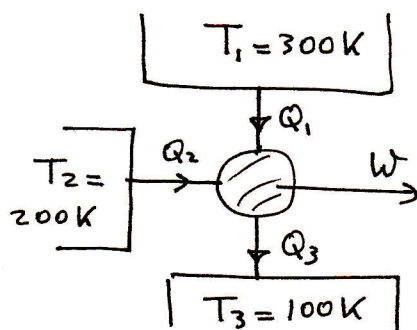


Problem 1

(a) Maximum efficiency is when no heat is absorbed from  $T_2$  reservoir,  $Q_2 = 0$

$$e = 1 - \frac{Q_3}{Q_1} = 1 - \frac{100}{300} = \boxed{0.667} \quad \left( e = \frac{W}{Q_1} = \frac{Q_1 - Q_3}{Q_1} = 1 - \frac{Q_3}{Q_1} = 1 - \frac{T_3}{T_1} \right)$$

(b) If  $Q_1 = 15\text{J}$ ,  $Q_2 = 20\text{J}$ . Maximum work is for reversible engine, where there is no change in the entropy of the universe:

$$\Delta S = 0 = -\frac{Q_1}{T_1} - \frac{Q_2}{T_2} + \frac{Q_3}{T_3} \Rightarrow Q_3 = T_3 \left( \frac{Q_1}{T_1} + \frac{Q_2}{T_2} \right) \Rightarrow$$

$$\Rightarrow Q_3 = T_3 \left( \frac{15}{300} + \frac{20}{200} \right) \frac{\text{J}}{\text{K}} = T_3 \frac{30 + 60}{600} \frac{\text{J}}{\text{K}} = 15\text{J} \Rightarrow$$

$$W = Q_1 + Q_2 - Q_3 = 15\text{J} + 20\text{J} - 15\text{J} \Rightarrow \boxed{W = 20\text{J}}$$

In this case, efficiency is  $e = \frac{W}{Q_1 + Q_2} = \frac{20\text{J}}{15\text{J} + 20\text{J}} = \boxed{0.571}$

(c) If  $W = 10\text{J}$ ,  $Q_1 = 15\text{J}$ ,  $Q_2 = 20\text{J}$ : then,

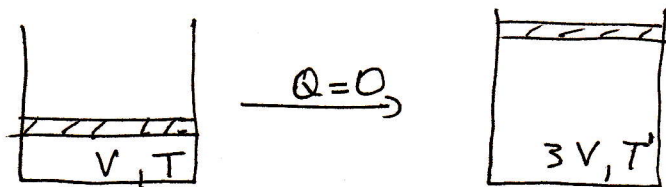
$$Q_3 = -W + Q_1 + Q_2 = -10\text{J} + 15\text{J} + 20\text{J} = 25\text{J}. \text{ For heat reservoirs,}$$

$$\Delta S_{\text{reserv}} = -\frac{Q_1}{T_1} - \frac{Q_2}{T_2} + \frac{Q_3}{T_3} = \frac{-15}{300} - \frac{20}{200} + \frac{25}{100} = \frac{-30 + 60 + 150}{600} = 0.3 \frac{\text{J}}{\text{K}}$$

$$\boxed{\Delta S_{\text{reserv}} = \Delta S_{\text{envir}} = 0.3 \frac{\text{J}}{\text{K}}} ; \quad \boxed{\Delta S_{\text{engine}} = 0 \text{ (since it's a cycle)}}$$

$$\Rightarrow \boxed{\Delta S_{\text{universe}} = 0.3 \frac{\text{J}}{\text{K}} \text{ in one cycle}}$$

## Problem 2



(a) Minimum temperature is reached when gas expands slowly doing work against external pressure = internal pressure. Since no heat is exchanged, the gas follows an adiabat

$$TV^{\gamma-1} = \text{const} \Rightarrow TV^{\gamma-1} = T'(3V)^{\gamma-1} \Rightarrow T' = \frac{1}{3^{\gamma-1}} T$$

$$\gamma = \frac{5}{3} \text{ for monatomic ideal gas, } \gamma-1 = \frac{2}{3} \Rightarrow \boxed{T' = 0.481T = 1443\text{K}}$$

(b) Maximum temperature is reached when it expands against zero external pressure, i.e. free expansion.  $Q=0$ ,  $W=0 \Rightarrow \Delta E_{\text{int}}=0$

$$\Rightarrow \Delta E_{\text{int}} = C_V(T' - T) = 0 \Rightarrow \boxed{T' = T = 300\text{K}}$$

(c) Work done by gas:  $\Delta E_{\text{int}} = Q - W = -W \Rightarrow \boxed{W = -\Delta E_{\text{int}}}$

$$\text{For case (b), } \Delta E_{\text{int}} = 0 \Rightarrow \boxed{W = 0}$$

$$\text{For case (a), } \Delta E_{\text{int}} = C_V(T' - T) = \frac{3}{2} R(1443\text{K} - 300\text{K}) = -1942\text{J}$$

$$\Rightarrow \boxed{\text{work done by gas} = 1942\text{J}}$$

Or, alternatively: calculate  $W = \int_V^{3V} P dV'$ , as follows:

$$PV^{\gamma} = c \text{ (constant)} \Rightarrow P = \frac{c}{V^{\gamma}}$$

$$W = \int_V^{3V} \frac{c dV'}{V'^{\gamma}} = \frac{c V'^{1-\gamma}}{1-\gamma} \Big|_V^{3V} = \frac{c}{\gamma-1} (V^{1-\gamma} - (3V)^{1-\gamma}) = \frac{c V^{1-\gamma}}{\gamma-1} (1 - \frac{1}{3^{\gamma-1}})$$

$$\text{Now } PV^{\gamma} = c, \text{ and } PV = RT \Rightarrow RTV^{\gamma-1} = c \Rightarrow$$

$$W = \frac{RTV^{\gamma-1} V^{1-\gamma}}{\gamma-1} (1 - \frac{1}{3^{\gamma-1}}) = \frac{RT}{\gamma-1} (1 - \frac{1}{3^{\gamma-1}}) = \boxed{1943\text{J}}$$

(d) For both processes  $Q=0 \Rightarrow$

$$\boxed{\Delta S_{\text{environment}} = 0}$$

$$(e) \Delta S_{\text{universe}} = \Delta S_{\text{gas}} + \Delta S_{\text{env}} = \Delta S_{\text{gas}}$$

$$\Delta S_{\text{gas}} = \int_{\text{rev}} \frac{\delta Q}{T}$$

where  $\delta Q$  is the heat absorbed by the gas in a reversible process going from initial to final state.

Process (a) is reversible  $\Rightarrow$  we can use that  $\delta Q=0 \Rightarrow$

$$\Delta S_{\text{gas}} = 0 \Rightarrow \boxed{\Delta S_{\text{universe}} = 0} \quad (a)$$

Process (b) is irreversible. A reversible process connecting the initial state with  $V, T$  and the final state with  $3V, T'=T$  is simply an isothermic reversible expansion where heat is absorbed and internal energy doesn't change

$$\Delta E_{\text{int}} = 0 = \delta Q - \delta W \quad \Rightarrow \quad \delta Q = \delta W = PdV$$

$$\Rightarrow \Delta S_{\text{gas}} = \int_V^{3V} \frac{PdV}{T} = \int_V^{3V} \frac{RT}{V \cdot T} dV = R \ln 3 = 9.13 \frac{\text{J}}{\text{K}}$$

$$\Rightarrow \boxed{\Delta S_{\text{univ}} = \Delta S_{\text{gas}} = 9.13 \frac{\text{J}}{\text{K}}} \quad (b)$$

Problem 3 :  $n = 3$  mol of He (ideal gas)

Gas is slowly compressed  $\Rightarrow$  work is being done on gas.

T stays same  $\Rightarrow$  Ent steps same  $\Rightarrow W = Q$ . Q goes into the ice-water-ice system.

$$W = \int P dV = \int_V^{V/2} \frac{nRT}{V} dV = -nRT \ln 2 = \text{work done by gas}$$

$\Rightarrow Q = nRT \ln 2$  melts a mass  $m$  of ice :

$$Q = nRT \ln 2 = m L_F \Rightarrow m = \frac{nRT \ln 2}{L_F} \Rightarrow$$

$$m = \frac{3 \text{ mol} \cdot 8.314 \frac{\text{J}}{\text{mol} \cdot \text{K}} \cdot \ln 2 \cdot 273 \text{ K}}{333,000 \text{ J/kg}} = 0.014 \text{ kg}$$

$\Rightarrow$   $m = 14 \text{ g}$  ice that melts

(b) Change in entropy of gas:

$$\Delta S = \int_V^{V/2} \frac{\delta Q}{T} = \int_V^{V/2} \frac{nRT}{VT} dV = -nR \ln 2 = -17.3 \frac{\text{J}}{\text{K}}$$

$\Rightarrow$  entropy of gas decreased by  $17.3 \frac{\text{J}}{\text{K}}$

entropy of ice-water system increased:  $\Delta S = \int \frac{\delta Q}{T} = +17.3 \frac{\text{J}}{\text{K}}$  (c)

(d) Entropy of the universe didn't change:

$$\Delta S_{\text{univ}} = -17.3 \frac{\text{J}}{\text{K}} + 17.3 \frac{\text{J}}{\text{K}} = 0$$

$\Rightarrow$  process is reversible, since  $\Delta S_{\text{univ}} = 0$