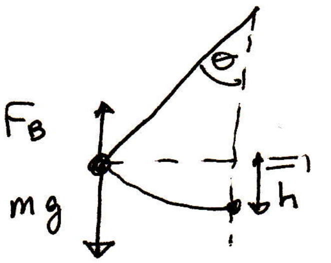
Physics 4B Final Exam Solutions Spring 2014

Problem 1

pendulum frequency in air: $\omega = \sqrt{\frac{g}{l}} = 2\pi f \Rightarrow$

$\Rightarrow l = \frac{g}{4\pi^2 f^2} = \frac{9.8}{4\pi^2} m = 0.248 m \Rightarrow \boxed{l = 24.8 cm}$
length of string

In water, there is buoyant force:



$F_B = \rho_w \cdot V \cdot g$, V = volume of ball

$F_B = \frac{\rho_w}{\rho_{ball}} \cdot mg$, $\rho_{ball} = \frac{m}{V} = \frac{11}{4}\frac{g}{m^3} = 2.75 g/cm^3$

so net force is $F = mg - F_B = mg\left(1 - \frac{\rho_w}{\rho_{ball}}\right)$

So g is reduced by the factor $1 - \frac{\rho_w}{\rho_{ball}} = 1 - \frac{1}{2.75} = 0.636$

So the frequency is reduced by factor $\sqrt{0.636} = 0.798 \simeq 0.8$

$\Rightarrow \boxed{f = 0.8 Hz \text{ in water}}$ (a)

(b) For damped oscillation, $\omega' = \sqrt{\omega^2 - \gamma^2}$, and $\theta(t) = Ae^{-\gamma t}\cos\omega' t$

For critical damping, $\boxed{\gamma = \omega = 2\pi f = 5.01 s^{-1}}$

For θ to go from 10° to 0.1° $\Rightarrow e^{\gamma t} = 100 \Rightarrow t = \frac{\ln 100}{\gamma} \Rightarrow$

$\Rightarrow \boxed{t = 0.92 s}$

(c) The initial energy is $E_{in} = mgh \times 0.636$

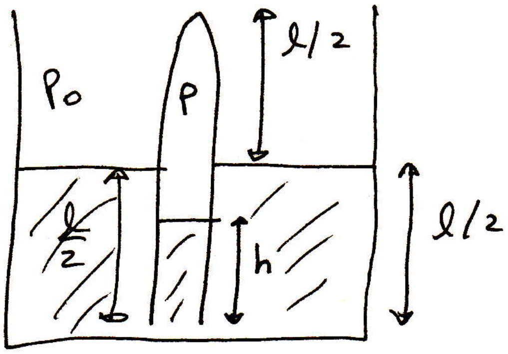
$h = l - l\cos\theta = 0.38 cm \Rightarrow E_{in} = 11 \times 10^{-3} \times 9.8 \times 0.636 \times 0.38 J = 0.026 J$

final energy $\sim 0 \Rightarrow E_{in}$ gets transformed into heat

$$\boxed{\Delta S = \frac{E_{in}}{T} = \frac{0.026 J}{393 K} = 6.6 \times 10^{-5} \frac{J}{K}}$$

Problem 2

$P_0 = 101{,}300\,N/m^2$ is atmospheric pressure



The gas in the tube occupies a smaller volume than initially ⇒ has higher pressure.

$PV = \text{constant} \Rightarrow \quad P(l-h) = P_0 l \Rightarrow \boxed{P = P_0 \frac{l}{l-h}}$

equating pressure at the tube opening:

$$P_0 + \rho g \frac{l}{2} = P + \rho g h = \frac{P_0 l}{l-h} + \rho g h \Rightarrow$$

$$\Rightarrow \frac{P_0}{\rho g} + \frac{l}{2} = \frac{P_0}{\rho g}\frac{l}{l-h} + h \Rightarrow \frac{P_0}{\rho g}(l-h) + \frac{l^2}{2} - \frac{lh}{2} = \frac{P_0}{\rho g} l + hl - h^2 \Rightarrow$$

$$\Rightarrow \cancel{\frac{P_0}{\rho g} l} - \frac{P_0}{\rho g} h + \frac{l^2}{2} - \frac{lh}{2} = \cancel{\frac{P_0}{\rho g} l} + lh - h^2 \Rightarrow$$

$$\Rightarrow h^2 - h\left(\frac{3}{2} l + \frac{P_0}{\rho g}\right) + \frac{l^2}{2} = 0$$

$$\frac{P_0}{\rho g} = \frac{101{,}300}{13.6 \times 1000 \times 9.8}\,m = 0.76\,m = 76\,cm \Rightarrow \text{with } l = 80\,cm$$

$$\frac{3}{2} l + \frac{P_0}{\rho g} = 196\,cm, \quad \frac{l^2}{2} = 3200\,cm^2$$

$$h^2 - 196 h + 3200 = 0 \Rightarrow$$

$$h = \left(\frac{196}{2} \pm \sqrt{\left(\frac{196}{2}\right)^2 - 3200}\right) cm = (98 \pm 80)\,cm \Rightarrow \boxed{h = 18\,cm}$$

Problem 3

Speed of sound $v = \sqrt{\frac{B}{\rho}}$, $B = -V \frac{\partial P}{\partial V}$

$\rho = \frac{\text{mass}}{\text{volume}} = \frac{nM}{V}$, with M = molecular mass = $\frac{4\,g}{mol}$ for He

Isothermic: $P = \frac{nRT}{V}$, $\frac{dP}{dV} = -\frac{nRT}{V^2} \Rightarrow B = \frac{nRT}{V} \Rightarrow$

$\Rightarrow \frac{B}{\rho} = \frac{nRT \cdot V}{V \cdot nM} = \frac{RT}{M} = \frac{8.314\, kg\, m^2}{s^2 K \cdot 4\times10^{-3} kg} \times 273K =$

$= 567{,}430.5 \frac{m^2}{s^2} \Rightarrow \boxed{v = 753.3\ m/s}$ (a)

(b) $PV^{\gamma} = \text{const} \Rightarrow P = \frac{\text{const}}{V^{\gamma}} \Rightarrow \frac{\partial P}{\partial V} = -\gamma \frac{\text{const}}{V^{\gamma+1}} = -\frac{\gamma P}{V} \Rightarrow$

$\Rightarrow B = \gamma P = \frac{\gamma nRT}{V} \Rightarrow$ same as before except for γ factor.

$\gamma = \frac{5}{3}$ for ideal gas $\Rightarrow$

$$\boxed{v = \sqrt{\frac{5}{3}} \times 753.3 \frac{m}{s} = 972.5\ m/s}$$

(c) Write $T = 273 + t$, with t the temperature in °C

$v = \sqrt{\frac{\gamma R}{M}(273+t)} = \sqrt{\frac{\gamma R}{M} \cdot 273}\sqrt{1 + \frac{t}{273}} \cong \sqrt{\frac{\gamma R}{M} \cdot 273}\left(1 + \frac{t}{2.273}\right) =$

$= 972.5\left(1 + \frac{t}{546}\right)\frac{m}{s} = \boxed{(972.5 + 1.78t)\frac{m}{s}}$

$\Rightarrow \boxed{b = 1.78}$

Problem 4

The instrument plays at frequency f. The frequency heard by G from the guitar in the moving car is

$$f' = \frac{1}{1-\frac{v_c}{v_s}} f$$, with v_c = speed of car. Hence

$$f'-f = \left(1 - \frac{1}{1-\frac{v_c}{v_s}}\right) f = \frac{v_c/v_s}{1-v_c/v_s} f \implies$$

$$f = \frac{1-v_c/v_s}{v_c/v_s}(f'-f)$$. With $f'-f = 25\,Hz \Rightarrow \boxed{f = 403.75\,Hz}$

The frequency heard by M from the guitar on the ground is

$$f' = \left(1+\frac{v_c}{v_s}\right) f \Rightarrow f'-f = \frac{v_c}{v_s} f = 23.54\,Hz$$

So M hears 23.54 beats/s

(b) $$\lambda f = v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{F_T \cdot l}{m}}$$, fundamental is $\lambda = 2l \Rightarrow$

$$\Rightarrow 2lf = \sqrt{\frac{F_T l}{m}} \Rightarrow 4l^2f^2 = \frac{F_T l}{m} \Rightarrow l = \frac{F_T}{4mf^2}$$

$$\Rightarrow l = \frac{100}{4 \cdot 0.5\times10^{-3} \times 403.75^2}\, m = 0.307\, m \Rightarrow \boxed{l = 30.7\, cm}$$

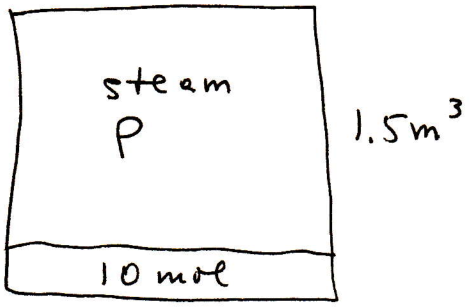
(c) Wavelength of fundamental string vibration

$$\boxed{\lambda = 2l = 61.34\ cm}$$

Wavelength of sound wave:

$$\lambda_s = \frac{v_s}{f} = \frac{343\ m/s}{403.75/s} = 0.85\, m \Rightarrow \boxed{\lambda_s = 85\, cm}$$

Problem 5



$80°C = 353 K$

$90°C = 363 K$

water has molecular weight 18 $\Rightarrow$ 1 mol $= 18 g = 18.10^{-3} kg$

At $80°C$, $P = 4.73 \times 10^4 N/m^2$. Using $PV = nRT$, $\Rightarrow$

$$n = \frac{PV}{RT} = \frac{4.73 \times 10^4 \times 1.5}{8.314 \times 353} = \boxed{24.175 \text{ mols of steam initially}}$$

(b) At $90°C$, if there is equilibrium between liquid and steam

$P = 7.01 \times 10^4 \frac{N}{m^2}$ $\Rightarrow$ the number of mols of steam is

$$n = 24.175 \times \frac{7.01}{4.73} = 35.83 \text{ mol} \Rightarrow \text{requires}$$

$35.83 - 24.175 = 11.65$ extra mols, but we have only 10 $\Rightarrow$

$\boxed{\text{all liquid evaporated}}$ $\Rightarrow$ pressure is smaller.

(c) $$P = \frac{nRT}{V} = \frac{34.175 \times 8.134 \times 363}{1.5} \frac{N}{m^2} = \boxed{6.88 \times 10^4 \frac{N}{m^2}}$$

(d) The heat in raising the T from $80°C$ to $90°C$ is

$$Q = 3000 \frac{J}{kg\,°C} \times 34.175 \text{ mol} \times 1.8 \times 10^{-3} \frac{kg}{mol} \times 10°C = \boxed{18{,}454 J}$$

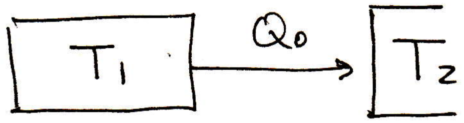
Heat of vaporization $Q_v = 2.26 \times 10^6 \frac{J}{kg} \times 10 \text{ mol} \times 18.10^{-3} \frac{kg}{mol} =$

$= 406{,}800 J$. Since $Q_v >> Q$, ignore Q

$$\boxed{\Delta S \sim \frac{Q_v}{\bar{T}} = \frac{406{,}800 J}{358 K} = 1136 \frac{J}{K}}$$

$\bar{T}$ is the average temperature, ok as an approximation

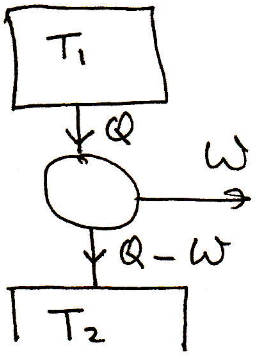
Problem 6



$$Q_0 = C(T_1 - T_2) \Rightarrow$$

$$\boxed{C = \frac{Q_0}{T_1 - T_2}}$$ is heat capacity of the body

With a heat engine



As the heat engine runs, the body's temperature gradually decreases. The change in entropy of the body is

$$\Delta S_b = \int \frac{dQ}{T} = \int_{T_1}^{T_2} \frac{C\,dT}{T} = C \ln \frac{T_2}{T_1} \Rightarrow$$

$$\Delta S_b = -C \ln \frac{T_1}{T_2} = -\frac{Q_0}{T_1 - T_2} \ln \frac{T_1}{T_2}$$

The total work done after many cycles is W, the total heat transfered to the heat reservoir is $Q_0 - W \Rightarrow$ its entropy change is

$$\Delta S_r = \frac{Q_0 - W}{T_2}$$ since its temperature doesn't change.

If the engine is reversible, $\Delta S_b + \Delta S_r = 0 \Rightarrow$

$$\Rightarrow \frac{Q_0 - W}{T_2} - \frac{Q_0}{T_1 - T_2} \ln \frac{T_1}{T_2} = 0 \Rightarrow Q_0 - W - \frac{Q_0 T_2}{T_1 - T_2} \ln \frac{T_1}{T_2} = 0 \Rightarrow$$

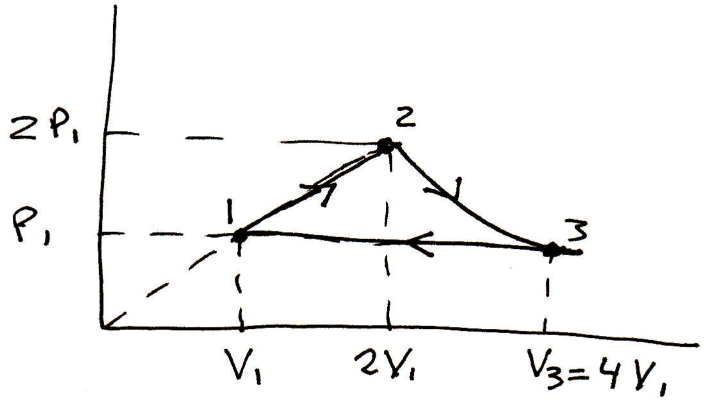
$$\Rightarrow W = Q_0 \left(1 - \frac{T_2}{T_1 - T_2} \ln \frac{T_1}{T_2}\right) \Rightarrow \boxed{\frac{W}{Q_0} = 1 - \frac{1}{\frac{T_1}{T_2} - 1} \ln \frac{T_1}{T_2}}$$

that's the maximum, if engine is irreversible, $\Delta S > 0$ and W/Q_0 is smaller.

(b) For $\frac{W}{Q_0} = 0.5 = \frac{1}{2} \Rightarrow \frac{1}{2} = \frac{1}{\frac{T_1}{T_2} - 1} \ln \frac{T_1}{T_2} \Rightarrow \boxed{\frac{T_1}{T_2} = 1 + 2 \ln \frac{T_1}{T_2}}$

Solving by iteration, $\boxed{\frac{T_1}{T_2} \simeq 3.5}$, so range of possible values is $\boxed{\frac{T_1}{T_2} \geq 3.5}$ ($>$ if engine is irreversible)

Problem 7



(a) Process 1–2

$P_1 V_1 = nRT_1$; $P_2 V_2 = 4P_1 V_1 = nRT_2 \Rightarrow \boxed{T_2 = 4T_1}$

$\Delta E_{int} = C_v \Delta T = \frac{3}{2} nR \cdot 3T_1 = \frac{9}{2} nRT_1$

$W = \frac{3}{2} P_1 \cdot (2V_1 - V_1) = \frac{3}{2} P_1 V_1 = \boxed{\frac{3}{2} nRT_1 \text{ did positive work}}$

$Q = \Delta E_{int} + W = \boxed{6nRT_1 \text{ absorbed heat}}$

(b) Process 2–3: isothermic, so $\Delta E_{int} = 0$

$W = \int_2^3 P\,dV = nRT_2 \ln \frac{V_3}{V_2} = 4nRT_1 \ln \frac{V_3}{V_2}$; $V_2 = 2V_1$;

$P_1 V_3 = nRT_2 = 4nRT_1 \Rightarrow \boxed{V_3 = 4V_1} \Rightarrow \boxed{W = 4nRT_1 \ln 2}$

$\boxed{Q = W = 4nRT_1 \ln 2 \text{ absorbed}}$

(c) Process 3–1 : $W = -P_1 \cdot 3V_1 = -3nRT_1$

$\Delta E_{int} = C_v \Delta T = \frac{3}{2} nR(T_1 - 4T_1) = -\frac{9}{2} nRT_1 \Rightarrow$

$\boxed{Q = \Delta E_{int} + W = -\frac{15}{2} nRT_1 \text{ released}}$ $\quad (n C_p \Delta T = -\frac{5}{2} nR \cdot 3T_1)$

Total work: $W_T = \frac{3}{2} nRT_1 + 4nRT_1 \ln 2 - 3nRT_1 \Rightarrow$

$\boxed{W_T = (4 \ln 2 - \frac{3}{2})\, nRT_1}$

Total heat absorbed: $Q_T = 6nRT_1 + 4nRT_1 \ln 2 = \boxed{(4 \ln 2 + 6)\, nRT_1}$

Efficiency:

$$\boxed{e = \frac{W+}{Q+} = \frac{4\ln 2 - 3/2}{4\ln 2 + 6} = 0.145}$$

(e) Efficiency is low because a lot of heat is absorbed at lower temperatures and released at higher temperatures than the maximum ($4T_1$) and minimum (T_1) temperatures.

The best efficiency results when all the heat absorbed is at the highest temperature, and all the heat released is at the lowest temperature, which is the Carnot cycle

$$\boxed{e_c = 1 - \frac{T_1}{4T_1} = 0.75 >> 0.145}$$

(f) For free expansion, $\Delta S_{gas} = nR\ln\frac{V_f}{V_i} = nR\ln 2$ here.

Everything except this step is reversible in this cycle, and the change of entropy of the gas in 1 cycle is 0 $\Rightarrow$

$$\boxed{\Delta S_{univ} = nR\ln 2}$$

If the expansion is reversible the environment decreases its entropy by $nR\ln 2$ in that process, in the free expansion instead the environment doesn't change its entropy.

Problem 8

E_A = energy of system A, E_B = energy of system B

$E_A + E_B = 3$, A has 2 particles, B has 3 particles

E_A	E_B	microstates of A	#	microstates of B	#	total # of microstates	energy/particle in A	energy/particle in B
3	0	3,0 2,1 1,2 0,3	4	0,0,0	1	4	1.5	0
2	1	2,0 1,1 0,2	3	1,0,0 0,1,0 0,0,1	3	9	1	0.33
1	2	1,0 0,1	2	2,0,0 0,2,0 0,0,2 1,1,0 1,0,1 0,1,1	6	12	0.5	0.66
0	3	0,0	1	3,0,0 0,3,0 0,0,3 2,1,0 2,0,1 1,2,0 0,2,1 1,0,2 0,1,2 1,1,1	10	10	0	1

(b) The most likely partition is $E_A = 1$, $E_B = 2$, giving 12 microstates.
The average energy per particle is 0.5 and 0.66 for A and B.

(c) Initially $S = k \ln W_m = k \ln 4 = 1.39k$

At the end $S = k \ln 12 = 2.48k$

$\Rightarrow \boxed{\Delta S = 1.10k > 0}$