Physics 4B Quiz 3 Solutions Spring 2014

Problem 1

(a) $\lambda f = v \quad , \quad v = \sqrt{\frac{F_T}{\mu}} \quad , \quad \mu = m/\ell \quad , \quad F_T = 200\,N$

$\lambda = 2\ell$ for the fundamental frequency $\Rightarrow$

$$2\ell f = \sqrt{\frac{F_T \cdot \ell}{m}} \Rightarrow 4\ell^{\cancel{2}} f^2 = \frac{F_T \cancel{\ell}}{m} \Rightarrow m = \frac{F_T}{4\ell f^2}$$

$$\Rightarrow m = \frac{200}{4 \cdot 0.5 \cdot 500^2}\,kg = 0.0004\,kg \Rightarrow \boxed{m = 0.4\,g}$$

(b) $$\frac{f'}{f} = \frac{v'}{v} = \sqrt{\frac{F_T'}{F_T}} \Rightarrow F_T' = F_T\left(\frac{f'}{f}\right)^2 \Rightarrow$$

$$\Rightarrow F_T' = \left(\frac{353.55}{500}\right)^2 F_T = \frac{1}{2} F_T \Rightarrow \boxed{F_T' = 100\,N}$$

(c) With a tension of 200 N, the string length increased from 49.9 cm to 50 cm, i.e. $\Delta\ell = 0.1\,cm$. A tension force $F_T' = 100\,N = 1/2 \cdot 200\,N$ corresponds to a change in length of $1/2 \times 0.1\,cm = 0.05\,cm = \Delta\ell' = \alpha\,\ell\,\Delta T$, $\Delta T = 100\,°C$

$$\Rightarrow \alpha = \frac{0.05\,cm}{50\,cm \cdot 100\,°C} \Rightarrow \boxed{\alpha = 10 \times 10^{-6}\,°C^{-1}}$$

Or: use $\Delta\ell = 1\,cm = \frac{F_T \cdot \ell}{EA} \Rightarrow EA = \frac{F_T\,\ell}{\Delta\ell} \Rightarrow$

$$\Rightarrow \Delta\ell' = \frac{F_T'\,\ell}{EA} = \frac{F_T'\,\ell}{F_T\,\ell}\,\Delta\ell = \frac{1}{2}\Delta\ell$$

Problem 2

Doppler shift from approaching source : $f' = \frac{1}{1-\frac{v_{source}}{v_{sound}}} f$

Doppler shift from receding observer: $f'' = \left(1 - \frac{v_{obs}}{v_{sound}}\right) f'$ $\Rightarrow$

$$\Rightarrow f'' = \frac{1-\frac{v_{obs}}{v_{sound}}}{1-\frac{v_{source}}{v_{sound}}} f \quad ; \quad \frac{v_{obs}}{v_{sound}} = \frac{85}{340} = \frac{1}{4} \Rightarrow$$

$$\frac{f''}{f} = \frac{1800}{1200} = \frac{3}{2}$$

$$\Rightarrow \frac{3}{2} = \frac{1-\frac{1}{4}}{1-\frac{v_{source}}{v_{sound}}} = \frac{\frac{3}{4}}{1-\frac{v_{source}}{v_{sound}}} \Rightarrow 1 - \frac{v_{source}}{v_{sound}} = \frac{3}{4}\cdot\frac{2}{3} = \frac{1}{2}$$

$$\Rightarrow \boxed{v_{source} = \frac{1}{2} v_{sound} = 170\,m/s} \quad (a)$$

(b) After police car passed, signs of velocities change:

$$f'' = \frac{1+\frac{v_{obs}}{v_{sound}}}{1+\frac{v_{source}}{v_{sound}}} f = \frac{1+\frac{1}{4}}{1+\frac{1}{2}} \cdot 1200\,Hz = \frac{5\cdot 2}{4\cdot 3} \times 1200\,Hz$$

$$\Rightarrow \boxed{f'' = \frac{5}{6} \times 1200\,Hz = 1000\,Hz} \quad (b)$$

Problem 3

(a) 1 mol at 0°C, 1 atm, occupies $22.4\times10^{-3}\,m^3$

$PV = nRT$, so for $P = 2\,atm$, $V = 3\,m^3$, we have

$$n = \frac{3\,m^3}{22.4\times10^{-3}\,m^3}\times 2 = \boxed{267.86\text{ mols}}$$

Or: $PV = nRT \Rightarrow n = \frac{PV}{RT} = \frac{2\times1.013\times3}{8.314\times273}\text{ mols} = \boxed{267.79\text{ mols}}$

(b) Mass of 1 mol of He is $4g = 4\times10^{-3}\,kg$

$$M_{He} = 267.86\times4\times10^{-3}\,kg = \boxed{1.07\,kg}$$

(c) The container floats $\Rightarrow$ buoyant force = weight of displaced air = weight of He + container $\Rightarrow$

$M_{air} = M_{container} + M_{He}$, $M_{air} = \rho_{air}\times V \Rightarrow$

$M_{air} = 1.29\times3\,kg = 3.87\,kg \Rightarrow \boxed{M_{container} = 2.8\,kg}$

(d) $v = \sqrt{\frac{B}{\rho}}$ $\quad B = -V\frac{dP}{dV} = V\frac{nRT}{V^2} = \frac{nRT}{V} = P$

So $B = P$, pressure inside = twice pressure outside

$\rho = \frac{M}{V}$, $\frac{\rho_{inside}}{\rho_{outside}} = 2\cdot\frac{M_{He}}{M_{air}} = 2\cdot\frac{4}{29} = \frac{8}{29} \Rightarrow$

$$\frac{v_{inside}}{v_{outside}} = \frac{\sqrt{\frac{B_{He}}{\rho_{He}}}}{\sqrt{\frac{B_{air}}{\rho_{air}}}} = \sqrt{\frac{B_{He}}{B_{air}}\cdot\frac{\rho_{air}}{\rho_{He}}} = \sqrt{\frac{P_{He}}{P_{air}}\cdot\frac{\rho_{air}}{\rho_{He}}} =$$

$$= \sqrt{2\cdot\frac{29}{8}} = \sqrt{\frac{58}{8}} \Rightarrow v_{inside} = \sqrt{\frac{58}{8}}\times340\,\frac{m}{s} = \boxed{915\,m/s}$$