PHYSICS 161 Prof: Kim Griest
HOMEWORK 3 DUE: Wed 23 Apr, 2014

1. Calculate the gravitational redshift of the Ly alpha line of Hydrogen (rest frame wavelength $\lambda_0 = 1215.67$ Angstroms) and also the wavelength as measured far away for a photon emitted at:
(a) the surface of the Earth
(b) the surface of a neutron star
(c) 1.2 Schwarzchild radius from center of a black hole.

2. A photon near the surface of the Earth travels a horizontal distance of 1.5 km. How far (in meters) does the photon "fall" in this time? (Hint: think equivalance principle).

3. Calculate the Schwarzchild radius and "density" for the following black hole (BH) masses. (density: $\rho = M/(\frac{4}{3}\pi r_{SC}^3)$)
(a) BH at center of M87 galaxy: $m = 7 \times 10^9 M_\odot$
(b) BH at center of Milky Way: $m = 4 \times 10^7 M_\odot$
(c) Smallest real BH: $m = 8M_\odot$
(d) Primordial: $m = 2 \times 10^{17}$ grams
Convert all densities to gm/cm^3, think of something that is the similar in size to each black hole and compare each density to the density of water.

4. Consider the 2-D metric of the surface of a sphere of constant radius r_0:

$$ds^2 = r_0^2 d\theta^2 + r_0^2 \sin^2\theta d\phi^2.$$

(a) For the 2-D surface of a sphere with metric above, calculate the geodesics using the Euler-Lagrange equations. *HINT: use s as the affine parameter; for part (b) solve the ϕ equation first and think about the constants for the path in question.*
(b) Show that the "shortest distance" between the north and south pole of the sphere is a "line of longitude", that is, find a geodesic that connects the north and south pole.

5. You shine your green laser pointer up to a building 150ft above you. Supposing its wavelength is exactly 5321 Angstroms when it leaves the laser, calculate the wavelength when it hits the building above. Assume you are at sea level in San Diego.

6. Consider the metric for the famous hyperbolic plane.

$$ds^2 = dx^2/y^2 + dy^2/y^2 \text{ (for } y \geq 0)$$

(a) Show that the distance between $(x, y) = (x_0, 0)$ and any point directly above it (x_0, y_0) is infinite. (Use the metric to calculate the distance).
(b) Use the Euler-Langrange equations to find the geodesic equations (Note in this part I am only asking for the equations that determine the geodesics, not the solutions of these equations).
(c) Show that vertical lines are geodesics (that is satisfy the equations from part (b)).
(d) For fun, (NOT FOR CREDIT), try and show that semicircles centered on the x axis (in the upper half plane) are also geodesics. Euclid said that given a line and a point, there is only one line through that point that does not intersect that line (the parallel line). This metric is a famous counter-example since there are an infinite number of geodesics ("straight lines" which are semicircles) through that point that do not interset a vertical geodesic.