**PHYSICS 239 : SUPERCONDUCTIVITY**
**HW ASSIGNMENT #2**

**(1)** Reread §1.3 of the lecture notes on London theory. Show that the Coulomb gauge $\boldsymbol{\nabla} \cdot \boldsymbol{A} = 0$ is compatible with Pippard's nonlocal electrodynamics. *Hint: Try writing the steady state continuity equation* $\boldsymbol{\nabla} \cdot \boldsymbol{j}_{\rm s} = 0$ *in Fourier space.*

**(2)** Analyze the problem of a bound state in the vicinity of the bottom of a band where the dispersion behaves as $\varepsilon(\boldsymbol{k}) \propto k^4$. Under what conditions do you expect a bound state to form for arbitrarily weak attractive interactions?

**(3)** Prove that the BCS ground state in Eqn. (3.71) of the notes satisfies $\gamma_{\boldsymbol{k}\sigma} \, | \, \mathrm{G} \, \rangle = 0$.

**(4)** Derive an expression, at $T = 0$, for the RMS number fluctuations $\langle (\Delta \hat{N})^2 \rangle$ in a BCS superconductor. Evaluate your expression for the model $V_{\boldsymbol{k}\boldsymbol{k}'}$ interaction from section 3.6 of the lecture notes.

**(5)** In §3.6.1 of the notes, we solved the BCS gap equation at $T = 0$ for a model attractive interaction. What about the particle number equation, $N = \sum_{\boldsymbol{k},\sigma} \langle c^\dagger_{\boldsymbol{k}\sigma} c_{\boldsymbol{k}\sigma} \rangle$ ? Investigate this equation and show how the two equations together determine the two unknowns $\mu$ and $\Delta$.