Formula sheet

Constants and Factors

Speed of light: $c = 299,792,458$ m/s exactly (about 3×10^8 meters/sec)
Newton's constant $G = 6.67 \times 10^{-11}$ m^3/s^2 kg
Earth constants: Acceleration of gravity at surface: $g = 9.8$m/s^2, Mass: $M_{\text{Earth}} = 5.97 \times 10^{24}$ kg, radius: $r_{\text{Earth}} = 6.37 \times 10^6$m
Mass of Sun and Moon: $M_\odot = 2 \times 10^{30}$ kg, $M_{moon} = 7.4 \times 10^{22}$ kg
Distance to Sun and Moon from Earth: $D_\odot = 150 \times 10^6$km, $D_{moon} = 384,000$ km
Mass of proton and neutron about 1.67 $\times 10^{-27}$ kg
Mass of electron: $m_e = 9.11 \times 10^{-31}$kg
Density of air: $\rho = 1.2$ kg/m^3; Density of water: $\rho = 1000$ kg/m^3;
1 mile = 1609 m; 1 foot = 0.3048 m; 1 foot = 12 inches; 1 mile = 5280 ft
1 pound (lb) = 4.448 Newton, corresponding to the weight from mass of 0.454 kg; 1 ton = 2000 lb
1 dyne = 10^{-5} Newton; Newton = kg m /s^2; Joule = Newton m; Watt = Joule/sec

Formulas

Velocity as a derivative of position: $\vec{v} = d\vec{r}/dt$
Acceleration as a derivative of velocity: $\vec{a} = d\vec{v}/dt = d^2\vec{r}/dt^2$
For **constant** acceleration: $\vec{v} = \vec{v}_0 + \vec{a}t$
For **constant** acceleration: $\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$
For **constant** acceleration in a straight line: $v^2 = v_0^2 + 2a(x - x_0)$
Frame of reference: $\vec{v}' = \vec{v} - \vec{V}$; $\vec{v}$ is velocity w.r.t. frame S, $\vec{v}'$ is w.r.t. frame S' which moves at $\vec{V}$ w.r.t. frame S
Projectile trajectory: (start at $x = 0$, $y = 0$, speed v_0, angle θ_0: $y = x\tan\theta_0 - gx^2/(2v_0^2\cos^2\theta_0)$
Range of projectile above: $x = (v_0^2/g)\sin 2\theta_0$
Circular motion at constant speed: $a = v^2/r$, toward center of circle
Non-uniform circular motion: radial: $a_r = v^2/r$, tangential: $a_t = dv/dt$

Newton's force law: $\vec{F}_{net} = d\vec{p}/dt$, where momentum is $\vec{p} = m\vec{v}$; or if constant mass: $\vec{F}_{net} = m\vec{a}$
Weight: $\vec{W} = m\vec{g}$
Hooke's law for a spring: $F = -kx$, where k is the spring constant
Friction: Static: $F_s \leq \mu_k N$; Kinetic: $F_k = \mu_k N$; N is the Normal Force
Drag Force: $F_D = \frac{1}{2}C_D\rho A v^2$; Terminal velocity: $v_t = \sqrt{2mg/(C_D\rho A)}$

Work (**constant force in 1-D**): $W = F_x\Delta x$; Work (**variable force in 3-D**): $W = \int_{r_1}^{r_2}\vec{F}\cdot d\vec{r}$
Kinetic Energy: $K = \frac{1}{2}mv^2$, Work-Energy theorem: $W_{\text{net}} = \Delta K$
Power: $P = dW/dt$; $P = \vec{F}\cdot\vec{v}$

Conservative forces: $\oint \vec{F}\cdot d\vec{r} = 0$; Difference in potental energy is independent of path taken.
Potential Energy: $\Delta U_{AB} = -\int_A^B \vec{F}\cdot d\vec{r}$; in 1-D: $F_x = -dU/dx$
Potential Energy: gravitational near Earth surface: $U = mgh$; spring elastic: $U = \frac{1}{2}kx^2$; gravitational in general $U = -GMm/r$
Newton's law of Gravity: $\vec{F} = -\frac{GMm}{r^2}\hat{r}$
Orbital period: $T^2 = 4\pi^2 r^3/(GM)$
Escape velocity: $v_{\text{esc}} = \sqrt{2GM/r}$

Center of Mass equations: $\vec{F}_{\text{net ext}} = M\frac{d^2\vec{R}}{dt^2} = M\vec{A}$; $\vec{R} = \frac{\sum m_i\vec{r}_i}{M} = \frac{1}{M}\int\vec{r}dm$
Right triangle with base, b, height h, CoM is 1/3 of way from long side (e.g. $X = b/3$, $Y = h/3$)
In the absence of external forces the center-of-mass velocity $\vec{V} = \sum m_i\vec{v}_i/M$ remains constant.
The total momentum is $\vec{P} = \sum m_i\vec{v}_i$, and $\vec{F}_{\text{net ext}} = d\vec{P}/dt$.
A rocket's speed is given by $Mdv/dt = -v_{exhaust}dM/dt$; or $v_f = v_i + v_{ex}\ln(M_i/M_f)$
The total kinetic energy of a system of particles is $K_{\text{total}} = K_{\text{cm}} + K_{\text{internal}}$, where center-of-mass kinetic energy is $K_{\text{cm}} = \frac{1}{2}MV^2$ and $K_{\text{internal}} = \sum\frac{1}{2}m_i\tilde{v}_i$, where $\tilde{v}_i$ is speed relative to center-of-mass.

Collision equations: Impulse: $I = \Delta\vec{p} = \int_{t_1}^{t_2} \vec{F}dt$; Average force during collison $F_{ave} = \Delta\vec{p}/dt$.
Momentum is conserved in collisions: Totally inelastic collision: $m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f$.
Kinetic Energy also conserved in *elastic* collisions:
$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$; $\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$
In 1-D final velocities can be found from initial velocities and masses:
$v_{1f} = \frac{m_1-m_2}{m_1+m_2}v_{1i} + \frac{2m_2}{m_1+m_2}v_{2i}$; $v_{2f} = \frac{2m_1}{m_1+m_2}v_{1i} + \frac{m_2-m_1}{m_1+m_2}v_{2i}$
Rotational equations: $\omega = d\theta/dt$, $\alpha = d\omega/dt$, $v_t = \omega r$, $a_t = \alpha r$
Torque: $\vec{\tau} = \vec{r} \times \vec{F} = d\vec{L}/dt = rF\sin\theta$; direction given by **Right Hand Rule**
Rotational analog of Newton's law: $\tau = I\alpha$, where moment of inertia, $I = \sum m_i r_i^2$ for discrete masses and $I = \int r^2 dm$ for continuous masses
Rotational kinetic energy: $K_{\text{rot}} = \frac{1}{2}I\omega^2$; $W_{\text{rot}} = \tau d\theta$
Some moment of inertias: Solid sphere about center: $I = \frac{2}{5}MR^2$;
Hollow sphere about center: $I = \frac{2}{3}MR^2$; Solid cylinder about axis: $I = \frac{1}{2}MR^2$;
Hollow cylinder about axis: $I = MR^2$; Thin rod about center:$I = \frac{1}{12}Ml^2$;
Thin rod about end $I = \frac{1}{3}Ml^2$
Angular momentum: $\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$; direction given by **Right Hand Rule**
$\vec{A} \times \vec{B} = \hat{i}(A_yB_z - A_zB_y) - \hat{j}(A_zB_x - A_xB_z) + \hat{k}(A_xB_y - A_yB_x) = AB\sin\theta$ (direction by RHR)

Static Equilibrium; $\sum \vec{F}_i = 0$ and $\sum \vec{\tau}_i = 0$