1. Using the 2-D Euclidean metric (line element) $ds^2 = dx^2 + dy^2$, find the distance from the origin to $x = 2$, $y = 4$, along the curve $y = x^2$.

2. Consider the 2-D metric (line element) for the surface of a sphere of radius a: $ds^2 = a^2 d\theta^2 + a^2 \sin^2 \theta d\phi^2$.
(a) By integrating this metric along a line of constant ϕ, Find the distance from the south pole the equator.
(b) Find the distance around the sphere at a a constant "latitude", $\theta = 33^0$.
(c) Find the area of a circle of "radius", r, drawn on the curved 2-D surface of this sphere. The radius, r, of the circle is the distance from the center of the circle to the circumference drawn along the curved space.
(d) Show that the area formula you find reduces to πr^2 when $r \ll a$.

3. (a) Write down the metric (line element) for a flat 5-dimensional space with coordinates: x, y, z, u, and v. (where x,y,z correspond to our normal space coordinates and u and v correspond to extra spatial directions).
(b) How far is the point $(x, y, z, u, v) = (2, 2, 2, 2, 2)$ from the origin?
(c) How far is the point $(4, 4, 4, 4, 4)$ from $(1, 1, 1, 1, 1)$?
(d) How far is the point $(1, 1, 1, 1, 1)$ from $(0, 1, 1, 0, 0)$?
(e) Suppose we can't see the extra 2 dimensions (We live on a 3 dimensional brane, with our coordinates x, y, and z and $u = v = 0$ inside this 5-D world.) Then in our world what does the straight line between $(0, 0, 0, 0, 0)$ and $(2, 2, 2, 2, 2)$ look like to us?
(f) What does the line between the point between $(1, 1, 1, 0, 0)$ and $(0, 0, 0, 0, 0)$ look like to us?

4. Consider a sphere with radius a. Draw examples of a triangle on the surface of this sphere which has (a) the sum of interior angles slightly greater than π. (b) the sum of its angles equal to 2π.

5. The surface of the Earth is not a perfect sphere. One quarter of the circumference around a great circle passing through the poles is 9,983 km. This is slightly less than one quarter the of equatorial circumference, 10,019 km, meaning the Earth is slightly squashed. Suppose the surface of the Earth is modeled by an axisymmetric surface with a metric (line element) of the kind in Hartle equation 2.21 with $f(\theta) = \sin \theta (1 + \epsilon \sin^2 \theta)$ for some small ϵ. What values of a and ϵ would best reproduce the known polar and equatorial circumferences? Comment: It is not an accident that one quarter of the polar circumference is almost exactly ten million meters. That was the original definition of the meter.

6. According to Hartle equation 2.4, what is the maximum that the sum of interior angles of a triangle drawn on a sphere can be? Draw a triangle on a sphere that achieves this maximum.