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Quantum Mechanics B (Physics 130B) Fall 2014
Quiz – Solutions

Problems

1. What is the wavelength of the so-called Balmer transition which involves an electron which transitions from an n = 5 level to an n = 2 for hydrogen?

   $\frac{1}{\lambda} = R(\frac{1}{n_f^2} - \frac{1}{n_i^2}) \approx 1.1 * 10^7[\text{m}^{-1}] * (\frac{1}{4} - \frac{1}{25}) \approx 2.3 * 10^6[\text{m}^{-1}] \implies \lambda \approx 430[\text{nm}]$

2. The energy of the hydrogen atom in its ground state is -13.6 eV. What is the ionization energy of the atom in the n = 4 state?

   $\Delta E = -\frac{m_e e^4}{2(4\pi\epsilon_0)^2\hbar^2}(\frac{1}{n_f^2} - \frac{1}{n_i^2}) \approx -13.6[\text{eV}](0 - \frac{1}{4^2}) \approx .85[\text{eV}]$

3. A muon is a subatomic particle that behaves like electron except that its mass is 207 times higher than that of the electron. If a muon were bound to a proton to make muonium how are the energy levels in the Bohr model related to those for Hydrogen?

   $E_n = -\frac{m_e e^4}{2(4\pi\epsilon_0)^2\hbar^2}\frac{1}{n^2}$ where m_e is approximately the mass of an electron.

   Replace this with $m_\mu = 207 m_e$.

4. Write down the wave functions for hydrogen including all three primary quantum numbers n, ℓ ,m and explain the physical meaning of each of these three quantum numbers.

   $\psi_{n\ell m}(r,\theta,\phi) = R_{n,\ell}(r)Y_{\ell,m}(\theta,\phi)$ by separation of variables.

   $Y_{\ell,m}$ are the spherical harmonics and depend on the orbital angular momentum ℓ and it's z-component m.

   $R_{n,\ell}$ are the associated Laguerre polynomials times an exponentially decaying factor of $e^{-\frac{r}{na}}$ where n is a quantum number associated with the energy levels.

5. A hydrogen atom in its n = 1 state absorbs a 12.09 eV photon. To what level is the electron promoted?

   $E_n = E_1 + E_{\text{photon}} \approx -13.6 + 12.1[\text{eV}] = -\frac{13.6}{n^2}[\text{eV}]$

   $\implies n^2 \approx \frac{13.6}{1.51} \approx 9$ so the electron is moved to $n = 3$

6. Lithium has atomic number 3. What is the energy needed to change a Li++ doubly-ionized ion in its ground state to a triply ionized lithium atom: Li+++?

   Doubly ionized Li has only one electron so this problem is equivalent to the ionization energy of the Bohr atom where $e_{\text{proton}} \to 3e_{\text{proton}}$ which appears as e^2 in the formula for the spectrum.

$E_{\text{ionization}} = Z^2 \frac{m_e e^4}{2(4\pi\epsilon_0)^2\hbar^2}\frac{1}{n^2} \approx 9 * 13.6[\text{eV}]$

7. If the principal quantum number for hydrogen is 5, which one of the following is not a permitted orbital angular quantum number? Explain your answer briefly.

   $\{\ell = 2,\ \ell = 5,\ \ell = 3,\ \ell = 4\}$

   $\ell = 5$ is not allowed when $n = 5$ as the range of permissible $\ell \in \{0, 1, \cdots, n-1\}$

8. What is the highest value of the orbital quantum number ℓ for an electron in Krypton's ($Z = 36$) outer shell when in its ground state?

   (Hint: Filled shells have $2n^2$ distinct states)

   The factor of 2 in the state counting is due to the fact elecrons have a spin quantum number in addition to n, ℓ, m

   Krypton should have 36 electrons and in its ground state they occupy the various orbitals/shells starting from the lowest energies and ascending.

   $n = 1, 2, 3$ are completely filled as there are $2 + 8 + 18 = 28$ states.

   The remaining 8 electrons occupy the $4s$ and $4p$ orbitals. This is the 'outer shell'. The $4p$ orbital has $\ell = 1$ so that is the highest ℓ occupied by any electron in this shell.

   Note that this completely fills the $4p$ orbital. This is to be expected as Krypton is a noble gas.

   One possible pitfall is to correctly conclude $n = 4$ is the outer shell but then say $\ell = n - 1 = 3$ is the highest value of ℓ.

   While certainly it is the largest angular momentum available to any electron at $n = 4$ the degeneracy in ℓ is broken by generic perturbation/interactions. Thus they will prefer lower ℓ states as they will be lower in energy. This can be quite dramatic; electrons will sooner occupy $4s$ than $3d$ for simple atoms.

   Partial credit was awarded for the answer of $\ell = 3$