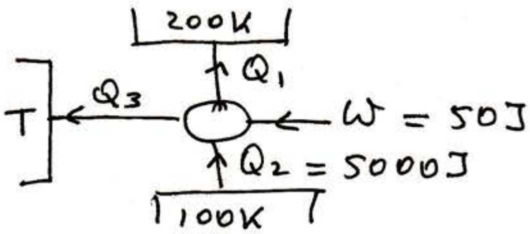
Physics 4B — Quiz 5 Solutions — Spring 2014

Problem 1



(a) $Q_1 = 5049\,J \Rightarrow Q_3 = 1\,J$, so $Q_2 + W = 5050\,J = Q_1 + Q_3$

Change in entropy of the universe has to be positive

$$\Delta S = -\frac{Q_2}{100K} + \frac{Q_1}{200K} + \frac{Q_3}{T} \geq 0 \Rightarrow \frac{Q_3}{T} \geq \frac{Q_2}{100} - \frac{Q_1}{200} \Rightarrow$$

$$\Rightarrow T \leq \frac{Q_3}{\frac{Q_2}{100} - \frac{Q_1}{200}} = \frac{1}{\frac{5000}{100} - \frac{5049}{200}} = 0.040K$$

$$\Rightarrow \boxed{0 < T \leq 0.040\,K} \text{ (a)}$$

(b) $T = 50K$. $\quad Q_1 + Q_3 = 5050\,J \Rightarrow Q_3 = 5050\,J - Q_1$

$$\Delta S \geq 0 \Rightarrow -\frac{Q_2}{100} + \frac{Q_1}{200} + \frac{Q_3}{50} \geq 0 \Rightarrow -\frac{Q_2}{100} + \frac{Q_1}{200} + \frac{5050 - Q_1}{50} \geq 0$$

$$\Rightarrow Q_1\left(\frac{1}{50} - \frac{1}{200}\right) \leq 101 - \frac{5000}{100} = 51 \Rightarrow Q_1 \leq 51 \cdot \frac{200}{3} = 3400\,J$$

So $\boxed{Q_1 \leq 3400\,J, \quad Q_3 \geq 1650\,J, \quad Q_1 + Q_3 = 5050\,J}$

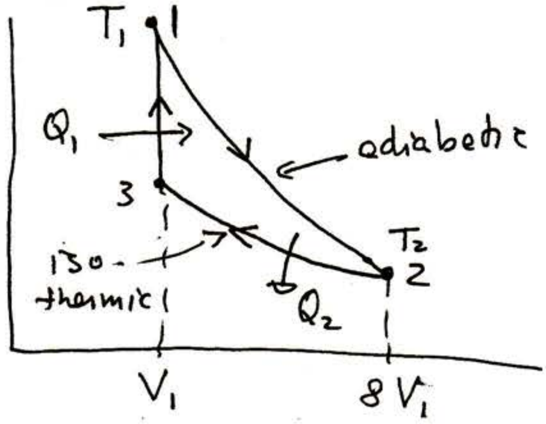
(c) To have the maximum T compatible with $\Delta S \geq 0$, need Q_3

as large as possible, so $Q_1 = 0 \Rightarrow Q_3 = 5050\,J \Rightarrow$

$$-\frac{5000}{100} + 0 + \frac{Q_3}{T} \geq 0 \Rightarrow T \leq \frac{Q_3}{50} = \frac{5050}{50} = 101$$

$$\boxed{T \leq 101K}$$

Problem 2



Q_1 = heat absorbed by gas during 3-1; Q_2 = heat released by gas during 2-3

(b) $TV^{\gamma-1} = \text{const} \Rightarrow T_1 V_1^{\gamma-1} = T_2 (8V_1)^{\gamma-1} \Rightarrow T_2 = \frac{1}{8^{\gamma-1}} T_1$

$\gamma = \frac{5}{3}$, $\gamma - 1 = \frac{2}{3} \Rightarrow T_2 = \frac{1}{2^{3\cdot\frac{2}{3}}} T_1 \Rightarrow \boxed{T_2 = \frac{1}{4} T_1}$

(c) $e = \frac{W}{Q_1}$, $W = Q_1 - Q_2 \Rightarrow \boxed{e = 1 - \frac{Q_2}{Q_1}}$

Find Q_2: $\Delta E_{int} = 0 = dQ - dW \Rightarrow dQ = dW = PdV \Rightarrow$

$$\Rightarrow Q_2 = \int_{V_1}^{8V_1} \frac{nRT_2}{V} dV = nRT_2 \ln 8 = \frac{nRT_1}{4} \ln 2^3 \Rightarrow \boxed{Q_2 = \frac{3}{4} nRT_1 \ln 2}$$

Find Q_1: $\Delta E_{int} = Q - W$, $W = 0$ (constant volume),

$$\Delta E_{int} = C_V \Delta T \Rightarrow Q_1 = C_V (T_1 - T_2) = \frac{3}{4} C_V T_1 = \frac{3}{4} \cdot \frac{3}{2} nRT_1$$

$$\Rightarrow \boxed{Q_1 = \frac{9}{8} nRT_1} \Rightarrow e = 1 - \frac{\cancel{3} \cdot \cancel{8}^{2}}{4 \cdot \cancel{9}_{3}} \ln 2 = 1 - \frac{2}{3} \ln 2 \Rightarrow$$

$\boxed{e = 0.538}$

(d) For Carnot engine, $e_C = 1 - \frac{T_2}{T_1} = 1 - \frac{1}{4} = \boxed{0.75} \Rightarrow$

$\boxed{e_C > e}$ that is because this engine absorbs heat also at temperatures smaller than T_1.

Problem 3

(a) Entropy change of the gas in reversible isothermic process

$$\Delta S = \int \frac{dQ}{T} = \int \frac{P\,dV}{T} = \int_V^{3V} \frac{nRT}{TV}\,dV = nR\ln 3$$

$\Delta E_{int} = Q - W$, no work in free expansion $\Rightarrow Q = 0 \Rightarrow$ no change in entropy of environment $\Rightarrow \Delta S_{univ} = \Delta S_{gas} + \Delta S_{env} = \Delta S_{gas} \Rightarrow$

$$\boxed{\Delta S_{univ} = nR\ln 3 = 1.099\,nR} \text{ (a)}$$

(b) Gas expands against pressure $P/3 \Rightarrow$ does work $W = \frac{P}{3}\Delta V$,

$\Delta V = 3V - V = 2V \Rightarrow W = \frac{2}{3}PV = \frac{2}{3}nRT$

$\Delta E_{int} = 0 = Q - W \Rightarrow$ gas absorbs heat $Q = W$.

$\Rightarrow \Delta S_{env} = -\frac{Q}{T} = -\frac{2}{3}nR$. $\Delta S_{gas} =$ same as before (same final state)

$$\Rightarrow \boxed{\Delta S_{univ} = nR\left(\ln 3 - \frac{2}{3}\right) = 0.432\,nR}$$

(c) Removing one weight, final volume is $V' = \frac{3}{2}V$ $\left(PV = \frac{2}{3}P \cdot \frac{3}{2}V\right)$

work done is $W_1 = \frac{2}{3}P \cdot \Delta V$, $\Delta V = \frac{3}{2}V - V = \frac{1}{2}V \Rightarrow W_1 = \frac{2}{3} \cdot \frac{1}{2}PV \Rightarrow$

$\Rightarrow \boxed{W_1 = \frac{1}{3}nRT}$ Removing second weight, gas expands against

pressure $\frac{P}{3}$, $\Delta V = 3V - \frac{3}{2}V = \frac{3}{2}V \Rightarrow W_2 = \frac{P}{3} \cdot \frac{3}{2}V = \frac{1}{2}PV \Rightarrow$

$\Rightarrow W_2 = \frac{1}{2}nRT \Rightarrow W_1 + W_2 = \left(\frac{1}{2} + \frac{1}{3}\right)nRT = \frac{5}{6}nRT.$

Gas absorbs $Q = W_1 + W_2$ from environment $\Rightarrow$

$\Delta S_{env} = -\frac{Q}{T} = -\frac{5}{6}nR$, $\Delta S_{gas} = nR\ln 3$

$$\Rightarrow \boxed{\Delta S_{univ} = nR(\ln 3 - 5/6) = 0.265\,nR}$$

(d) in this limit, $\Delta S_{env} = -\Delta S_{gas} \Rightarrow \boxed{\Delta S_{univ} \cong 0}$