PHYSICS 210B : NONEQUILIBRIUM STATISTICAL PHYSICS
HW ASSIGNMENT #4 : DIFFUSION

(1) A diffusing particle is confined to the interval $[0, L]$. The diffusion constant is D and the drift velocity is $v_{\rm D}$. The boundary at $x = 0$ is absorbing and that at $x = L$ is reflecting.

(a) Calculate the mean and mean square time for the particle to get absorbed at $x = 0$ if it starts at $t = 0$ from $x = L$. Examine in detail the cases $v_{\rm D} > 0$, $v_{\rm D} = 0$, and $v_{\rm D} < 0$.

(b) Compute the Laplace transform of the distribution of trapping times for the cases $v_{\rm D} > 0$, $v_{\rm D} = 0$, and $v_{\rm D} < 0$, and discuss the asymptotic behaviors of these distributions in the limits $t \to 0$ and $t \to \infty$.

(2) Consider a continuum model of a polymer, where the position $\boldsymbol{R}(s) = \big(a/\sqrt{d}\big)\,\boldsymbol{W}(s)$, where $\boldsymbol{W}(s) = \{W_1(s), \ldots, W_d(s)\}$ is a d-dimensional Wiener process, with $s \in [0, N]$, where N is the length of the polymer in units of the persistence length a. The density, in units of mass per persistence length, is

$$\rho(\boldsymbol{r}) = \int\limits_0^N \! ds\, \delta\big(\boldsymbol{r} - \boldsymbol{R}(s)\big) \quad .$$

Show that the structure factor $S(\boldsymbol{k}) = N^{-1}\big\langle\big|\hat\rho(\boldsymbol{k})\big|^2\big\rangle$, where $\hat\rho(\boldsymbol{k})$ is the Fourier transform of the density, is of the Debye form,

$$S(\boldsymbol{k}) = 2\,(R_0/a)^2\, f(k^2 R_0^2/2d) \quad ,$$

where $f(x) = \big(e^{-x} - 1 + x\big)/x^2$.

(3) Verify that the distribution

$$\varPi\big[h(x)\big] = \exp\left\{ -\frac{D}{\varGamma} \int\limits_{-\infty}^{\infty} \! dx \left(\frac{\partial h}{\partial x}\right)^{\!2} \right\}$$

solves the functional Fokker-Planck equation for the one-dimensional KPZ equation.

(4) Consider the Mullins equation,

$$\frac{\partial h}{\partial t} = -\nu\, \nabla^4 h + \eta\ ,$$

where $\nabla^4 = (\boldsymbol{\nabla}^2)^2$.

(a) Use dimensional analysis and linearity to show how the interface width $w(t)$ scales with the parameters and time. For what dimensions does the noise roughen the interface?

(b) Compute the interface width and the two point correlation function in dimensions $d = 1$, $d = 2$, and $d = 3$.