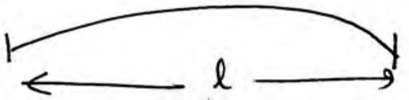
Physics 4B — Quiz 3 Solutions — Spring 2013

Problem 1



$l = \lambda/2$, $f = 400\ Hz$, $F_T = 100\ N$, $m = 0.5\ g = 0.5\times10^{-3}\ kg$

$$v = \sqrt{\frac{F_T}{\mu}} = \lambda f \quad , \quad \mu = \frac{m}{l}$$

(a) λ stays fixed, so the new f is

$$f(F_T = 150\ N) = f(F_T = 100\ N)\cdot\sqrt{\frac{150}{100}} = 1.225\times 400\ Hz \Rightarrow$$

$$\boxed{f(F_T = 150\ N) = 490\ Hz}$$

(b) $v = \lambda f = \sqrt{\frac{F_T}{m}\cdot l} \Rightarrow$ using $\lambda = 2l \Rightarrow 2lf = \sqrt{\frac{F_T l}{m}} \Rightarrow$

$$4l^2f^2 = \frac{F_T l}{m} \Rightarrow l = \frac{F_T}{4mf^2} = \frac{100\ N}{4\cdot 0.5\times10^{-3}\ kg\cdot 400^2}\cdot s^2 = 0.3125\ m \Rightarrow$$

$$\Rightarrow \boxed{l = 31.25\ cm}$$

(c) On a hot day, both the brass body and the steel string expand, but since $\alpha_{brass} > \alpha_{steel}$, body expands more and the string is under additional tension force $\Rightarrow$ larger $F_T \Rightarrow$ $\boxed{\text{larger frequency}}$

(it is true that l expands also, but the effect of this is negligible).
$\underbrace{\text{expands also}}_{(\Rightarrow \mu \text{ decreases})}$

(d) With $l_0 = 31.25\ cm$ we have $\Delta l_{brass} = \alpha_{brass}\, l_0\, \Delta T$, $\Delta l_{steel} = \alpha_{steel}\, l_0\, \Delta T$

$\Rightarrow (\Delta l)_{net} = (\alpha_{brass} - \alpha_{steel})\, l_0\, \Delta T$. The force needed for this is

$$\frac{F}{A} = E\cdot\frac{\Delta l}{l_0} \Rightarrow F = E\cdot\frac{\Delta l}{l_0}\cdot A = E\cdot A\cdot \Delta T\cdot(\alpha_{brass} - \alpha_{steel}) \Rightarrow$$

$$\Rightarrow F = 200\times10^9\ \frac{N}{m^2}\cdot 10^{-6}\ m^2 \times 20^\circ C\times 7\cdot10^{-6}\ (^\circ C)^{-1} \boxed{= 28\ N}$$

(e) The new frequency is

$$f = \frac{1}{\lambda}\sqrt{\frac{F_T}{\mu}} = \frac{1}{2l}\sqrt{\frac{F_T \cdot l}{m}} = \sqrt{\frac{F_T}{4lm}} \Rightarrow$$

$$f = \sqrt{\frac{128}{4 \cdot 0.3125 \cdot 0.5\times10^{-3}}}\ Hz = 452.5\ Hz$$

Or, simpler:

$$\boxed{f = \sqrt{\frac{128}{100}} \cdot 400\ Hz = 452.5\ Hz}$$

(f) Check that the change in length of the string is negligible.

$$\Delta l = \alpha_{brass} \cdot l_0\ \Delta T = 19\times10^{-6}\times0.3125\times20\ m = 0.12\ mm$$

So the percentual change is

$$\boxed{\frac{\Delta l}{l_0} = \frac{0.12\ mm}{312.5\ mm} = 0.04\ \%}$$

which is much smaller than the percentual change in the tension force

$$\boxed{\frac{\Delta F_T}{F_T} = \frac{28\ N}{100\ N} = 28\ \%}$$

Problem 2

Molly is moving towards Richard at speed v_M, emitting with frequency f_M. The frequency that R hears is given by the Doppler formula for moving source:

$$f_R = \frac{f_M}{1 - \frac{v_{source}}{v_{snd}}}$$

$f_M = 23{,}510\ Hz$

$f_R = 24{,}000\ Hz$

Since Richard hears no beats, f_R is the same as the frequency at which R emits, $f_R = 24{,}000\ Hz$. So

$$1 - \frac{v_{source}}{v_{snd}} = \frac{f_M}{f_R} \Rightarrow v_{source} = v_{snd}\left(1 - \frac{f_M}{f_R}\right) \Rightarrow$$

$$\Rightarrow v_{source} = 343\ \frac{m}{s}\left(1 - \frac{23{,}510}{24{,}000}\right) = 7\ m/s = v_M$$

$\boxed{\text{So M flies at speed } \cancel{\ } \text{ 7 m/s towards R}}$

(b) The frequency that M hears from R is (Doppler for moving observer)

$$f'_M = f_R\left(1 + \frac{v_{obs}}{v_{snd}}\right) = f_R\left(1 + \frac{7}{343}\right) = 24{,}490\ Hz$$

So $\boxed{\Delta f = f'_M - f_M = 24{,}490\ Hz - 23{,}510\ Hz = 980\ Hz}$

the time interval between maxima is

$$\boxed{\Delta t = \frac{1}{\Delta f} \approx 1\ msec}$$

Problem 3

$$PV = Nk_BT$$

(a) We have $T_A = 2T_B$, $N_A = 2N_B$

$$P_AV = N_Ak_BT_A$$
$$P_BV = N_Bk_BT_B$$
$$\Rightarrow \boxed{\frac{P_A}{P_B} = \frac{N_A}{N_B}\frac{T_A}{T_B} = 4} \text{ (a)}$$

(b) If $P_A = P_B$ and $T_A = 2T_B \Rightarrow$

$$P_AV = N_Ak_BT_A$$
$$P_AV = N_Bk_BT_B$$
$$\Rightarrow N_AT_A = N_BT_B \Rightarrow \boxed{\frac{N_A}{N_B} = \frac{T_B}{T_A} = \frac{1}{2}}$$

Since the He atom is 4 times heavier than the H atom

$$\boxed{\frac{m_A}{m_B} = \frac{1}{4}\frac{N_A}{N_B} = \frac{1}{8}} \text{ (b)}$$

(c) Sound velocity is $v = \sqrt{\frac{B}{\rho}}$, B is same for both

$$\frac{v_A}{v_B} = \sqrt{\frac{\rho_B}{\rho_A}} = \sqrt{\frac{m_B}{m_A}}$$ since $\rho = m/V$ and V is the same.

We have as in (b) that $m_B/m_A = 8 \Rightarrow$

(i) $\boxed{\frac{v_A}{v_B} = \sqrt{8} = 2.83}$

(ii) If both gases are hydrogen, $\frac{m_B}{m_A} = \frac{N_B}{N_A} = 2 \Rightarrow$

$$\boxed{\frac{v_A}{v_B} = \sqrt{2} = 1.41}$$