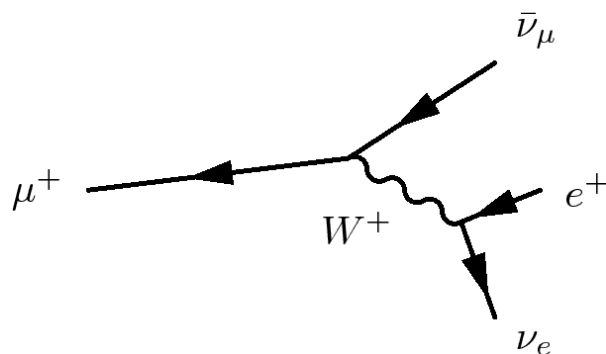


Muon Spin Relaxation Technique (μSR)

History of Muon

Before we get into details of the technique, it is worth discussing the origin and property of subatomic particle muon. Anderson and Neddermeyer were the first to discovered muon in 1937, when they were observing particles from cosmic rays. They observed a particle that followed a curved trajectory in magnetic field (, which means it is charged) but the curvature of the trajectory was less than that of electrons and more than that of protons. It meant it is heavier than electron but lighter than proton. Due to its mass, at first people thought this particle was something that's holding the nucleus together. Later people realized it was actually a lepton. The lepton family has electron, muon, and tau, also their neutrino counterpart (ν_e, ν_μ and ν_τ). Muon is very unstable, it has a lifetime of 2.2 microsecond. It decays rapidly into electron or positron through weak interaction.



$$m_\mu \approx 207 m_e$$

$$\text{Charge} = \pm e$$

$$\text{Spin} = \frac{1}{2}$$

$$\text{gyromagnetic ratio } \gamma_\mu = 851.6$$

$$\text{lifetime} = 2.197 \mu\text{s}$$

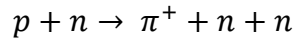
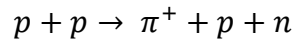
(Feynman Diagram of the positive Muon decay)

The cosmic ray was traveling at nearly the speed of light. It enhanced the lifetime of muon by time dilation, thus Anderson and Neddermeyer were able to detect the particles on Earth.

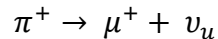
In the last 60s, T.D. Lee and C.N. Yang proposed the idea that parity may not be conserved in weak interaction. Eventually it was discovered parity is indeed variant in weak interaction which enabled us to realize polarized beam of muons from weak decay was possible. Garwin et al was the first to purpose using muon in probing local magnetic property of atoms [2]. I don't really know the concept of parity in depth; common conception was physics should be the same even if spatial coordinates were inverted in a system. So I supposed if parity is conserved, muons are going to be equally distributed in spin up and spin down. Now weak interaction doesn't conserve parity, the muons are going to be all polarized in a particular spin direction. So why would polarization matter? Let's get into the detail of the technique.

The Experimental Technique

In order to create muon, we need a pion first. A pion is composed of a quark and an antiquark. Experimentalists bombard carbon nucleus with 600~800 MeV energetic proton beams. The protons will go in and interact with other protons and neutrons to create pions.



In the process, only positive pions are produced. Pions are highly unstable. Pions will decay into muons within nanoseconds.

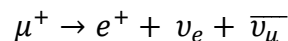


Looking at first two equations, we can tell pions are actually spinless. The two protons produce proton and neutron, the spins have already given away. The remaining spin is zero and that goes to pion. When pion decay into muon and neutrino, we know for sure they get opposite spin. Neutrino has a helicity of -1, that means its spin direction is precisely opposite from its momentum direction. Although it's very hard for us to detect neutrino, when we see the direction of muon beam we can immediately deduce the spin orientation of the muons. Also the muons are polarized, which means their spins are all pointed in the same direction. Muons are also subjected to Larmor precession under the influence of external field. Now we have polarized muons and their spin react to magnetic field. We basically have bunch of tiny magnetometers that can be sent into literally anything. If the muons weren't polarized, we will get two opposing signals of the interaction. The two opposing signals will leave us with no important information.

Muon Implementation

The process of sending muons into material is called muon implementation. Energetic muons will go into lattice and they will be "implemented", in other words they will stuck to the lattice site until their decay. Inelastic scattering with electrons, electron capture (bound state of electron and muon) and ionization of atoms can lead to the loss of muon's kinetic energy. Once muons lose all their kinetic energy, they will stay and interact with the local magnetic field of the lattice. The direction of the muon's spin is influenced by the strength of local magnetic field. More precisely, the precession frequency of the spin is directly determined by the local field strength.

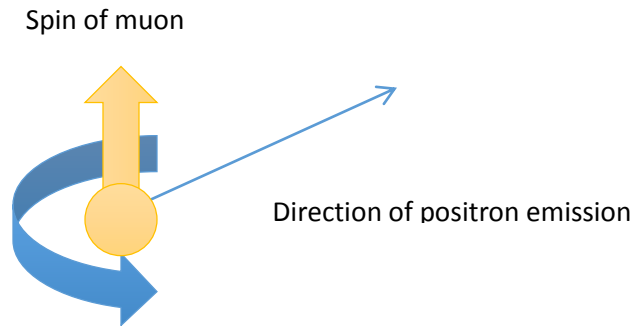
First of all, the muons will decay after its implementation:



The products are positrons, electron neutrino and anti-muon neutrino.

The direction of positron emission is dependent on the direction of muon's spin. The probability of positron emission is given by

$$W(\theta) d\theta = 1 + \frac{\cos \theta}{3} P(t) d\theta$$



The angle between the spin and emission direction is θ

The function $P(t)$ is the polarization of muon as a function of time, with $P(0) = 1$

The equation actually suggests the emission of positron (weak decay) is anisotropic. That means some regions of the detector will pick up higher density of positron emission. Obviously, the directions along 0 and 2π have most intense positron signals. However, knowing the direction of positron emission isn't enough. At most this tells us direction of muon spin (up & down) along an axis but it doesn't tell us any useful information of the local magnetic field we try to measure.

Now we turn to Larmor precession equation

The Evolution of Muon Spin and its Polarization

As soon as the muon sits in the lattice, the local magnetic field will exert torque to align them. However, muons spins can't really align with magnetic field due to uncertainty principle. The spins will precess around some direction along $+z$ or $-z$ direction. Eventually it will become depolarize and the net moment will be zero. The rate at which muon spin rotates along perpendicular direction of local magnetic field before it gets depolarizes or "relaxes" is determined by Larmor precession equation.

The time derivative of angular momentum is equal to the torque exerted on the spin.

$$\frac{d\vec{m}_\mu}{dt} = \gamma_\mu [m_\mu(t) \times B_{loc}(t)]$$

The moment of muon is proportional to its polarization (magnetization)

$$\frac{d\vec{P}}{dt} = \gamma_\mu [P(t) \times B_{loc}(t)]$$

A simple solution to the equation assuming we set up transverse field experiment

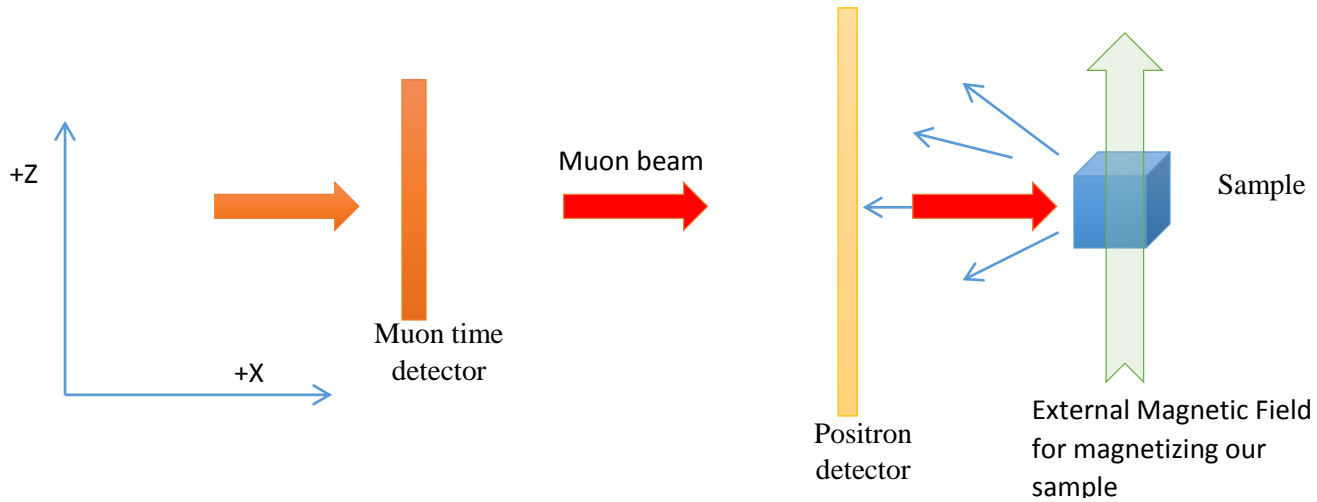
$$P(t) = P(0) \hat{z} + P(0)(\cos[\gamma_\mu B_{loc}(t)t] \hat{x} - \sin[\gamma_\mu B_{loc}(t)t] \hat{y})$$

There's also the exponential spin relaxation factor, which gives important information of local magnetic property as well. Depending on the nature of the experiment, in this derivation, the

most important piece of information is the precession frequency. The precession frequency will dictate the frequency of positron detection oscillation, which tells us the exact $B_{loc}(t)$.

$$\text{Precession frequency } \omega(t) = \gamma_{\mu} B_{loc}(t)$$

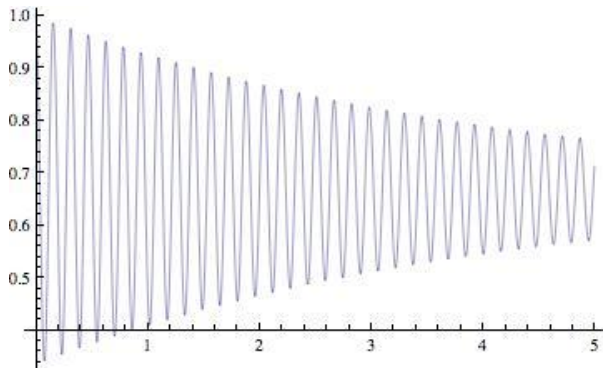
Transverse Field μSR



Initially the muons' spins are polarized in negative x-direction, because it is moving right along positive x-direction. Along the detector, we should expect positron concentrates at $z = 0$. However, as time goes on, the muons are depolarized by the local magnetic field. Muons will no longer point their spins along x-direction. The signal for positron along x-direction will weaken over time.

$$N(t) \propto \frac{1}{3} e^{-\lambda_{damp} t} + \frac{2}{3} e^{-\lambda_{relaxation} t} \cos \omega t$$

The density of positron observed along the x-direction is a function of time, precession frequency, and damping/relaxation factor.



The precession frequency is illustrated in this graph. The small oscillation within the exponential decay curve is what tells us the property of local magnetic field in materials. In fact the relaxation factor has important information about the nucleus and various other magnetic contributions of the lattice system.

The above discussion is actually fairly simplified. We haven't really discussed the distribution of field inside the physical sample. For real experiment, we need to assume a Gaussian field distribution for the internal field of the sample in order to derive a relaxation rate as a function of time.

Supposed before we apply any field to our sample. The probability of magnetization of the internal nuclear magnetic dipoles within the sample follows a random Gaussian distribution.

$$P(H_i) = \frac{1}{\sqrt{2\pi}\Delta} \text{Exp}\left(-\frac{H_i^2}{2\Delta^2}\right) \quad i = x, y, z$$

As soon as we implement muons into the system, muons will depolarize by the field exhibit by these nuclear dipoles.

Supposed, initially all muons are polarized in x-direction. Depolarization in a Gaussian field as a function of time is the following

$$G(t) = \iiint S_x(t) P(H_x)P(H_y)P(H_z) dH_x dH_y dH_z$$

If an external field is applied along z-axis then we get the following equation

$$G(t, \omega) = 1 - \frac{2\sigma^2}{\omega^2} \left(1 - \cos(\omega t) \exp\left(-\frac{\sigma^2 t^2}{2}\right)\right) + \frac{2\sigma^4}{\omega^3} \int_0^t \sin(\omega\tau) \exp\left(-\frac{\sigma^2 \tau^2}{2}\right) d\tau$$

$$\sigma = \gamma\Delta$$

$$\omega = \gamma B_{\text{external}}$$

The depolarization curve (from observing positron detections) and the frequency at which relaxation occurs will tell us vital information of the field and dipole distribution inside the sample.

“Field and Temperature Dependence of the Superfluid Density in LaFeAsO(x)F(x) Superconductors: A Muon Spin Relaxation Study” by Luetkens et al

This particular study utilized transverse field muon spin relaxation to study the nature of vortices in iron-based superconductors as they change the doping pnictides ratio in the compound [3]. They applied a small field around 40 Gauss, just so the sample is in its vortex state.

The study has determined the values of Gaussian relaxation rate σ for contributions from nuclear dipoles and contributions from vortex distributions.

Due to its technical aspect, there are few important concepts I do not understand, such as commensurate single wave density, static single wave density magnetism, and its instability. However, utilizing muon relaxation rates in the sample, the study has found that spin-density wave is suppressed upon doping of fluorine. The key signature of SDW suppression is the decreasing in Gaussian relaxation rate σ from superconducting state contribution as doping ratio increases. It suggests that superfluidity is very dilute within the sample.

Reference

1. *Note on Nature of Cosmic Ray Particles* Seth H. Neddermeyer, Phys. Rev., Vol.51, 884 (1937)
2. *Observations of the Failure of Conservation of Parity and Charge Conjugation in Meson Decays: the Magnetic Moment of the Free Muon* R. L. Garwin, L.M. Lederman, M. Weinrich, Phys. Rev. 105, 1415 (1957)
3. *Field and Temperature Dependence of the Superfluid Density in LaFeAsOF Superconductors: A Muon Spin Relaxation Study* H. Luetkens et al, Phys.Rev.Lett. 101, 097009 (2008)