2a) $y = l(1-\cos\phi)$, $\dot{y} = l\sin\phi\,\dot{\phi}$

$x = l\sin\phi + x_0\cos\omega t$, $\dot{x} = l\cos\phi\,\dot{\phi} - x_0\omega\sin\omega t$

$$L = \frac{m}{2}\left[(l\cos\phi\,\dot{\phi} - x_0\omega\sin\omega t)^2 + l^2\sin^2\phi\,\dot{\phi}^2\right] - mgl(1-\cos\phi)$$

$$= \frac{m}{2}\left(l^2\dot{\phi}^2 - 2lx_0\omega\dot{\phi}\cos\phi\sin\omega t + x_0^2\omega^2\sin^2\omega t\right) - mgl(1-\cos\phi)$$

$$\frac{\partial L}{\partial\dot{\phi}} = ml^2\dot{\phi} - mlx_0\omega\cos\phi\sin\omega t$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\phi}}\right) = ml^2\ddot{\phi} + mlx_0\omega\dot{\phi}\sin\phi\sin\omega t - mlx_0\omega^2\cos\phi\cos\omega t$$

$$\frac{\partial L}{\partial\phi} = +mlx_0\omega\dot{\phi}\sin\phi\sin\omega t - mgl\sin\phi$$

EoM: $$ml^2\ddot{\phi} = mlx_0\omega^2\cos\phi\cos\omega t - mgl\sin\phi$$

$$\ddot{\phi} = \frac{\omega^2 x_0}{l}\cos\phi\cos\omega t - \frac{g}{l}\sin\phi$$

Match with $\ddot{\phi} = -\frac{d}{d\phi}U(\phi) + f_1(\phi)\cos\omega t$

$\Rightarrow$ $U(\phi) = -\frac{g}{l}\cos\phi$, $f_1(\phi) = \frac{\omega^2 x_0}{l}\cos\phi$

Decompose ϕ into slow and fast solution:

$$\phi = \langle\phi\rangle + \tilde{\phi}$$

And take time-average $\langle\;\rangle$:

$$\langle\ddot{\phi}\rangle + \ddot{\tilde{\phi}} = -\frac{d}{d\phi}U(\phi)\Big|_{\langle\phi\rangle} + f_1(\langle\phi\rangle)\cos\omega t + \frac{\partial f_1}{\partial\phi}\Big|_{\langle\phi\rangle}\langle\tilde{\phi}\cos\omega t\rangle$$

Fast: $\ddot{\tilde{\phi}} = f_1\cos\omega t \Rightarrow \tilde{\phi} = -\frac{f_1}{\omega^2}\cos\omega t$

$$\therefore \langle\tilde{\phi}\cos\omega t\rangle = -\frac{f_1}{\omega^2}\langle\cos^2\omega t\rangle = -\frac{f_1}{2\omega^2}$$

Slow: $\langle\ddot\phi\rangle = -\frac{d}{d\phi}\left(U(\langle\phi\rangle) + \frac{f_1^2(\langle\phi\rangle)}{4\omega^2}\right)$ $\qquad f_1(\langle\phi\rangle) = \frac{\omega^2 x_0}{l}\cos\langle\phi\rangle$

$= -\frac{d}{d\phi} U_{eff}$

$\Rightarrow U_{eff} \simeq -\frac{g}{l}\cos\phi + \frac{\omega^2 x_0^2}{4l^2}\cos^2\phi = -\frac{g}{l}\cos\phi + \frac{\omega^2 x_0^2}{8l^2}(1+\cos 2\phi)$

Equilibrium $U'_{eff} = 0 = \frac{g}{l}\sin\phi - \frac{\omega^2 x_0^2}{4\cancel{4}l^2}\sin 2\phi = \frac{g}{l}\sin\phi\left[1 - \frac{\omega^2 x_0^2}{2gl^2}\cos\phi\right]$

$\Rightarrow \phi = 0, \pi, \phi_0 \quad$ where $\phi_0 = \cos^{-1}\left(\frac{2gl}{\omega^2 x_0^2}\right)$ if $\frac{2gl}{\omega^2 x_0^2} < 1$

$U''_{eff} = \frac{g}{l}\cos\phi - \frac{\omega^2 x_0^2}{2l^2}\cos 2\phi$

For $\phi = 0$, $U''_{eff} = \frac{g}{l} - \frac{\omega^2 x_0^2}{2l^2} \Rightarrow$ stable when $\boxed{\frac{\omega^2 x_0}{2gl} < 1}$

OR $U''_{eff} > 0$

For $\phi = \pi$, $U''_{eff} = -\left(\frac{g}{l} - \frac{\omega^2 x_0^2}{2l^2}\right) \Rightarrow$ stable when

$\frac{\omega^2 x_0}{2gl} > 1$

For $\phi = \phi_0$, given $\frac{\omega^2 x_0^2}{2gl} > 1$, $\cos\phi_0 = \frac{2gl}{\omega^2 x_0^2}$

$$U''_{eff}\Big|_{\phi_0} = \frac{g}{l}\cos\phi_0 - \frac{\omega^2 x_0}{2l^2}\cos 2\phi_0 = \frac{g}{l}\cos\phi_0 - \frac{\omega^2 x_0^2}{2l^2}(2\cos^2\phi_0 - 1)$$

$$= \frac{g}{l}\left(\frac{2gl}{\omega^2 x_0^2}\right) - \frac{\omega^2 x_0^2}{\cancel{2}l^2}\left(\frac{2gl}{\omega^2 x_0^2}\right)^2 + \frac{\omega^2 x_0^2}{2l^2}$$

$$= \frac{2g^2}{\omega^2 x_0^2} - \frac{4g^2}{\omega^2 x_0^2} + \frac{\omega^2 x_0^2}{2l^2} = -\frac{2g^2}{\omega^2 x_0^2} + \frac{\omega^2 x_0^2}{2l^2}$$

$$= -\frac{2g^2}{\omega^2 x_0^2}\left[1 - \left(\frac{\omega^2 x_0^2}{2gl}\right)^2\right] > 0$$

$\Rightarrow \phi = \phi_0$ stable //

3 $x = l\sin\phi + r_o\cos\omega t \quad , \quad \dot{x} = l\dot{\phi}\cos\phi - r_o\omega\sin\omega t$

$$y = l(1-\cos\phi) + r_o\sin\omega t \quad , \quad \dot{y} = l\dot{\phi}\sin\phi + r_o\omega\cos\omega t$$

$$L = \frac{m}{2}(\dot{x}^2 + \dot{y}^2) - mgy$$

$$= \frac{m}{2}\left(l^2\dot{\phi}^2 + r_o^2\omega^2 - 2lr_o\omega\dot{\phi}\cos\phi\sin\omega t + 2lr_o\omega\dot{\phi}\sin\phi\cos\omega t\right)$$

$$- mgl(1-\cos\phi) - mgr_o\sin\omega t$$

$$\frac{\partial L}{\partial\dot{\phi}} = ml^2\dot{\phi} + mlr_o\omega(\sin\phi\cos\omega t - \cos\phi\sin\omega t)$$

$$= ml^2\dot{\phi} + mlr_o\omega\sin(\phi - \omega t)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\phi}}\right) = ml^2\ddot{\phi} + mlr_o\omega(\dot{\phi} - \omega)\cos(\phi - \omega t)$$

$$\frac{\partial L}{\partial\phi} = mlr_o\omega\dot{\phi}(\cos\phi\cos\omega t + \sin\phi\sin\omega t) - mgl\sin\phi$$

$$= mlr_o\omega\dot{\phi}\cos(\phi - \omega t) - mgl\sin\phi$$

$$\Rightarrow ml^2\ddot{\phi} = mlr_o\omega^2\cos(\phi - \omega t) - mgl\sin\phi$$

$$\ddot{\phi} = -\frac{g}{l}\sin\phi + \frac{\omega^2 r_o}{l}\cos(\phi - \omega t)$$

identify

$\Rightarrow$ $U(\phi) = -\frac{g}{l}\cos\phi \qquad f = f_1\cos\omega t + f_2\sin\omega t$

$$f_1 = \frac{\omega^2 r_o}{l}\cos\phi \qquad f_2 = \frac{\omega^2 r_o}{l}\sin\phi$$

Let $\phi = \langle\phi\rangle + \hat{\phi}$, and take $\langle\ \rangle$:

$$\langle\ddot{\phi}\rangle + \ddot{\hat{\phi}} = -\frac{d}{d\phi}U\Big|_{\langle\phi\rangle} + f_1(\langle\phi\rangle)\cos\omega t + f_2(\langle\phi\rangle)\sin\omega t$$

$$+ \frac{\partial f_1}{\partial\phi}\Big|_{\langle\phi\rangle}\langle\hat{\phi}\cos\omega t\rangle + \frac{\partial f_2}{\partial\phi}\Big|_{\langle\phi\rangle}\langle\hat{\phi}\sin\omega t\rangle$$

3 Cont'd

Since $\ddot{\tilde{\phi}} = f(\langle\phi\rangle) = f_1 \cos\omega t + f_2 \sin\omega t$

$\Rightarrow \tilde{\phi} = -\frac{f}{\omega^2}$

$$\Rightarrow \begin{cases} \langle \tilde{\phi} \cos\omega t \rangle = -\frac{f_1}{\omega^2} \langle \cos^2\omega t \rangle = -\frac{f_1}{2\omega^2} \\ \langle \tilde{\phi} \sin\omega t \rangle = -\frac{f_2}{\omega^2} \langle \sin^2 \omega t \rangle = -\frac{f_2}{2\omega^2} \end{cases} \qquad \langle \cos\omega t \sin\omega t \rangle = 0$$

So, $\langle \ddot{\phi} \rangle = -\frac{d}{d\phi}\left(U(\phi) + \frac{f_1^2 + f_2^2}{4\omega^2} \right)\Big|_{\langle\phi\rangle}$

$$\Rightarrow U_{eff}(\phi) = U(\phi) + \frac{f_1^2 + f_2^2}{4\omega^2}$$
$$= -\frac{g}{l}\cos\phi + \frac{1}{4\omega^2}\left(\frac{\omega^2 r_0}{l}\right)^2 = -\frac{g}{l}\cos\phi + \frac{\omega^2 r_0^2}{4 l^2}$$

$U'_{eff} = \frac{g}{l}\sin\phi = 0$ when $\phi = 0, \pi$

$U''_{eff} = +\frac{g}{l}\cos\phi$

$\Rightarrow \phi = 0$ ~~un~~stable, $\phi = \pi$ unstable equilibrium. for all ω.

4. $\underline{B} = B(t)\hat{z}$

$\partial_t \underline{B} = \dot{B}\hat{z} = -c\nabla\times\underline{E}$ $\Rightarrow$ Assume $E_z = 0$

$$\Rightarrow \begin{cases} \dot{B} = -c\left(\dfrac{\partial E_y}{\partial x} - \dfrac{\partial E_x}{\partial y}\right) \\ 0 = \dfrac{\partial E_x}{\partial z} \\ 0 = -\dfrac{\partial E_y}{\partial z} \end{cases} \Rightarrow \underline{E} = -\frac{\dot{B}}{2c}(x\hat{y} - y\hat{x})$$

$$E_x = \frac{\dot{B}y}{2c}, \quad E_y = -\frac{\dot{B}x}{2c}$$

Equation of motion of ion: $m\ddot{\vec{x}} = +e(\vec{E} + \vec{v}\times\vec{B}/c)$

$B_x = B_y = 0$

z: $m\ddot{z} = +e\cancel{E_z} + \frac{e}{c}\cancel{(\underline{v}\times\underline{B})_z} = 0$

x: $m\ddot{x} = +eE_x + \frac{e}{c}(v_y B_z - \cancel{v_z B_y}) = +\frac{e\dot{B}}{2c}y + \frac{eB}{c}\dot{y}$

y: $m\ddot{y} = +eE_y + \frac{e}{c}(\cancel{v_z B_x} - v_x B_z) = -\frac{e\dot{B}}{2c}x - \frac{eB}{c}\dot{x}$

let $\Omega \equiv \frac{eB}{mc}$ ~~and z~~

$$\ddot{x} = +\frac{\dot{\Omega}}{2}y + \Omega\dot{y} \quad \text{and} \quad \ddot{y} = -\frac{\dot{\Omega}}{2}x - \Omega\dot{x}$$

let $z = x + iy$

$$\Rightarrow \ddot{z} = -\frac{\dot{\Omega}}{2}(-y + ix) + \Omega(\dot{y} - i\dot{x}) = -i\Omega\dot{z} - i(\dot{\Omega}/2)z$$

Set $z(t) = w(t)e^{-i\int(\Omega/2)dt}$ ← integrating factor method

$$\dot{z} = (\dot{w} - i\Omega/2\, w)e^{-i\int(\Omega/2)dt}$$

$$\ddot{z} = \left[\ddot{w} - i(\dot{\Omega}/2)w - \frac{\Omega^2}{4}w - i\Omega\dot{w}\right]e^{-i\int(\Omega/2)dt}$$

$$i\Omega\dot{z} = \left[i\Omega\dot{w} + \Omega^2/2\, w\right]e^{-i\int(\Omega/2)dt}$$

$$\Rightarrow \ddot{z} + i\Omega\dot{z} = \left[\ddot{w} + \frac{\Omega^2}{4}w\right]e^{-i\int\Omega/2\,dt} - i(\dot{\Omega}/2)z$$

4. Cont'd:

$$\ddot{z} + i\Omega_b\dot{z} + i(\dot{\Omega}_b/2)z = 0 = (\ddot{w} + \frac{\Omega_b^2}{4}w)e^{-i\int(\Omega_b/2)t}$$

$$\Rightarrow \ddot{w} + \frac{\Omega_b^2}{4}w = 0 \quad //$$

b) Let B be linear in time $\Rightarrow B = \alpha t$

$$\Rightarrow \frac{d^2w}{dt^2} + \frac{\alpha^2 t^2}{4}w = 0$$

To get to Bessel's Equation, we want something in the form $t^2\ddot{u} + t\dot{u} + \ldots$ for $\dot{u}$ and $\ddot{u}$:

Try $w = ut^{1/2}$

$\dot{w} = \dot{u}t^{1/2} + \frac{1}{2}u/t^{1/2}$

$\ddot{w} = \ddot{u}t^{1/2} + \dot{u}/t^{1/2} - \frac{1}{4}u/t^{3/2}$

$$\therefore \ddot{w} + \frac{\alpha^2t^2}{4}w = \ddot{u}t^{1/2} + \dot{u}/t^{1/2} - \frac{1}{4}u/t^{3/2} + \frac{\alpha^2t^2}{4}ut^{1/2} = 0$$

$$\Rightarrow t^2\ddot{u} + t\dot{u} + [(\tfrac{\alpha}{2})^2t^4 - \tfrac{1}{4}]u = 0 \quad (*)$$

Compare to simple Bessel's Eq: $x^2u'' + xu' + [x^2 - n^2]u = 0$

We should transform $x = t^2$

$$\frac{dx}{dt} = 2t = 2\sqrt{x}$$

$$\Rightarrow \frac{du}{dt} = \frac{dx}{dt}\frac{du}{dx} = 2\sqrt{x}u', \quad \frac{d^2u}{dt^2} = 2\sqrt{x}\frac{d}{dx}(2\sqrt{x}u') = 2u' + 4xu''$$

$$(*) \Rightarrow x(2u' + 4xu'') + \sqrt{x}(2\sqrt{x}u') + [\frac{\alpha^2x^2}{4} - \frac{1}{4}]u = 0$$

$$\Rightarrow x^2u'' + xu' + [\frac{\alpha^2x^2}{16} - \frac{1}{16}]u = 0$$

Let $x \to \frac{\alpha}{4}x \quad \Rightarrow x^2u'' + xu' + (x^2 - \frac{1}{4^2})u = 0$

4 Cont'd, $x^2 u'' + x u' + (x^2 - \frac{1}{4^2}) u = 0$

$\Rightarrow u(x) = c_1 J_{1/4}(x)$

Bessel function of 1^{st} kind (2^{nd} kind is singular at $t=0$ or $x=0$ and so dropped)

Thus, solution is

$$w(t) = u(t)\, t^{1/2}$$
$$= c_1 t^{1/2} J_{1/4}\left(\frac{\alpha}{4} t^2\right)$$

Consider magnetic moment, $\vec{m} = \frac{1}{2} q \underline{r} \times \underline{v}$

$$m_z = \frac{e}{2}(x\dot{y} - y\dot{x}) = \frac{e}{2} Im(z\bar{\dot{z}})$$ where $\bar{\dot{z}}$ is complex conj. of $\dot{z}$

$$\dot{z} = \left(\dot{w} - i\frac{\Omega}{2} w\right) e^{-i\int \Omega/2\, dt}$$
$$\bar{\dot{z}} = \left(\dot{w} + \frac{i\Omega}{2} w\right) e^{i\int \Omega/2\, dt}$$

$$\Rightarrow Im(z\bar{\dot{z}}) = Im\left[w\left(\dot{w} + \frac{i\Omega}{2} w\right)\right] = \frac{\Omega}{2} w^2$$
$$= \frac{\alpha c_1}{2} t^2 J_{1/4}^2\left(\frac{\alpha}{4} t^2\right)$$

For large $|t|$ $(t \to \pm\infty)$ $J_{1/4}(z) = \sqrt{\frac{2}{\pi z}}\left[\cos\left(z - \frac{\pi}{8} - \frac{\pi}{4}\right) + \mathcal{O}(|z|^{-1})\right]$

$$\Rightarrow t^2 J_{1/4}^2 \simeq t^2 \frac{2}{\pi t^2}\left(\frac{4}{\alpha}\right) \cos^2\left(\frac{\alpha}{4} t^2 - \frac{\pi}{8} - \frac{\pi}{4}\right)$$

$$\Rightarrow m_z \propto \cos^2\left(\frac{\alpha t^2}{4} - \delta_0\right)$$ ~~fin~~ oscillating with finite amplitude..

$\Rightarrow$ "constant"

6. $H = H_0(q,p) + V(q)\frac{d^2A}{dt^2}$, $A(t)$ is periodic with period $\tau \ll T$. Assume $H_0 = \frac{p^2}{2m} + V_0(q)$

a) Since $A(t)$ is periodic, we write $A(t) = \sum_n A_n \cos n\omega t$ where $\omega \equiv 2\pi/\tau$

$$\Rightarrow \ddot{A}(t) = -\sum_n n^2\omega^2 A_n \cos n\omega t$$

$$\Rightarrow H = \frac{p^2}{2m} + V_0(q) - V\sum_n n^2\omega^2 A_n \cos n\omega t$$

$$\dot{p} = -\frac{\partial H}{\partial q} = -\frac{\partial}{\partial q}V_0(q) + \frac{dV}{dq}\sum_n n^2\omega^2 A_n \cos n\omega t$$
$$= m\ddot{q}$$
$$= -\frac{\partial V_0}{\partial q} + \sum_n f_n(q)\cos n\omega t \text{ , where } \boxed{f_n = n^2\omega^2 A_n \frac{\partial V}{\partial q}}$$

Let $q = \langle q\rangle + \tilde{q}$ where $\langle q\rangle$ is short-time average $\Rightarrow \tilde{q}$ fast

$$\Rightarrow m\langle\ddot{q}\rangle + m\ddot{\tilde{q}} = -\frac{dV_0}{dq}\Big|_{\langle q\rangle} + \sum_n f_n(\langle q\rangle)\cos n\omega t$$
$$+ \sum_n \frac{\partial f_n}{\partial q}\Big|_{\langle q\rangle}\langle\tilde{q}\cos n\omega t\rangle$$

Fast: $m\ddot{\tilde{q}} = \sum_n f_n(\langle q\rangle)\cos n\omega t \Rightarrow \tilde{q} = -\sum_n \frac{f_n}{mn^2\omega^2}\cos n\omega t$

$$\Rightarrow \langle\tilde{q}\cos n\omega t\rangle = -\frac{f_n}{2mn^2\omega^2}$$ with $\langle\cos p\omega t\cos n\omega t\rangle = \frac{1}{2}\delta_{p,n}$

$$\Rightarrow \text{Last term} = \sum_n \frac{\partial f_n}{\partial q}\Big|_{\langle q\rangle}\left(-\frac{f_n}{2mn^2\omega^2}\right)$$
$$= -\frac{\partial}{\partial q}\left\{\sum_n \frac{f_n^2}{4mn^2\omega^2}\right\}$$

6 Cont'd

$$\{\cdots\} = \sum_n \frac{f_n^2}{4mn^2\omega^2} = \frac{1}{4m}\left(\sum_n n^2\omega^2 A_n^2\right)\left(\frac{\partial V}{\partial q}\right)^2$$

Recall $\dot{A}(t) = -\sum_n n\omega A_n \sin(n\omega t)$

$$\sum_n n^2\omega^2 A_n^2 = \langle |\dot{A}|^2 \rangle$$

$$\Rightarrow m\langle \ddot{q} \rangle = -\frac{d}{dq}\left[V_0 + \frac{1}{4m}\langle \left|\frac{dA}{dt}\right|^2 \rangle \left(\frac{dV}{dq}\right)^2 \right]$$

$\Rightarrow K = H_0(p,q) + \frac{1}{4m}\langle \left|\frac{dA}{dt}\right|^2 \rangle \left(\frac{dV}{dq}\right)^2$ is the eff. Hamiltonian

7. a) Energy of one particle moving in one direction:

$E_{\alpha,i} = \frac{1}{2} m_\alpha v_{\alpha,i}^2$ $\quad \alpha = 1 \dots N$ $\quad$ N-particle

$i = x, y, z$

$E_\alpha = E_{\alpha,x} + E_{\alpha,y} + E_{\alpha,z}$ (energy in 3D)

Total energy for the gas $= E = \sum_\alpha E_\alpha \simeq 3 \sum_\alpha E_{\alpha,x}$

To find pressure:

Force on one side wall (e.g $x=L$) $= \Delta p \,/$ box-crossing time

$$= 2m_\alpha v_{\alpha,x} \Big/ \left(\frac{2L}{v_{\alpha,x}}\right) = \frac{m_\alpha v_{\alpha,x}^2}{L} = \frac{2E_{\alpha,x}}{L}$$

$$\text{Pressure} = \frac{\text{total force}}{\text{area}} = \sum_\alpha \frac{2E_{\alpha,x}}{AL} = \frac{2}{3}\frac{E}{V}$$

$V = AL = L^3$

b) Adiabatic invariant: $I_{\alpha,i} = \oint p_{\alpha,i}\, dq_{\alpha,i}$

$$= \sqrt{2m E_{\alpha,i}} \cdot 2L \simeq \text{const}$$

$\Rightarrow E_x L^2 \propto$ const.

$\Rightarrow \frac{dE_x}{E_x} = -2\frac{dL}{L} \Rightarrow dE_x = -\frac{2E_x}{L} dL$ (only in x direction)

7 b) Cont'd

In thermal dynamics:
$$dE = dW + dQ$$
$$= -p\,dV$$
$$= -\frac{2E}{3V}\,dV$$
$$= -\frac{2E}{L}\,dL$$

$V = L^3$

$dV = 3L^2 dL$

In terms of E_x (KE in x-direction)

$p = \frac{2}{3}\frac{E}{V} = \frac{2E_x}{V}$, Expansion of walls $\Rightarrow dV = \cancel{L^2 d}\, A\,dL$
$= L^2 dL$

$\therefore dE_x = dW + dQ = -p\,dV = -\frac{2E_x}{V}\,dV = -\frac{2E_x}{L}\,dL$ //

8 a) Euler Equations:

$$I_1 \dot{\Omega}_1 = \Omega_2 \Omega_3 (I_2 - I_3)$$
$$I_2 \dot{\Omega}_2 = \Omega_3 \Omega_1 (I_3 - I_1)$$
$$I_3 \dot{\Omega}_3 = \Omega_1 \Omega_2 (I_1 - I_2)$$

b) $\Omega_2 \simeq \Omega_0 \Rightarrow \begin{cases} \dot{\Omega}_1 = \dfrac{I_2 - I_3}{I_1}\,\Omega_0 \Omega_3 \\ \dot{\Omega}_3 = \dfrac{I_1 - I_3}{I_3}\,\Omega_0 \Omega_1 \end{cases}$

$\Rightarrow \ddot{\Omega}_1 = -\lambda^2 \Omega_1$ where $\lambda^2 = -\frac{(I_2 - I_3)(I_1 - I_2)}{I_1 I_3}\,\Omega_0^2$

$\Omega_1 \propto e^{i\lambda t}$

Since $I_1 < I_2 < I_3 \Rightarrow \lambda^2 < 0 \Rightarrow$ unstable

Similarly, for $\Omega_1 \simeq \Omega_0 \Rightarrow \lambda^2 = -\frac{(I_1 - I_2)(I_3 - I_1)}{I_2 I_3} > 0 \Rightarrow$ stable

for $\Omega_3 \simeq \Omega_0 \Rightarrow \lambda^2 = -\frac{(I_2 - I_3)(I_3 - I_1)}{I_1 I_2} > 0 \Rightarrow$ stable

//