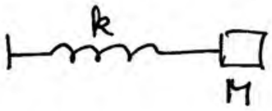
Physics 4B   **Quiz 2 Solutions**   Spring 2014

Problem 1



$$, \ \omega_0 = \sqrt{\frac{k}{M}} \quad , \quad \gamma = \frac{b}{2M}$$

The motion is $\quad x(t) = A e^{-\gamma t} \cos \omega' t \quad , \ \omega' = \sqrt{\omega_0^2 - \gamma^2} = 0.1 \text{ rad/s}$

(a) At time t_1, amplitude has decreased to $1/2$; at time t_2, it has decreased to $1/8$, $\Rightarrow$

$$e^{-\gamma t_1} = \frac{1}{2} \Rightarrow \gamma t_1 = \ln 2$$
$$e^{-\gamma t_2} = \frac{1}{8} \Rightarrow \gamma t_2 = \ln 8$$
$$\Rightarrow \boxed{t_2 = \frac{\ln 8}{\ln 2} t_1 = 3 t_1}$$

So if after 10 oscillations amplitude is $1/2$, after $\boxed{\text{20 more oscillations}}$ amplitude is $1/8$.

(b) $\omega' = \sqrt{\omega_0^2 - \gamma^2} \Rightarrow \omega_0 = \sqrt{\omega'^2 + \gamma^2}$. We need γ.

t_1 = time for 10 oscillations $= 10\,T$, T = period $= \frac{2\pi}{\omega'} \Rightarrow$

$$\Rightarrow t_1 = \frac{20\pi}{\omega'} \ , \ \gamma = \frac{\ln 2}{t_1} = \frac{\ln 2}{20\pi}\omega' = 0.001103 \text{ s}^{-1}$$

$$\boxed{\omega_0 = \sqrt{0.1^2 + 0.0011^2} = 0.100006 \text{ rad/s}}$$

(c) Critical damping is for $\omega_0' = \gamma'$; $\omega_0' = \sqrt{\frac{k}{m}}$, $\gamma' = \frac{b}{2m}$

$$\Rightarrow \frac{k}{m} = \frac{b^2}{4m^2} \Rightarrow m = \frac{b^2}{4k} = \left(\frac{b}{2M}\right)^2 \cdot \frac{1}{4} \cdot \frac{4M^2}{k} = \frac{\gamma^2}{\omega_0^2} M \Rightarrow$$

$$\Rightarrow \boxed{m = \frac{0.0011^2}{0.1^2} \times 1000\,g = 0.12\,g}$$

Problem 2

The speed of transverse waves in a string is $v = \sqrt{\frac{F_T}{\mu}}$; $\mu = \frac{m}{l}$

For standing waves, $\lambda = \frac{2l}{n}$, with $n = 1, 2, 3, \ldots$

Wavelength, frequency and velocity satisfy $\lambda f = v \Longrightarrow$

(a) $\frac{2l}{n} f = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{F_T\, l}{m}} \Rightarrow \frac{4l^2}{n^2} f^2 = \frac{F_T\, l}{m} \Rightarrow$

$\Rightarrow l = \frac{F_T}{4mf^2} n^2$; with $F_T = 300\,N$, $m = 1.5\,kg$, $f = 2\,Hz$,

$l = \frac{300}{4 \times 1.5 \times 2^2}\,m \cdot n^2 \Rightarrow l = 12.5\,m \times n^2 \Rightarrow$ for $n = 1$, $n = 2$,

smallest possible lengths are $\boxed{l = 12.5\,m \text{ and } l = 50\,m}$

(b) Assuming $l = 50\,m$, the frequency $f = 2\,Hz$ corresponds to $n = 2$, so from $f_n = n f_1$ we conclude that

$\boxed{1\,Hz,\ 2\,Hz \text{ and } 3\,Hz \text{ are the three lowest frequencies}}$

(c) The displacement of a point as a function of time can be written as: $D(t) = A \sin \omega t = A \sin(2\pi f t)$

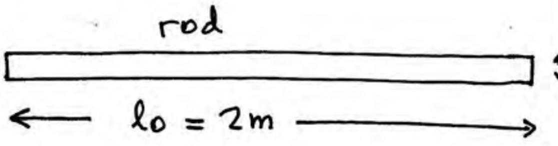
then, at time $t = 0$, $D = 0$. At $t_1 = 0.1\,s$ it is $1\,cm \Rightarrow$

$A \sin(2 f t_1) = 1\,cm \Rightarrow A = 1\,cm / \sin(2 f t_1) \Rightarrow$ at time

$t_2 = 0.2\,s$, $D = A \sin(2\pi f t_2) = \frac{\sin(2\pi f t_2)}{\sin(2\pi f t_1)} \times 1\,cm$

$\Rightarrow D = \frac{\sin(4\pi \times 0.2)}{\sin(4\pi \times 0.1)} \times 1\,cm = \frac{0.588}{0.951} \times 1\,cm \Rightarrow \boxed{D(t_2) = 0.62\,cm}$

Problem 3



$A = 1 cm^2 = 10^{-4} m^2$

$m = 1.5 \, kg$

$E = 10^{11} \frac{N}{m^2}$ elastic modulus

(a)

Change in length under force F: $\Delta l = \frac{F}{A} l_0 \frac{1}{E}$ $\Rightarrow$

$$\Delta l = \frac{1000 \times 2}{10^{-4} \times 10^{11}} m = 0.0002 m \Rightarrow \boxed{\Delta l = 0.2 \text{ mm}}$$

(b) Speed of longitudinal waves: $v = \sqrt{\frac{E}{\rho}}$

density is $\rho = \frac{m}{V} = \frac{1.5 \, kg}{2m \cdot 10^{-4} m^2} \Rightarrow \rho = 7500 \frac{kg}{m^3}$

$$\boxed{v = \sqrt{\frac{10^{11}}{7500}} \frac{m}{s} = 3651 \, m/s}$$

(c) Wavenumber $k = 1 cm^{-1}$; $k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{k}$

$\Rightarrow \boxed{\lambda = 6.28 \text{ cm}}$ wavelength

$\lambda f = v \Rightarrow f = v/\lambda = \frac{3651 m/s}{6.28 \times 10^{-2} m} \Rightarrow \boxed{f = 58{,}137 \, Hz}$ frequency

(d) The displacement of the atoms is $D(x,t) = A \sin(kx - \omega t)$

The speed is $\frac{dD}{dt} = -\omega A \cos(kx - \omega t)$

Maximum displacement = amplitude = $0.01 mm = 10^{-5} m$

Maximum speed = $\omega A = 2\pi f \cdot A = 2\pi \cdot 58{,}137 \times 10^{-5} \frac{m}{s}$ =

$= \boxed{3.65 \, m/s}$ maximum speed