PHYSICS 161 Prof: Kim Griest
HOMEWORK 7 DUE: Wed 21 May, 2014

1. By making the change of coordinates given in class (and in the notes), show that the Kruskal-Szekeres line element and the Schwartzchild line element are the same.

2. Suppose black holes really were described by the Kruskal extension instead of produced by collapsing stars.
a) Draw the Kruskal-Szekeres diagram for a black hole. Be sure to label the singularities, the regions I,II,III, and IV, and the horizons.
b) Using the Kruskal diagram, decide whether an observer could enter the black hole and while inside, see light from stars from the other asymptotically flat region? Draw the light worldlines and explain. Could they tell anyone about what they saw?
c) Using a Kruskal coordinate diagram explain why it would not be possible to travel from one asymptotically flat region outside the black hole (region I) to the other asymptotically flat region outside the hole (region III). Draw the world line of someone trying to do this and explain the problem with this world line.

3a) Write the Kerr metric from class for a maximally rotating black hole. (use only r, θ, ϕ, t, and r_S).
3b) Using this metric find the formula for a small radial proper distance, dl_r, going out from directly above the north pole ($\theta = 0$) away from the hole, and for a small radial proper distance going out from above the equator ($\theta = \pi/2$) away from the hole.
3c) Suppose you are sitting still at $r = 4r_S$ above a black hole. If you move 1 foot in coordinate distance r toward the hole how many actual feet have you moved for a) a Schwarzschild hole, b) maximally rotating black hole above the north pole, c) maximally rotating black hole above the equator.
3d) Thus in which case above is space "curved" the most?

4. Estimate the Kerr parameter $a = J/M$ for the Sun, the Earth, and Jupiter. Look up the mass and radii of these objects, but assume the Sun spins about once every 24 days, Jupiter spins once per 0.45 days, and the Earth spins once per day. To find J, approximate the objects as uniform spheres, and remember your mechanics from Physics 2 or 4: (i.e. use the momement of inertia for a solid sphere, angular momentum $J = I\omega$, etc.) Be sure to add in factors of c to get the units right!
b. Estimate the Schwarzschild radii of these objects and compare to a. Make the comparison in units of cm.

5. The Hawking area theorem says the area of the horizon of a black hole can never decrease. Using this theorem show that a black hole can never bifurcate into 2 black holes preserving total mass. First calculate the area of the horizon of a non-rotating black hole using the Schwarzschild metric (we did this at the beginning of the quarter), then compare this area to the area of two smaller black holes with masses M_1 and M_2 that add up to make the first black hole mass, M.

6. In class we found that the maximum angular momentum of an uncharged black hole is $J_{max} = GM^2/c$. Compare this maximum for a black hole of mass $1.4M_{\odot}$ with the angular momentum of the fastest known pulsar, which rotates with a period of 0.00156 sec. Assume the pulsar is a $1.4M_{\odot}$ uniform sphere with a radius of 9 km.