Physics 4B — Quiz 1 Solutions — Spring 2013

Problem 1

By Archimedes principle, the buoyant force = weight of fluid displaced. Since the body floats in water, the buoyant force (F_B) in water = weight of body. So if V_{body}, m_{body} are the volume and mass

$$F_B = \rho_{H_2O} \cdot 0.70 \cdot V_{body} \cdot \cancel{g} = m_{body} \cdot \cancel{g}$$

$$\Rightarrow V_{body} = \frac{m_{body}}{0.70 \cdot \rho_{H_2O}} = \frac{2.80\ g \cdot cm^3}{0.70 \cdot 1 g} = 4\ cm^3$$

So (a) $\boxed{V_{body} = 4\ cm^3}$

(b) In the unknown liquid, the body's weight − the buoyant force = apparent weight. So,

$$m_{body} \cdot g - F_B = m_{apparent} \cdot g \Rightarrow F_B = (m_{body} - m_{apparent})\, g$$

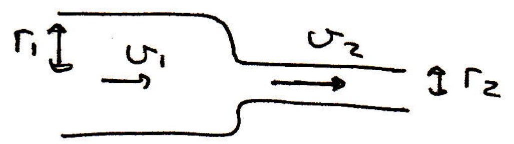
$$F_B = \rho_{liquid} \cdot V_{body} \cdot g = (m_{body} - m_{apparent})\, g$$

$$\Rightarrow \rho_{liquid} = \frac{m_{body} - m_{apparent}}{V_{body}} = \frac{2.8 g - 2 g}{4\ cm^3} = \frac{0.8\ g}{4\ cm^3}$$

$$\Rightarrow \boxed{\rho_{liquid} = 0.2\ g/cm^3}$$

Problem 2

$r_1 = 5\,cm$, $r_2 = 3\,cm$



(a) flow is faster in narrow part of the pipe.

Faster flow $\Rightarrow$ lower pressure $\Rightarrow$ $\boxed{\text{left side has higher pressure}}$

(b) From Bernoulli's equation:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow v_2^2 - v_1^2 = \frac{2(P_1 - P_2)}{\rho} = \frac{2\Delta P}{\rho}$$

From continuity equation, $v_1 A_1 = v_2 A_2$

$$A_1 = \pi r_1^2,\ A_2 = \pi r_2^2 \Rightarrow v_1 = \frac{A_2}{A_1} v_2 = \frac{r_2^2}{r_1^2} v_2 \Rightarrow$$

$$v_2^2\left(1 - \left(\frac{r_2}{r_1}\right)^4\right) = \frac{2\Delta P}{\rho} \Rightarrow \boxed{v_2 = \sqrt{\frac{2\Delta P}{\rho\left(1 - \left(\frac{r_2}{r_1}\right)^4\right)}}}$$

(c) $\Delta P = 6000\,Pa$, $\rho = 1000\,kg/m^3\ (= 1\,g/cm^3) \Rightarrow$

$$v_2 = \sqrt{\frac{2 \cdot 6000}{1000\left(1 - \frac{81}{625}\right)}}\ \frac{m}{s} = 3.71\,\frac{m}{s}\ ;\ v_1 = \frac{9}{25} v_2 = 1.34\,\frac{m}{s}$$

$$\boxed{v_2 = 3.71\,\frac{m}{s},\ v_1 = 1.34\,\frac{m}{s}}$$

Volume rate of flow $= A_1 v_1 = A_2 v_2 = \pi \times 0.05^2 \times 1.34\,\frac{m}{s} = 0.105\,m^3/s$

$= \pi \times 0.03^2 \times 3.71\,\frac{m}{s} = 0.105\,m^3/s$

$$\boxed{\text{Volume rate of flow} = 0.105\,\frac{m^3}{s}}$$

Problem 3

The pressure of the Hg column in the tube is $\rho_{Hg} \cdot g \cdot h$, it equals the pressure at the top of the container, which is atmospheric pressure P_0 + pressure due to the weights. With $A = 100\,cm^2 = 10^{-2}\,m^2$ the area of the container, $m_w = 50\,kg$ the mass of each weight:

$$P_0 + \frac{2 m_w \cdot g}{A} = \rho_{Hg} \cdot g \cdot h \Rightarrow h = \frac{P_0 + 2 m_w \cdot g / A}{\rho_{Hg} \cdot g} \Rightarrow$$

$$h = \frac{1.013 \times 10^5 + 2 \times 50 \times 10^2 \times 9.81}{13.6 \times 1000 \times 9.81}\,m = 1.49\,m$$

$$\boxed{h = 149\,cm}$$

(b) with one of the weights removed, replace $2 \cdot m_w$ by m_w above $\Rightarrow$

$$\boxed{h = 113\,cm}$$

The height change was $149 - 113 = \boxed{36\,cm}$. The volume of Hg there, given the cross-sectional area of tube of $1\,cm^2$, is $36\,cm^3$. Since the cross section of the container is $100\,cm^2$, piston goes up by

$$\boxed{\Delta h = \frac{36\,cm^3}{100\,cm^2} = 0.36\,cm}$$

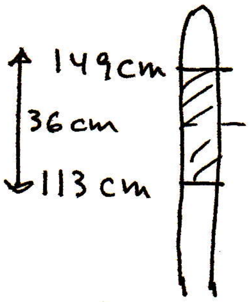
(d) Work is both to lift the remaining 50 kg mass and against P_0:

$$W = m_w g \Delta h + P_0 . A . \Delta h =$$

$$= (50 \times 9.81 + 1.013 \times 10^5 . 10^{-2}) . 0.36 \times 10^{-2}\,J = 5.41\,J$$

$$\boxed{W = 5.41\,J}$$

(e) The energy came from the potential energy of the Hg in the glass tube that dropped. As an estimate,



--- 131 cm = average height = $\bar{h}$

The mass is $m = \rho_{Hg} \times 1\,cm^2 \times 36\,cm = 490\,g = 0.49\,kg$

The potential energy of 0.49 kg at height $\bar{h} = 131$ cm is

$$U_{pot} = m g \bar{h} = 0.49 \times 9.81 \times 1.31\,J = 6.30\,J$$

$\boxed{U_{pot} = 6.30\,J}$ which is larger than 5.41 J, as needed.

This energy, 6.30 J, is the work required to push up a mass of 75 kg a distance Δh, including the effect of P_0:

$$\boxed{6.30\,J = P_0 \cdot A \cdot \Delta h + \frac{3}{2} m_w \cdot g \cdot \Delta h}$$

So where does the difference $6.30\,J - 5.41\,J = 0.89\,J$ go?