

**Problem 1** (10 pts + 2 pts extra credit)

A transverse traveling wave on a stretched cord with tension 3.2N is represented by  
 $D(x,t) = 0.1\sin(6x + 24t)$

where D is the displacement of the cord in direction perpendicular to x, in meters. x is in meters and t is in seconds.

- In which direction is this wave traveling, and at what speed (in m/s)?
- Give the wavelength (in m), the frequency (in Hz) and the maximum speed of the particles of the cord (in m/s).
- If the length of the cord is 3m, what is its mass (in kg)?
- Give the equation describing a standing wave in this cord that has the same frequency and wavelength as the traveling wave given above.
- What is the minimum length of this cord (in m) that you need in order to set up such a standing wave so that the ends are fixed? Can you also do it with a longer cord? Explain.

**Problem 2** (10 pts)

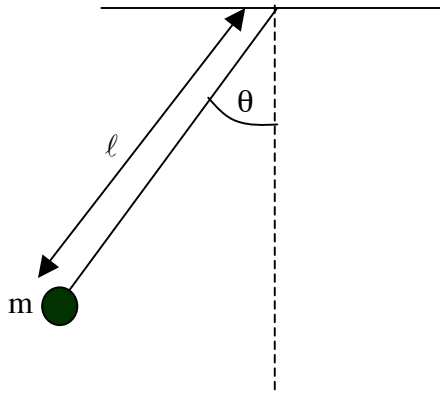
A physical system is described by a time-dependent variable  $w(t)$  that obeys the differential equation

$$2 \frac{dw(t)}{dt} = -w(t) - \frac{d^2w(t)}{dt^2}$$

Assume  $w(t)$  and  $t$  are dimensionless.

- Show by direct substitution that the function  $w(t)$  given by  
 $w(t) = te^{-\lambda t}$   
is a solution of that differential equation for one or more values of  $\lambda$ . Find those/that value/s of  $\lambda$ . Show your work (no credit otherwise, even if  $\lambda$ 's are right).
- Show by direct substitution that the function  $w(t)$  given by  
 $w(t) = e^{-\alpha t}$   
is also a solution of that differential equation for one or more values of  $\alpha$ . Find those/that value/s of  $\alpha$ .
- Calling  $w_1(t)$  and  $w_2(t)$  the solutions in (a) and (b), is  $w_1(t)+w_2(t)$  also a possible solution? Explain, without doing a calculation.
- Assume in the differential equation given above, the factor 2 on the left side is replaced by  $2a$ , with a  $a$  positive number that is either (i)  $a < 1$ , or (ii)  $a > 1$ . Show that the solution of the form (b) is still valid, but in one of the two cases (i) or (ii)  $\alpha$  is not a real number. Which of the two cases? And what does it mean physically?

**Problem 3** (10 pts + 3 pts extra credit)



The simple pendulum shown in the figure has mass  $m=100\text{kg}$  and cord length  $l = 61.25$  centimeters, It starts at rest from an initial angle  $\theta_0=0.2$  rads. Gravity acceleration is  $g=9.8\text{m/s}^2$ . Assume there is no damping.

- How high is the initial position relative to the lowest possible position? Give your answer in meters. Using this, find the initial potential energy of this mass, in Joules, relative to the potential energy when it is at its lowest position, i.e. when the pendulum is vertical (by "relative" I mean the difference, not the ratio). Your answer for the potential energy should be accurate to two decimal places.
- Give an equation for the time evolution of the angle  $\theta(t)$  satisfying the initial condition  $\theta(t=0)=\theta_0$ . Using that equation, find how long (in s) it takes for the mass to first reach its lowest position.
- Find how fast the mass is moving when it is at its lowest position, in m/s, using the equation for  $\theta(t)$  given in (b).
- Find the kinetic energy when the mass is at the lowest position, in Joules, using the answer in (c), accurate to two decimal places.
- Compare the answers for the energies found in (a) and (d). They should not be identical. Explain why. State which is the better answer, and explain why.