PHYSICS 215A: PARTICLES AND FIELDS, FALL 2014

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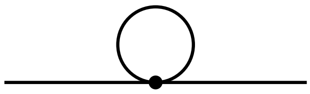
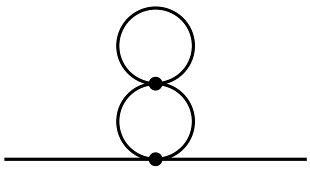
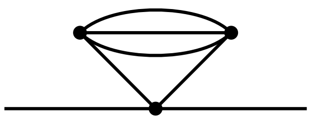
Assignment 5 - For extra credit

December 3, 2013 (Due 11:30 A.M., Thursday December 18, 2014 at MHA 5301)

1. Consider a quantum field theory of a single real scalar field ϕ with interaction Lagrangian density $\mathscr{L}' = -\frac{1}{4!}\lambda\phi^4$. The Feynman rules for this theory were derived in lecture. The last of the rules given states the graph may require a correction factor $1/S$ that must be applied to compensate for over-counting. For the following Feynman graphs determine the symmetry factor S that one must divide by to obtain the correct Green function.

(a) 

(b) 

(c) 

(i) Do this by applying Wick's theorem to the perturbative expansion of the Green function.

(ii) Do this again, but now using the formula

$$S = g \prod_{n=2,3,\ldots} 2^{\beta}(n!)^{\alpha_n} ,$$

where g is the number of possible permutations of vertices which leave un-changed the diagram with fixed external lines, α_n is the number of vertex pairs connected by n identical lines, and β is the number of lines connecting a vertex with itself.

2. Consider a system consisting of a complex scalar field ψ and a real scalar field ϕ described by the Lagrangian density

$$\mathscr{L} = \partial_\mu\psi(x)^*\partial^\mu\psi(x) - m^2\psi(x)^*\psi(x) + \tfrac{1}{2}\partial_\mu\phi(x)\partial^\mu\phi(x) - \tfrac{1}{2}M^2\phi(x)^2 + g\phi(x)\psi(x)\psi(x)^* .$$

We will assume g is small so that this system can be analyzed using perturbation theory. Call "p" the particle created by ψ^*, and $\bar{p}$ it's antiparticle, and let "π" be the particle created by ϕ. For the processes listed below (a) determine under what conditions the process is kinematically allowed and (b) calculate, to lowest non-trivial order in perturbation theory, the scattering amplitude for the process. Label the 4-momenta as follows: in $a+b\to c+d$ use $p_a = k_1$, $p_b = k_2$, $p_c = k_1'$ and $p_d = k_2'$. Express your answer in terms of the variables $s=(k_1+k_2)^2=(k_1'+k_2')^2$, $t=(k_1-k_1')^2=(k_2-k_2')^2$ and $u=(k_1-k_2')^2=(k_2-k_1')^2$ (these are not all independent, $s+t+u=\sum_{i=1}^{2}(k_i^2+k_i'^2)$, so pick the most convenient).

(i) $p\bar{p}\to p\bar{p}$.

(ii) $pp\to pp$.

(iii) $\bar{p}\bar{p}\to\bar{p}\bar{p}$.

(iv) $\pi p\to\pi p$.

(v) $\pi p\to\pi\bar{p}$.

(vi) $\pi\pi\to p\bar{p}$.

3. Calculate the total cross section (at some fixed center of mass energy, or equivalently, at fixed s) for each of the cases of problem 2 above. Note: in lecture we determined the phase space $d\Phi_2$ for 2-particle final states when the two particles have equal mass. You may use this result for the cases in problem 2 that have equal mass final states. For the other cases you will have to derive a formula for the integral of $d\Phi_2$ for the unequal mass case.

4. Consider the following Lagrangian density:

$$\mathscr{L}=\tfrac{1}{2}\partial_\mu A(x)\partial^\mu A(x)-\tfrac{1}{2}M^2A(x)^2+\tfrac{1}{2}\partial_\mu B(x)\partial^\mu B(x)-\tfrac{1}{2}m^2B(x)^2+\tfrac{1}{2}\partial_\mu C(x)\partial^\mu C(x)+gA(x)B(x)C(x)^2,$$

describing the interactions of particles a, b and c (which are quanta of the real fields A, B and C, respectively).

(i) Assuming $M>m$ compute to lowest order in perturbation theory the differential decay width of particle a, $d\Gamma/de_1\,de_2$, where e_1 and e_2 are two kinematic variables of your choosing.
Note: 3-particle phase space $d\Phi_3$ has $9-4=5$ unintegrated variables. However, in the CM-frame, which is the rest frame of the decaying particle, the 3-momenta of the three resulting particles lie on a plane. The orientation of this plane corresponds to two of the integrals. Moreover, within this plane the orientation of one of the 3-momenta correspond to one more integral. In the decay of a spin-less particle these orientations must be equally probable, so the integrals can be made even before knowing the form of the decay-amplitude. This leaves $5-3=2$ variables: these are what I have called e_1 and e_2.
(*If you cannot compute the phase space integral, try $m=0$, it is simpler*).

(ii) Find the lifetime of particle a in terms of g, M and m (to lowest order in perturbation theory).

(iii) If $g=1$, $M=400$ MeV and $m=200$ MeV, what is the lifetime of a in seconds?