Wakes - Supplement

→ Revisit turbulent wake, using turbulent viscosity, i.e.

$$W \sim (\nu x / U)^{1/2} \qquad (\nu \to D_T)$$

$$\to (D_T x / U)^{1/2}$$

i.e. is width of turbulent wake set by turbulent diffn. following Blasius Law.

but $D_T \sim W \tilde{V}$ ⇒ turbulent viscosity at mixing length level.

$$\sim W \left(F_d / \rho U W^2 \right)$$

$$\sim F_d / \rho U W \quad \sim \quad \text{const} / W$$

⇒

$$W \sim \left(F_d x / \rho U^2 W \right)^{1/2}$$

$$W^{3/2} \sim \left(F_d / \rho U^2 \right)^{1/2} x^{1/2} \sim (C_D R^2)^{1/2} x^{1/2}$$

$$\sim C_D^{1/2} R \, x^{1/2}$$

$$W \sim (C_D)^{1/3} R^{2/3} x^{1/3}$$

$$\Rightarrow \boxed{w/R \sim C_D^{1/3} (x/R)^{1/3}}$$ agrees ✓.

Now, $D_T \sim \tilde{v} w$

$$\sim \frac{(\tilde{v} w^2)}{w}$$

$$\sim \frac{\rho U \tilde{v} w^2}{\rho U w}$$

$$\sim \frac{Q}{w} \sim Q/R\,(x/R)^{1/3}$$

i.e. - Point is that turbulent viscosity mixing drops downstream, relative to constant viscous mixing.

- follows from $\tilde{v} w \sim \overset{\to\,\text{const.}}{Q}/w$

- explains why turbulent wake spreads more slowly than laminar wake.

→ Some Observations re: Wake Flows

→ note,

$$F_x = -\rho U \int_{\text{Wake}} V_x \, dy \, dz$$

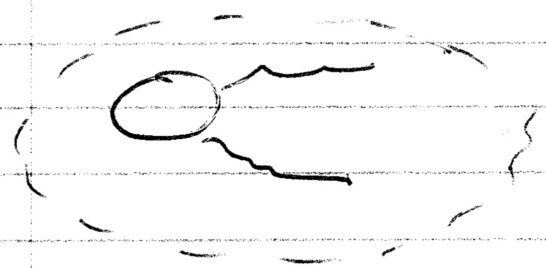
Now $Q = \rho \int V_x \, dy \, dz$ [redacted]

↓

mass flow due wak

⇒ deficit.

→ but if encircle body



$$\rho \int \underline{v} \cdot d\underline{a} = 0$$ i.e. continuity!

Now total $\underline{V}$ → $\left\{ \begin{array}{l} \text{velocity field} \\ \text{departure from } \underline{U} \end{array} \right.$

= $\overset{\text{vertical}}{\text{wake flow}}$ + potential flow.

so, must have $\underline{V}_{\substack{pot\\flow}}$ s/t

$$\int \underline{V} \cdot da = Q/\rho$$

then, for area at r:

$$v \pi r^2 \sim Q/\rho$$

$$\Rightarrow \quad v \sim Q/r^2$$

$$\phi \sim Q/r$$

global adjustment in potential flow due wake/viscosity (localized)

Message:

$$\boxed{\text{A little } \nu \text{ forces a global adjustment in flow structure.}}$$