac{I_0}{I} = \frac{91}{99},\ \frac{\omega}{\omega_0} = \frac{91}{99}.$$

Plugging these into the ratio of kinetic energies, we have

$$\frac{K}{K_0} = \frac{I\omega^2/2}{I_0\omega_0^2/2} = \frac{I}{I_0}\left(\frac{\omega}{\omega_0}\right)^2 = \frac{99}{91}\left(\frac{91}{99}\right)^2 = 0.92.$$

58. The initial rotational inertia of the system is $I_i = I_{\text{disk}} + I_{\text{student}}$, where $I_{\text{disk}} = 300\ \text{kg}\cdot\text{m}^2$ (which, incidentally, does agree with Table 10-2(c)) and $I_{\text{student}} = mR^2$ where $m = 60\ \text{kg}$ and $R = 2.0$ m.

The rotational inertia when the student reaches $r = 0.5$ m is $I_f = I_{\text{disk}} + mr^2$. Angular momentum conservation leads to

$$I_i\omega_i = I_f\omega_f \Rightarrow \omega_f = \omega_i \frac{I_{\text{disk}} + mR^2}{I_{\text{disk}} + mr^2}$$

which yields, for $\omega_i = 1.5$ rad/s, a final angular velocity of $\omega_f = 2.6$ rad/s.

59. By angular momentum conservation (Eq. 11-33), the total angular momentum after the explosion must be equal to that before the explosion:

$$L'_p + L'_r = L_p + L_r$$

$$\left(\tfrac{L}{2}\right) m v_{\text{p}} + \tfrac{1}{12} ML^2\ \omega' = I_{\text{p}}\ \omega + \tfrac{1}{12} ML^2\ \omega$$

where one must be careful to avoid confusing the length of the rod ($L = 0.800$ m) with the angular momentum symbol. Note that $I_{\text{p}} = m(L/2)^2$ by Eq.10-33, and

$$\omega' = v_{\text{end}}/r = (v_{\text{p}} - 6)/(L/2),$$

where the latter relation follows from the penultimate sentence in the problem (and "6" stands for "6.00 m/s" here). Since $M = 3m$ and $\omega = 20$ rad/s, we end up with enough information to solve for the particle speed: $v_{\text{p}} = 11.0$ m/s.

60. (a) With r = 0.60 m, we obtain $I = 0.060 + (0.501)r^2 = 0.24$ kg · m^2.

(b) Invoking angular momentum conservation, with SI units understood,

$$\ell_0 = L_f \;\Rightarrow\; mv_0 r = I\omega \;\Rightarrow\; (0.001)v_0(0.60) = (0.24)(4.5)$$

which leads to $v_0 = 1.8 \times 10^3$ m/s.

61. We make the unconventional choice of *clockwise* sense as positive, so that the angular velocities in this problem are positive. With r = 0.60 m and I_0 = 0.12 kg · m^2, the rotational inertia of the putty-rod system (after the collision) is

$$I = I_0 + (0.20)r^2 = 0.19 \text{ kg} \cdot \text{m}^2.$$

Invoking angular momentum conservation $L_0 = L_f$ or $I_0\omega_0 = I\omega$, we have

$$\omega = \frac{I_0}{I}\omega_0 = \frac{0.12 \text{ kg}\cdot\text{m}^2}{0.19 \text{ kg}\cdot\text{m}^2}(2.4\,\text{rad/s}) = 1.5\,\text{rad/s}.$$

62. The aerialist is in extended position with $I_1 = 19.9\,\text{kg}\cdot\text{m}^2$ during the first and last quarter of the turn, so the total angle rotated in t_1 is $\theta_1 = 0.500$ rev. In t_2 he is in a tuck position with $I_2 = 3.93\,\text{kg}\cdot\text{m}^2$, and the total angle rotated is $\theta_2 = 3.500$ rev. Since there is no external torque about his center of mass, angular momentum is conserved, $I_1\omega_1 = I_2\omega_2$. Therefore, the total flight time can be written as

$$t = t_1 + t_2 = \frac{\theta_1}{\omega_1} + \frac{\theta_2}{\omega_2} = \frac{\theta_1}{I_2\omega_2/I_1} + \frac{\theta_2}{\omega_2} = \frac{1}{\omega_2}\left(\frac{I_1}{I_2}\theta_1 + \theta_2\right).$$

Substituting the values given, we find ω_2 to be

$$\omega_2 = \frac{1}{t}\left(\frac{I_1}{I_2}\theta_1 + \theta_2\right) = \frac{1}{1.87\text{ s}}\left(\frac{19.9\text{ kg}\cdot\text{m}^2}{3.93\text{ kg}\cdot\text{m}^2}(0.500\text{ rev}) + 3.50\text{ rev}\right) = 3.23\text{ rev/s}.$$

63. This is a completely inelastic collision, which we analyze using angular momentum conservation. Let m and v_0 be the mass and initial speed of the ball and R the radius of the merry-go-round. The initial angular momentum is

$$\vec{\ell}_0 = \vec{r}_0 \times \vec{p}_0 \Rightarrow \ell_0 = R(mv_0)\cos 37°$$

where ϕ = 37° is the angle between $\vec{v}_0$ and the line tangent to the outer edge of the merry-go-around. Thus, $\ell_0 = 19\,\text{kg}\cdot\text{m}^2/\text{s}$. Now, with SI units understood,

$$\ell_0 = L_f \quad \Rightarrow \quad 19\ \text{kg}\cdot\text{m}^2 = I\omega = \left(150 + (30)R^2 + (1.0)R^2\right)\omega$$

so that ω = 0.070 rad/s.

64. We treat the ballerina as a rigid object rotating around a fixed axis, initially and then again near maximum height. Her initial rotational inertia (trunk and one leg extending outward at a $90°$ angle) is

$$I_i = I_{\text{trunk}} + I_{\text{leg}} = 0.660\ \text{kg}\cdot\text{m}^2 + 1.44\ \text{kg}\cdot\text{m}^2 = 2.10\ \text{kg}\cdot\text{m}^2.$$

Similarly, her final rotational inertia (trunk and *both* legs extending outward at a $\theta = 30°$ angle) is

$$I_f = I_{\text{trunk}} + 2I_{\text{leg}}\sin^2\theta = 0.660\ \text{kg}\cdot\text{m}^2 + 2(1.44\ \text{kg}\cdot\text{m}^2)\sin^2 30° = 1.38\ \text{kg}\cdot\text{m}^2,$$

where we have used the fact that the effective length of the extended leg at an angle θ is $L_\perp = L\sin\theta$ and $I \sim L_\perp^2$. Once airborne, there is no external torque about the ballerina's center of mass and her angular momentum cannot change. Therefore, $L_i = L_f$ or $I_i\omega_i = I_f\omega_f$, and the ratio of the angular speeds is

$$\frac{\omega_f}{\omega_i} = \frac{I_i}{I_f} = \frac{2.10\ \text{kg}\cdot\text{m}^2}{1.38\ \text{kg}\cdot\text{m}^2} = 1.52.$$

65. **THINK** If we consider a short time interval from just before the wad hits to just after it hits and sticks, we may use the principle of conservation of angular momentum. The initial angular momentum is the angular momentum of the falling putty wad.

EXPRESS The wad initially moves along a line that is $d/2$ distant from the axis of rotation, where d is the length of the rod. The angular momentum of the wad is $mvd/2$ where m and v are the mass and initial speed of the wad. After the wad sticks, the rod has angular velocity ω and angular momentum $I\omega$, where I is the rotational inertia of the system consisting of the rod with the two balls (each having a mass M) and the wad at its end. Conservation of angular momentum yields $mvd/2 = I\omega$ where $I = (2M + m)(d/2)^2$. The equation allows us to solve for ω.

ANALYZE (a) With M = 2.00 kg, d = 0.500 m, m = 0.0500 kg, and v = 3.00 m/s, we find the angular speed to be

$$\omega = \frac{mvd}{2I} = \frac{2mv}{(2M + m)d} = \frac{2(0.0500\ \text{kg})(3.00\ \text{m/s})}{\left(2(2.00\ \text{kg}) + 0.0500\ \text{kg}\right)(0.500\ \text{m})}$$
$$= 0.148\ \text{rad/s}.$$

(b) The initial kinetic energy is $K_i = \frac{1}{2}mv^2$, the final kinetic energy is $K_f = \frac{1}{2}I\omega^2$, and their ratio is

$$K_f/K_i = I\omega^2/mv^2 .$$

When $I = (2M + m)d^2/4$ and $\omega = 2mv/(2M + m)d$ are substituted, the ratio becomes

$$\frac{K_f}{K_i} = \frac{m}{2M+m} = \frac{0.0500\text{ kg}}{2(2.00\text{ kg}) + 0.0500\text{ kg}} = 0.0123.$$

(c) As the rod rotates, the sum of its kinetic and potential energies is conserved. If one of the balls is lowered a distance *h*, the other is raised the same distance and the sum of the potential energies of the balls does not change. We need consider only the potential energy of the putty wad. It moves through a 90° arc to reach the lowest point on its path, gaining kinetic energy and losing gravitational potential energy as it goes. It then swings up through an angle θ, losing kinetic energy and gaining potential energy, until it momentarily comes to rest. Take the lowest point on the path to be the zero of potential energy. It starts a distance $d/2$ above this point, so its initial potential energy is $U_i = mg(d/2)$. If it swings up to the angular position θ, as measured from its lowest point, then its final height is $(d/2)(1 - \cos\theta)$ above the lowest point and its final potential energy is

$$U_f = mg(d/2)(1 - \cos\theta).$$

The initial kinetic energy is the sum of that of the balls and wad:

$$K_i = \frac{1}{2}I\omega^2 = \frac{1}{2}(2M+m)(d/2)^2\,\omega^2.$$

At its final position, we have $K_f = 0$. Conservation of energy provides the relation:

$$U_i + K_i = U_f + K_f \quad \Rightarrow \quad mg\frac{d}{2} + \frac{1}{2}(2M+m)\left(\frac{d}{2}\right)^2 \omega^2 = mg\frac{d}{2}(1-\cos\theta).$$

When this equation is solved for $\cos\theta$, the result is

$$\begin{aligned}\cos\theta &= -\frac{1}{2}\left(\frac{2M+m}{mg}\right)\left(\frac{d}{2}\right)\omega^2 \\ &= -\frac{1}{2}\left(\frac{2(2.00\text{ kg}) + 0.0500\text{ kg}}{(0.0500\text{ kg})(9.8\text{ m/s}^2)}\right)\left(\frac{0.500\text{ m}}{2}\right)(0.148\text{ rad/s})^2 \\ &= -0.0226.\end{aligned}$$

Consequently, the result for θ is 91.3°. The total angle through which it has swung is 90° + 91.3° = 181°.

LEARN This problem is rather involved. To summarize, we calculated ω using angular momentum conservation. Some energy is lost due to the inelastic collision between the putty wad and one of the balls. However, in the subsequent motion, energy is conserved, and we apply energy conservation to find the angle at which the system comes to rest momentarily.

66. We make the unconventional choice of *clockwise* sense as positive, so that the angular velocities (and angles) in this problem are positive. Mechanical energy conservation applied to the particle (before impact) leads to

$$mgh = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gh}$$

for its speed right before undergoing the completely inelastic collision with the rod. The collision is described by angular momentum conservation:

$$mvd = \left(I_{\text{rod}} + md^2\right)\omega$$

where I_{rod} is found using Table 10-2(e) and the parallel axis theorem:

$$I_{\text{rod}} = \frac{1}{12}Md^2 + M\left(\frac{d}{2}\right)^2 = \frac{1}{3}Md^2.$$

Thus, we obtain the angular velocity of the system immediately after the collision:

$$\omega = \frac{md\sqrt{2gh}}{(Md^2/3) + md^2}$$

which means the system has kinetic energy $\left(I_{\text{rod}} + md^2\right)\omega^2/2$, which will turn into potential energy in the final position, where the block has reached a height H (relative to the lowest point) and the center of mass of the stick has increased its height by $H/2$. From trigonometric considerations, we note that $H = d(1 - \cos\theta)$, so we have

$$\frac{1}{2}\left(I_{\text{rod}} + md^2\right)\omega^2 = mgH + Mg\frac{H}{2} \;\Rightarrow\; \frac{1}{2}\frac{m^2d^2\left(2gh\right)}{(Md^2/3) + md^2} = \left(m + \frac{M}{2}\right)gd\left(1 - \cos\theta\right)$$

from which we obtain

$$\theta = \cos^{-1}\left(1 - \frac{m^2 h}{(m+M/2)\,(m+M/3)}\right) = \cos^{-1}\left(1 - \frac{h/d}{(1+M/2m)\,(1+M/3m)}\right)$$
$$= \cos^{-1}\left(1 - \frac{(20\text{ cm}/\ 40\text{ cm})}{(1+1)(1+2/3)}\right) = \cos^{-1}(0.85)$$
$$= 32°.$$

67. (a) We consider conservation of angular momentum (Eq. 11-33) about the center of the rod:

$$L_i = L_f \;\Rightarrow\; -dmv + \frac{1}{12}ML^2\omega = 0$$

where negative is used for "clockwise." Item (*e*) in Table 11-2 and Eq. 11-21 (with $r_\perp = d$) have also been used. This leads to

$$d = \frac{ML^2\,\omega}{12\,m\,v} = \frac{M(0.60\text{ m})^2\,(80\text{ rad/s})}{12(M/3)(40\text{ m/s})} = 0.180\text{ m}.$$

(b) Increasing *d* causes the magnitude of the negative (clockwise) term in the above equation to increase. This would make the total angular momentum negative before the collision, and (by Eq. 11-33) also negative afterward. Thus, the system would rotate clockwise if *d* were greater.

68. (a) The angular speed of the top is $\omega = 30\text{ rev/s} = 30(2\pi)\text{ rad/s}$. The precession rate of the top can be obtained by using Eq. 11-46:

$$\Omega = \frac{Mgr}{I\omega} = \frac{(0.50\text{ kg})(9.8\text{ m/s}^2)(0.040\text{ m})}{(5.0\times10^{-4}\text{ kg}\cdot\text{m}^2)(60\pi\text{ rad/s})} = 2.08\text{ rad/s} \approx 0.33\text{ rev/s}.$$

(b) The direction of the precession is clockwise as viewed from overhead.

69. The precession rate can be obtained by using Eq. 11-46 with *r* = (11/2) cm = 0.055 m. Noting that $I_{\text{disk}} = MR^2/2$ and its angular speed is

$$\omega = 1000\text{ rev/min} = \frac{2\pi(1000)}{60}\text{ rad/s} \approx 1.0\times10^2\text{ rad/s},$$

we have

$$\Omega = \frac{Mgr}{(MR^2/2)\omega} = \frac{2gr}{R^2\omega} = \frac{2(9.8\text{ m/s}^2)(0.055\text{ m})}{(0.50\text{ m})^2(1.0\times10^2\text{ rad/s})} \approx 0.041\text{ rad/s}.$$

70. Conservation of energy implies that mechanical energy at maximum height up the ramp is equal to the mechanical energy on the floor. Thus, using Eq. 11-5, we have

$$\frac{1}{2}mv_f^2 + \frac{1}{2}I_{\text{com}}\omega_f^2 + mgh = \frac{1}{2}mv^2 + \frac{1}{2}I_{\text{com}}\omega^2$$

where $v_f = \omega_f = 0$ at the point on the ramp where it (momentarily) stops. We note that the height h relates to the distance traveled along the ramp d by $h = d\sin(15°)$. Using item (*f*) in Table 10-2 and Eq. 11-2, we obtain

$$mgd\sin 15° = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2.$$

After canceling m and plugging in $d = 1.5$ m, we find $v = 2.33$ m/s.

71. **THINK** The applied force gives rise to a torque that causes the cylinder to rotate to the right at a constant angular acceleration.

EXPRESS We make the unconventional choice of *clockwise* sense as positive, so that the angular acceleration is positive (as is the linear acceleration of the center of mass, since we take rightwards as positive). We approach this in the manner of Eq. 11-3 (*pure rotation* about point P) but use torques instead of energy. The torque (relative to point P) is $\tau = I_P\alpha$, where

$$I_P = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

with the use of the parallel-axis theorem and Table 10-2(c). The torque is due to the F_{app} force and can be written as $\tau = F_{\text{app}}(2R)$. In this way, we find

$$\tau = I_P\alpha = \left(\frac{3}{2}MR^2\right)\alpha = 2RF_{\text{app}}.$$

The equation allows us to solve for the angular acceleration α, which is related to the acceleration of the center of mass as $\alpha = a_{\text{com}}/R$.

ANALYZE (a) With $M = 10\text{ kg}$, $R = 0.10$ m and $F_{\text{app}} = 12$ N, we obtain

$$a_{\text{com}} = \alpha R = \frac{2R^2F_{\text{app}}}{3MR^2/2} = \frac{4F_{\text{app}}}{3M} = \frac{4(12\text{ N})}{3(10\text{ kg})} = 1.6\text{ m/s}^2.$$

(b) The magnitude of the angular acceleration is

$$\alpha = a_{\text{com}}/R = (1.6\text{ m/s}^2)/(0.10\text{ m}) = 16\text{ rad/s}^2.$$

(c) Applying Newton's second law in its linear form yields $(12\,\text{N}) - f = Ma_{\text{com}}$. Therefore, $f = -4.0$ N. Contradicting what we assumed in setting up our force equation, the friction force is found to point *rightward* with magnitude 4.0 N, i.e., $\vec{f} = (4.0\ \text{N})\hat{\text{i}}$.

LEARN As the cylinder rolls to the right, the frictional force also points to the right to oppose the tendency to slip.

72. The rotational kinetic energy is $K = \frac{1}{2}I\omega^2$, where $I = mR^2$ is its rotational inertia about the center of mass (Table 10-2(a)), $m = 140$ kg, and $\omega = v_{\text{com}}/R$ (Eq. 11-2). The ratio is

$$\frac{K_{\text{transl}}}{K_{\text{rot}}} = \frac{\frac{1}{2}mv_{\text{com}}^2}{\frac{1}{2}(mR^2)(v_{\text{com}}/R)^2} = 1.00.$$

73. This problem involves the vector cross product of vectors lying in the *xy* plane. For such vectors, if we write $\vec{r}\,' = x'\hat{\text{i}} + y'\hat{\text{j}}$, then (using Eq. 3-30) we find

$$\vec{r}\,' \times \vec{v} = (x'v_y - y'v_x)\hat{\text{k}}.$$

(a) Here, $\vec{r}\,'$ points in either the $+\hat{\text{i}}$ or the $-\hat{\text{i}}$ direction (since the particle moves along the x axis). It has no y' or z' components, and neither does $\vec{v}$, so it is clear from the above expression (or, more simply, from the fact that $\hat{\text{i}} \times \hat{\text{i}} = 0$) that $\vec{\ell} = m(\vec{r}\,' \times \vec{v}) = 0$ in this case.

(b) The net force is in the $-\hat{\text{i}}$ direction (as one finds from differentiating the velocity expression, yielding the acceleration), so, similar to what we found in part (a), we obtain $\tau = \vec{r}\,' \times \vec{F} = 0$.

(c) Now, $\vec{r}\,' = \vec{r} - \vec{r}_{\text{o}}$ where $\vec{r}_{\text{o}} = 2.0\hat{\text{i}} + 5.0\hat{\text{j}}$ (with SI units understood) and points from (2.0, 5.0, 0) to the instantaneous position of the car (indicated by $\vec{r}$, which points in either the $+x$ or $-x$ directions, or nowhere (if the car is passing through the origin)). Since $\vec{r} \times \vec{v} = 0$ we have (plugging into our general expression above)

$$\vec{\ell} = m(\vec{r}\,' \times \vec{v}) = -m(\vec{r}_{\text{o}} \times \vec{v}) = -(3.0)\big((2.0)(0) - (5.0)(-2.0t^3)\big)\hat{\text{k}}$$

which yields $\vec{\ell} = (-30t^3\hat{\text{k}})\ \text{kg}\cdot\text{m/s}^2$.

(d) The acceleration vector is given by $\vec{a} = \frac{d\vec{v}}{dt} = -6.0t^2\hat{\text{i}}$ in SI units, and the net force on the car is $m\vec{a}$. In a similar argument to that given in the previous part, we have

$$\vec{\tau} = m(\vec{r}\,' \times \vec{a}) = -m(\vec{r}_{\text{o}} \times \vec{a}) = -(3.0)\big((2.0)(0) - (5.0)(-6.0t^2)\big)\hat{\text{k}}$$

which yields $\vec{\tau} = (-90t^2\hat{\mathrm{k}})$ N·m.

(e) In this situation, $\vec{r}\,' = \vec{r} - \vec{r}_o$ where $\vec{r}_o = 2.0\hat{\mathrm{i}} - 5.0\hat{\mathrm{j}}$ (with SI units understood) and points from (2.0, –5.0, 0) to the instantaneous position of the car (indicated by $\vec{r}$, which points in either the $+x$ or $-x$ directions, or nowhere (if the car is passing through the origin)). Since $\vec{r} \times \vec{v} = 0$ we have (plugging into our general expression above)

$$\vec{\ell} = m(\vec{r}\,' \times \vec{v}) = -m(\vec{r}_o \times \vec{v}) = -(3.0)\big((2.0)(0) - (-5.0)(-2.0t^3)\big)\hat{\mathrm{k}}$$

which yields $\vec{\ell} = (30t^3\hat{\mathrm{k}})$ kg·m²/s.

(f) Again, the acceleration vector is given by $\vec{a} = -6.0t^2\hat{\mathrm{i}}$ in SI units, and the net force on the car is $m\vec{a}$. In a similar argument to that given in the previous part, we have

$$\vec{\tau} = m(\vec{r}\,' \times \vec{a}) = -m(\vec{r}_o \times \vec{a}) = -(3.0)\big((2.0)(0) - (-5.0)(-6.0t^2)\big)\hat{\mathrm{k}}$$

which yields $\vec{\tau} = (90t^2\hat{\mathrm{k}})$ N·m.

74. For a constant (single) torque, Eq. 11-29 becomes

$$\vec{\tau} = \frac{d\vec{L}}{dt} = \frac{\Delta\vec{L}}{\Delta t}.$$

Thus, we obtain

$$\Delta t = \frac{\Delta L}{\tau} = \frac{600\ \mathrm{kg\cdot m^2/s}}{50\ \mathrm{N\cdot m}} = 12\ \mathrm{s}\,.$$

75. **THINK** No external torque acts on the system consisting of the child and the merry-go-round, so the total angular momentum of the system is conserved.

EXPRESS An object moving along a straight line has angular momentum about any point that is not on the line. The magnitude of the angular momentum of the child about the center of the merry-go-round is given by Eq. 11-21, mvR, where R is the radius of the merry-go-round.

ANALYZE (a) In terms of the radius of gyration k, the rotational inertia of the merry-go-round is $I = Mk^2$. With M = 180 kg and k = 0.91 m, we obtain

$$I = (180\ \mathrm{kg})\,(0.910\ \mathrm{m})^2 = 149\ \mathrm{kg\cdot m^2}.$$

(b) The magnitude of angular momentum of the running child about the axis of rotation of the merry-go-round is

$$L_{\text{child}} = mvR = (44.0 \text{ kg})(3.00 \text{ m/s})(1.20 \text{ m}) = 158 \text{ kg}\cdot\text{m}^2/\text{s}.$$

(c) The initial angular momentum is given by $L_i = L_{\text{child}} = mvR$; the final angular momentum is given by $L_f = (I + mR^2)\ \omega$, where ω is the final common angular velocity of the merry-go-round and child. Thus $mvR = (I + mR^2)\omega$ and

$$\omega = \frac{mvR}{I + mR^2} = \frac{158 \text{ kg}\cdot\text{m}^2/\text{s}}{149 \text{ kg}\cdot\text{m}^2 + (44.0 \text{ kg})(1.20 \text{ m})^2} = 0.744 \text{ rad/s}.$$

LEARN The child initially had an angular velocity of

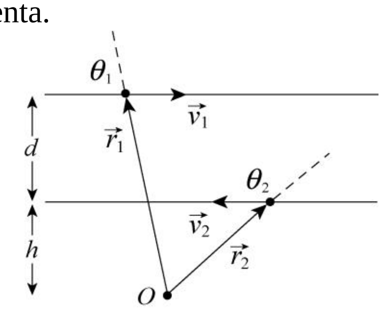
$$\omega_0 = \frac{v}{R} = \frac{3.00 \text{ m/s}}{1.20 \text{ m}} = 2.5 \text{ rad/s}.$$

After he jumped onto the merry-go-round, the rotational inertia of the system (merry-go-round + child) increases, so the angular velocity decreases by angular momentum conservation.

76. Item (*i*) in Table 10-2 gives the moment of inertia about the center of mass in terms of width *a* (0.15 m) and length *b* (0.20 m). In using the parallel axis theorem, the distance from the center to the point about which it spins (as described in the problem) is $\sqrt{(a/4)^2 + (b/4)^2}$. If we denote the thickness as *h* (0.012 m) then the volume is *abh*, which means the mass is ρabh (where ρ = 2640 kg/m^3 is the density). We can write the kinetic energy in terms of the angular momentum by substituting $\omega = L/I$ into Eq. 10-34:

$$K = \frac{1}{2}\frac{L^2}{I} = \frac{1}{2}\frac{(0.104)^2}{\rho abh((a^2 + b^2)/12 + (a/4)^2 + (b/4)^2)} = 0.62 \text{ J}.$$

77. **THINK** Our system consists of two particles moving in opposite directions along parallel lines. The angular momentum of the system about a point is the vector sum of the two individual angular momenta.



EXPRESS The diagram above shows the particles and their lines of motion. The origin is marked *O* and may be anywhere. We set up our coordinate system in such a way that

+x is to the right, +y up and +z out of the page. The angular momentum of the system about O is

$$\vec{\ell} = \vec{\ell}_1 + \vec{\ell}_2 = \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2 = m(\vec{r}_1 \times \vec{v}_1 + \vec{r}_2 \times \vec{v}_2)$$

since $m_1 = m_2 = m$.

ANALYZE (a) With $\vec{v}_1 = v_1\hat{\text{i}}$, the angular momentum of particle 1 has magnitude

$$\ell_1 = mvr_1 \sin\theta_1 = mv\left(d+h\right)$$

and is in the –z-direction, or into the page. On the other hand, with $\vec{v}_2 = -v_2\hat{\text{i}}$, the angular momentum of particle 2 has magnitude $\ell_2 = mvr_2 \sin\theta_2 = mvh$, and is in the +$z$-direction, or out of the page. The net angular momentum has magnitude

$$\ell = mv(d+h) - mvh = mvd$$

which depends only on the separation between the two lines and not on the location of the origin. Thus, if O is midway between the two lines, the total angular momentum is

$$\ell = mvd = (2.90\times10^{-4}\ \text{kg})(5.46\ \text{m/s})(0.042\ \text{m}) = 6.65\times10^{-5}\ \text{kg}\cdot\text{m}^2/\text{s}$$

and is into the page.

(b) As indicated above, the expression does not change.

(c) Suppose particle 2 is traveling to the right. Then

$$\ell = mv(d+h) + mvh = mv(d+2h).$$

This result now depends on h, the distance from the origin to one of the lines of motion. If the origin is midway between the lines of motion, then $h = -d/2$ and $\ell = 0$.

(d) As we have seen in part (c), the result depends on the choice of origin.

LEARN Angular momentum is a vector quantity. For a system of many particles, the total angular momentum about a point is

$$\vec{\ell} = \vec{\ell}_1 + \vec{\ell}_2 + \cdots = \sum_i \vec{\ell}_i = \sum_i m_i \vec{r}_i \times \vec{v}_i\ .$$

78. (a) Using Eq. 2-16 for the translational (center-of-mass) motion, we find

$$v^2 = v_0^2 + 2a\Delta x \Rightarrow a = -\frac{v_0^2}{2\Delta x}$$

which yields a = –4.11 for v_0 = 43 and $\Delta x = 225$ (SI units understood). The magnitude of the linear acceleration of the center of mass is therefore 4.11 m/s^2.

(b) With R = 0.250 m, Eq. 11-6 gives

$$|\alpha| = |a| / R = 16.4\,\text{rad/s}^2.$$

If the wheel is going rightward, it is rotating in a clockwise sense. Since it is slowing down, this angular acceleration is counterclockwise (opposite to ω) so (with the usual convention that counterclockwise is positive) there is no need for the absolute value signs for α.

(c) Equation 11-8 applies with Rf_s representing the magnitude of the frictional torque. Thus,

$$Rf_s = I\alpha = (0.155\ \text{kg}\cdot\text{m}^2)\,(16.4\ \text{rad/s}^2) = 2.55\ \text{N}\cdot\text{m}.$$

79. We use $L = I\omega$ and $K = \frac{1}{2}I\omega^2$ and observe that the speed of points on the rim (corresponding to the speed of points on the belt) of wheels A and B must be the same (so $\omega_A R_A = \omega_B r_B$).

(a) If $L_A = L_B$ (call it L) then the ratio of rotational inertias is

$$\frac{I_A}{I_B} = \frac{L/\omega_A}{L/\omega_B} = \frac{\omega_A}{\omega_B} = \frac{R_A}{R_B} = \frac{1}{3} = 0.333.$$

(b) If we have $K_A = K_B$ (call it K) then the ratio of rotational inertias becomes

$$\frac{I_A}{I_B} = \frac{2K/\omega_A^2}{2K/\omega_B^2} = \left(\frac{\omega_B}{\omega_A}\right)^2 = \left(\frac{R_A}{R_B}\right)^2 = \frac{1}{9} = 0.111.$$

80. The total angular momentum (about the origin) before the collision (using Eq. 11-18 and Eq. 3-30 for each particle and then adding the terms) is

$$\vec{L}_i = [(0.5\ \text{m})(2.5\ \text{kg})(3.0\ \text{m/s}) + (0.1\ \text{m})(4.0\ \text{kg})(4.5\ \text{m/s})]\hat{k}.$$

The final angular momentum of the stuck-together particles (after the collision) measured relative to the origin is (using Eq. 11-33)

$$\vec{L}_f = \vec{L}_i = (5.55\ \text{kg}\cdot\text{m}^2/\text{s})\hat{k}.$$

81. **THINK** As the wheel rolls without slipping down an inclined plane, its gravitational potential energy is converted into translational and rotational kinetic energies.

EXPRESS As the wheel-axel system rolls down the inclined plane by a distance d, the change in potential energy is $\Delta U = -mgd\sin\theta$. By energy conservation, the total kinetic energy gained is

$$-\Delta U = \Delta K = \Delta K_{\text{trans}} + \Delta K_{\text{rot}} \quad \Rightarrow \quad mgd\sin\theta = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 .$$

Since the axel rolls without slipping, the angular speed is given by $\omega = v/r$, where r is the radius of the axel. The above equation then becomes

$$mgd\sin\theta = \frac{1}{2}I\omega^2\left(\frac{mr^2}{I}+1\right) = \Delta K_{\text{rot}}\left(\frac{mr^2}{I}+1\right).$$

ANALYZE (a) With m=10.0 kg, d = 2.00 m, r = 0.200 m, and $I = 0.600\ \text{kg}\cdot\text{m}^2$, the rotational kinetic energy may be obtained as

$$\Delta K_{\text{rot}} = \frac{mgd\sin\theta}{\dfrac{mr^2}{I}+1} = \frac{(10.0\ \text{kg})(9.80\ \text{m/s}^2)(2.00\ \text{m})\sin 30.0°}{\dfrac{(10.0\ \text{kg})(0.200\ \text{m})^2}{0.600\ \text{kg}\cdot\text{m}^2}+1} = 58.8\ \text{J} .$$

(b) The translational kinetic energy is $\Delta K_{\text{trans}} = \Delta K - \Delta K_{\text{rot}} = 98\ \text{J} - 58.8\ \text{J} = 39.2\ \text{J}$.

LEARN One may show that $mr^2/I = 2/3$, which implies that $\Delta K_{\text{trans}}/\Delta K_{\text{rot}} = 2/3$. Equivalently, we may write $\Delta K_{\text{trans}}/\Delta K = 2/5$ and $\Delta K_{\text{rot}}/\Delta K = 3/5$. So as the wheel rolls down, 40% of the kinetic energy is translational while the other 60% is rotational.

82. (a) We use Table 10-2(e) and the parallel-axis theorem to obtain the rod's rotational inertia about an axis through one end:

$$I = I_{\text{com}} + Mh^2 = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \frac{1}{3}ML^2$$

where L = 6.00 m and M = 10.0/9.8 = 1.02 kg. Thus, the inertia is $I = 12.2\ \text{kg}\cdot\text{m}^2$.

(b) Using ω = (240)(2π/60) = 25.1 rad/s, Eq. 11-31 gives the magnitude of the angular momentum as

$$I\omega = \left(12.2\ \text{kg}\cdot\text{m}^2\right)\left(25.1\text{rad/s}\right) = 308\ \ \text{kg}\cdot\text{m}^2/\text{s} .$$

Since it is rotating clockwise as viewed from above, then the right-hand rule indicates that its direction is down.

83. We note that its mass is M = 36/9.8 = 3.67 kg and its rotational inertia is $I_{\text{com}} = \frac{2}{5}MR^2$ (Table 10-2(f)).

(a) Using Eq. 11-2, Eq. 11-5 becomes

$$K = \frac{1}{2}I_{\text{com}}\omega^2 + \frac{1}{2}Mv_{\text{com}}^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v_{\text{com}}}{R}\right)^2 + \frac{1}{2}Mv_{\text{com}}^2 = \frac{7}{10}Mv_{\text{com}}^2$$

which yields K = 61.7 J for v_{com} = 4.9 m/s.

(b) This kinetic energy turns into potential energy Mgh at some height $h = d \sin \theta$ where the sphere comes to rest. Therefore, we find the distance traveled up the θ = 30° incline from energy conservation:

$$\frac{7}{10}Mv_{\text{com}}^2 = Mgd\sin\theta \quad \Rightarrow \quad d = \frac{7v_{\text{com}}^2}{10g\sin\theta} = 3.43\,\text{m}.$$

(c) As shown in the previous part, M cancels in the calculation for d. Since the answer is independent of mass, then it is also independent of the sphere's weight.

84. (a) The acceleration is given by Eq. 11-13:

$$a_{\text{com}} = \frac{g}{1 + I_{\text{com}}/MR_0^2}$$

where upward is the positive translational direction. Taking the coordinate origin at the initial position, Eq. 2-15 leads to

$$y_{\text{com}} = v_{\text{com},0}t + \frac{1}{2}a_{\text{com}}t^2 = v_{\text{com},0}t - \frac{\frac{1}{2}gt^2}{1 + I_{\text{com}}/MR_0^2}$$

where y_{com} = – 1.2 m and $v_{\text{com},0}$ = – 1.3 m/s. Substituting $I_{\text{com}} = 0.000095\ \text{kg}\cdot\text{m}^2$, M = 0.12 kg, R_0 = 0.0032 m, and g = 9.8 m/s^2, we use the quadratic formula and find

$$\begin{aligned} t &= \frac{\left(1 + \frac{I_{\text{com}}}{MR_0^2}\right)\left(v_{\text{com},0} \mp \sqrt{v_{\text{com},0}^2 - \frac{2gy_{\text{com}}}{1+I_{\text{com}}/MR_0^2}}\right)}{g} \\ &= \frac{\left(1 + \frac{0.000095}{(0.12)(0.0032)^2}\right)\left(-1.3 \mp \sqrt{(1.3)^2 - \frac{2(9.8)(-1.2)}{1+0.000095/(0.12)(0.0032)^2}}\right)}{9.8} \\ &= -21.7 \text{ or } 0.885 \end{aligned}$$

where we choose t = 0.89 s as the answer.

(b) We note that the initial potential energy is $U_i = Mgh$ and h = 1.2 m (using the bottom as the reference level for computing U). The initial kinetic energy is as shown in Eq. 11-5, where the initial angular and linear speeds are related by Eq. 11-2. Energy conservation leads to

$$\begin{aligned} K_f &= K_i + U_i = \frac{1}{2} m v_{\text{com},0}^2 + \frac{1}{2} I \left(\frac{v_{\text{com},0}}{R_0} \right)^2 + Mgh \\ &= \frac{1}{2}(0.12 \text{ kg})(1.3 \text{ m/s})^2 + \frac{1}{2}\left(9.5\times10^{-5} \text{ kg}\cdot\text{m}^2\right)\left(\frac{1.3 \text{ m/s}}{0.0032 \text{ m}}\right)^2 + (0.12 \text{ kg})\left(9.8 \text{ m/s}^2\right)(1.2 \text{ m}) \\ &= 9.4 \text{ J}. \end{aligned}$$

(c) As it reaches the end of the string, its center of mass velocity is given by Eq. 2-11:

$$v_{\text{com}} = v_{\text{com},0} + a_{\text{com}} t = v_{\text{com},0} - \frac{gt}{1 + I_{\text{com}}/MR_0^2}.$$

Thus, we obtain

$$v_{\text{com}} = -1.3 \text{ m/s} - \frac{\left(9.8 \text{ m/s}^2\right)(0.885 \text{ s})}{1 + \dfrac{0.000095 \text{ kg}\cdot\text{m}^2}{(0.12 \text{ kg})(0.0032 \text{ m})^2}} = -1.41 \text{ m/s}$$

so its linear speed at that moment is approximately 1.4 m/s.

(d) The translational kinetic energy is

$$\tfrac{1}{2} m v_{\text{com}}^2 = \tfrac{1}{2}(0.12 \text{ kg})(-1.41 \text{ m/s})^2 = 0.12 \text{ J}.$$

(e) The angular velocity at that moment is given by

$$\omega = -\frac{v_{\text{com}}}{R_0} = -\frac{-1.41 \text{ m/s}}{0.0032 \text{ m}} = 441 \text{ rad/s} \approx 4.4\times10^2 \text{ rad/s}.$$

(f) And the rotational kinetic energy is

$$\frac{1}{2} I_{\text{com}} \omega^2 = \frac{1}{2}\left(9.50\times10^{-5} \text{ kg}\cdot\text{m}^2\right)(441 \text{ rad/s})^2 = 9.2 \text{ J}.$$

85. The initial angular momentum of the system is zero. The final angular momentum of the girl-plus-merry-go-round is $(I + MR^2)\,\omega$, which we will take to be positive. The final angular momentum we associate with the thrown rock is negative: $-mRv$, where v is the speed (positive, by definition) of the rock relative to the ground.

(a) Angular momentum conservation leads to

$$0 = \left(I + MR^2\right)\omega - mRv \quad \Rightarrow \quad \omega = \frac{mRv}{I + MR^2}.$$

(b) The girl's linear speed is given by Eq. 10-18:

$$R\omega = \frac{mvR^2}{I + MR^2}.$$

86. (a) Interpreting *h* as the height increase for the center of mass of the body, then (using Eq. 11-5) mechanical energy conservation, $K_i = U_f$, leads to

$$\frac{1}{2}mv_{\text{com}}^2 + \frac{1}{2}I\omega^2 = mgh \quad \Rightarrow \quad \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 = mg\left(\frac{3v^2}{4g}\right)$$

from which *v* cancels and we obtain $I = \frac{1}{2}mR^2$.

(b) From Table 10-2(c), we see that the body could be a solid cylinder.