Physics 4B — Quiz 4 Solutions — Spring 2013

Problem 1

$V = 2\,m^3$

(a) $PV = nRT$, $T = 343\,K$, $P = 3.12\times10^4\,N/m^2$. $R = 8.314\,J/mol\,K$

$$\Rightarrow n = \frac{PV}{RT} = \frac{3.12\times10^4\times 2}{8.314\times343} = \boxed{21.88\ mol}$$

(b) Total water + steam $= 11\,mol + 21.88\,mol = \boxed{32.88\ mol.}$

If all the liquid evaporates, pressure would be ($T = 353K$ now)

$$P = \frac{nRT}{V} = \frac{32.88\times8.314\times353}{2}\frac{N}{m^2} = 4.83\times10^4\frac{N}{m^2}$$

Since P is larger than the saturated vapor pressure at $T = 80°C$ ($4.73\times10^4\,N/m^2$) $\Rightarrow$ $\boxed{\text{not all the liquid evaporates}}$

(c) The # of mol of steam to give the saturated vapor pressure is

$$n = \frac{PV}{RT} = \frac{4.73\times10^4\times2}{8.314\times353} = 32.33$$

Since we have 32.88 mol total $\Rightarrow$ $\boxed{\text{0.65 mol liquid remain}}$

(d) Most of the heat is latent heat of vaporization

# of mol that evaporated: $32.33 - 21.88 = 10.45\,mol$ ($= 11\,mol - 0.65\,mol$)

heat of vaporization $= 2.26\times10^6\frac{J}{kg} = 2.26\times10^6\frac{J}{kg}\cdot\frac{0.018\,kg}{mol} = 4.07\frac{J}{mol}\cdot10^4$

So $Q = 4.07\times10^4\times10.45\,J = \boxed{4.25\times10^5\,J}$

Note: the heat used in heating 32.88 mol from 70°C to 80°C is

$$32.88\,mol\times3000\frac{J}{kg\,°C}\times\frac{0.018\,kg}{mol}\times10°C = 1.78\times10^4\,J,$$

i.e. 4% of the heat of vaporization.

Problem 2

The change in momentum when an atom collides is

$$\Delta p_x = 2 m_{He} U_x$$

It takes time $\Delta t = \frac{2l}{U_x}$ for the atom to go back and forth, $l = 0.5\,m$. So

$$\frac{\Delta p_x}{\Delta t} = \frac{2 m_{He} U_x}{\frac{2l}{U_x}} = \frac{m_{He} U_x^2}{l}$$. Force due to N atoms is

$$F = \frac{N \Delta p_x}{\Delta t} = \frac{N m_{He} U_x^2}{l} = \frac{1.2\times10^{24} \times 4\times1.66\cdot10^{-27}\times 500^2}{0.5}\,N$$

$$\Rightarrow \boxed{F = 3984\,N}$$

(b) In equilibrium, speeds will be $U_x'^2 = U_y'^2 = U_z'^2$. By energy conservation, $U_x'^2 = \frac{1}{3}U_x^2$ (since $\frac{1}{2} m_{He} U_x^2 = \frac{1}{2} m_{He}(U_x'^2 + U_y'^2 + U_z'^2)$)

By equipartition, $\frac{1}{2} m_{He} U^2 = \frac{3}{2} k_B T = \frac{3}{2} m_{He} U_x'^2 \Rightarrow T = \frac{1}{3}\frac{m_{He} U_x^2}{k_B}$

$$\Rightarrow T = \frac{1}{3}\frac{N_A m_{He} U_x^2}{R} = \frac{1}{3}\cdot\frac{6.02\times10^{23}\times 4\times1.66\times10^{-27}\cdot 500^2}{8.314}\,K$$

$$\Rightarrow \boxed{T = 40.95\,K}$$

(c) The speed in the x direction has dropped,

i.e. $U_x^2 \longrightarrow U_x'^2 = \frac{1}{3}U_x^2$

so $\boxed{\text{the force dropped by a factor of } 3}$

Problem 3

(a) $\frac{P_2 V_2}{T_2} = \frac{P_1 V_1}{T_1}$, since $V_2 = V_1$, $P_2 = 2P_1 \Rightarrow$

$\Rightarrow \boxed{T_2 = 2T_1 = 600\text{ K}}$; $\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3}$, since $T_2 = T_3$, $P_3 = P_1 = \frac{P_2}{2} \Rightarrow$

$\Rightarrow P_2 V_2 = \frac{P_2}{2} V_3 \Rightarrow \boxed{V_3 = 2V_2 = 2V_1 = 12\text{ m}^3}$

(b) $Q = C_V \Delta T$, $C_V = \frac{3}{2} nR \Rightarrow$

$Q_{12} = \frac{3}{2} nR(T_2 - T_1) = \frac{3}{2} nRT_1 = \frac{3}{2} \cdot 5 \cdot 8.314 \times 300\text{ J} = 18,707\text{ J}$

$\boxed{Q_{12} = 18,707\text{ J} \text{ is absorbed in } 1 \to 2}$

(c) $\Delta E_{int} = Q_{23} - W_{23} = 0 \Rightarrow \boxed{Q_{23} = W_{23}}$

$W_{23} = \int_{V_2}^{V_3} P\,dV = nRT_2 \ln \frac{V_3}{V_2} = nRT_2 \ln 2 = nRT_1 \cdot 2\ln 2$

Note: $\frac{3}{2} nRT_1 = 18.707\text{ J} \Rightarrow \boxed{nRT_1 = 12,471\text{ J}} \Rightarrow \boxed{W_{23} = 17,288\text{ J}} \boxed{= Q_{23}}$

(d) $\Delta E_{int}^{31} = Q_{31} - W_{31} = C_V(T_1 - T_3) = -C_V T_1 = -\frac{3}{2} nRT_1$

$W_{31} = P_1(V_1 - V_3) = -P_1 V_1 = -nRT_1$

$\Rightarrow \boxed{Q_{31} = \Delta E_{int}^{31} + W_{31} = -\frac{5}{2} nRT_1 = -31,178\text{ J}}$

(e) Over entire cycle: $Q_{tot} = Q_{12} + Q_{23} + Q_{31} \Rightarrow$

$Q_{tot} = \frac{3}{2} nRT_1 + nRT_1 \cdot 2\ln 2 - \frac{5}{2} nRT_1 = nRT_1(2\ln 2 - 1)$

$\Rightarrow \boxed{Q_{tot} = nRT_1(2\ln 2 - 1) = 12,471\text{ J} \times 0.386 = 4817\text{ J}}$

$W_{tot} = W_{12} + W_{23} + W_{31} = 0 + nRT_1 \cdot 2\ln 2 - nRT_1 \Rightarrow$

$\Rightarrow \boxed{W_{tot} = nRT_1(2\ln 2 - 1) = 4817\text{ J}}$

$\boxed{Q_{tot} = W_{tot} \text{ since } \Delta E_{int} = 0 \text{ in a cycle, and } \Delta E_{int} = Q - W \text{ by 1st law}}$